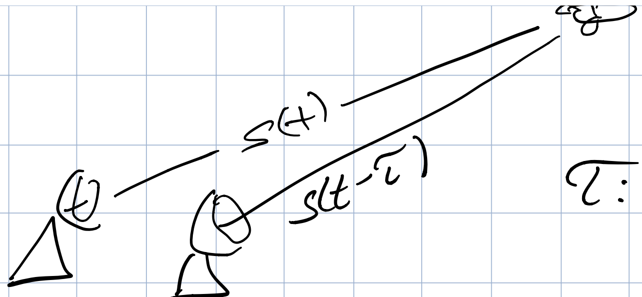



$\tau$: parameter $\quad \tau : \mathbb{R}^+$

$\boxed{H(f)} \rightarrow \underline{X}(\tau)$ statistic

$\hookrightarrow r(t) = s(t-\tau) + n_w(t)$ white noise

filter & sample $\hookrightarrow X_i = r(iT_s) = s(iT_s - \tau) + n_i$

sampling rate $\quad 1 \le i \le n$

Block processing

statistic:

$$\underline{X} = [X_1, X_2, X_3, \dots\dots X_n]$$

Assume $n_i \sim \mathcal{N}(0, \sigma^2)$

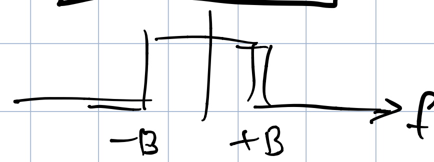
We need

$$\frac{d \ln(f_{\underline{X}}(\underline{x}; \tau))}{d\tau} = ?$$

$$f_{\underline{X}}(\underline{x}; \tau) \cong \prod_{i=1}^{n} e^{\frac{-|x_i - s_i(\tau)|^2}{2\sigma^2}}$$

$T_s = \frac{1}{2B} \cdot i$

$n_W(t) \longrightarrow \boxed{B} \longrightarrow n_i = n(iT_s)$




$n_i, n_j$ are indep.

$$f_{\underline{x}}(\underline{x};\tau) \cong e^{-\sum_{i=1}^{n} \frac{|x_i - s_i(\tau)|^2}{2\sigma^2}}$$

$$\ln(f_{\underline{x}}(\underline{x};\tau)) = -\sum_{i=1}^{n} \frac{|x_i - s_i(\tau)|^2}{2\sigma^2} + c$$

$$\frac{d \ln f_{\underline{x}}(\underline{x};\tau)}{d\tau} = +\frac{1}{\cancel{2}\sigma^2} \sum_{i=1}^{n} (x_i - s_i(\tau)) \cdot \frac{ds_i(\tau)}{d\tau}$$

$E[x_i] = s_i(\tau)$

$x_i = s_i(\tau) + n_i$

$$\frac{d^2 \ln(f_{\underline{x}}(\underline{x};\tau))}{d\tau^2} = \frac{1}{\sigma^2} \sum_{i=1}^{n} (x_i - s_i(\tau)) \frac{d^2 s_i(\tau)}{d\tau^2} - \left(\frac{ds_i(\tau)}{d\tau}\right)^2$$

$$I(\tau) = -\int_{\underline{x}} \frac{d^2 \ln(f_{\underline{x}}(\underline{x};\tau))}{d\tau^2} \cdot f_{\underline{x}}(\underline{x};\tau)$$

$$I(\tau) = \frac{1}{\sigma^2} \sum^{n} \left(\frac{ds_i(\tau)}{d\tau}\right)^2 \stackrel{?}{=} I(\hat{\tau} - \tau)$$

$$\sigma^2 \sum_{i=1} ( \ldots )$$

CRLB: any estimator

$$E\left((\hat{\tau}-\tau)^2\right) \geq \frac{1}{I(\tau)}$$

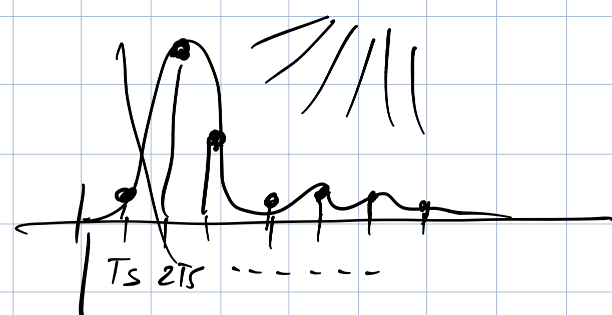
does such estimator exist?

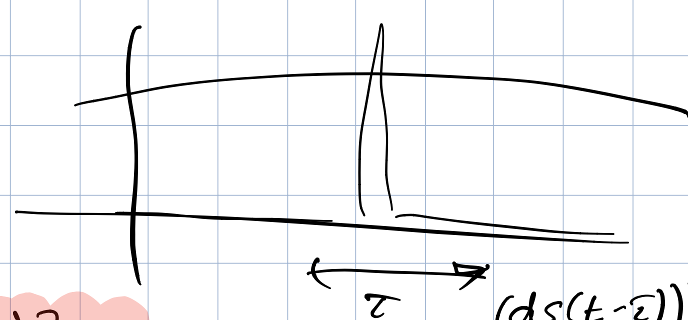
**not known**

look at:

$$\sum_{i=1}^{n}\left(\frac{ds_i(\tau)}{d\tau}\right)^2 = \qquad s_i(\tau) = s(iT_s-\tau)$$

$$\sum_{i=1}^{n}\left(\frac{ds(iT_s-\tau)}{d\tau}\right)^2 =$$




$t$

$$2B\sum_{i=1}^{n}\left(\frac{ds(iT_s-\tau)}{d\tau}\right)^2 \cdot \frac{1}{2B}$$

$$\left(\frac{ds(t-\tau)}{d\tau}\right)^2$$

$T_s = \frac{1}{2B}$

$$\approx \int^{T}\left(\frac{ds(t-\tau)}{d\tau}\right)^2 \cdot dt$$

$$\int_0 \ldots dt$$

$0 \quad T$

fourier $\frac{d}{dt} \longleftrightarrow 2\pi j f$

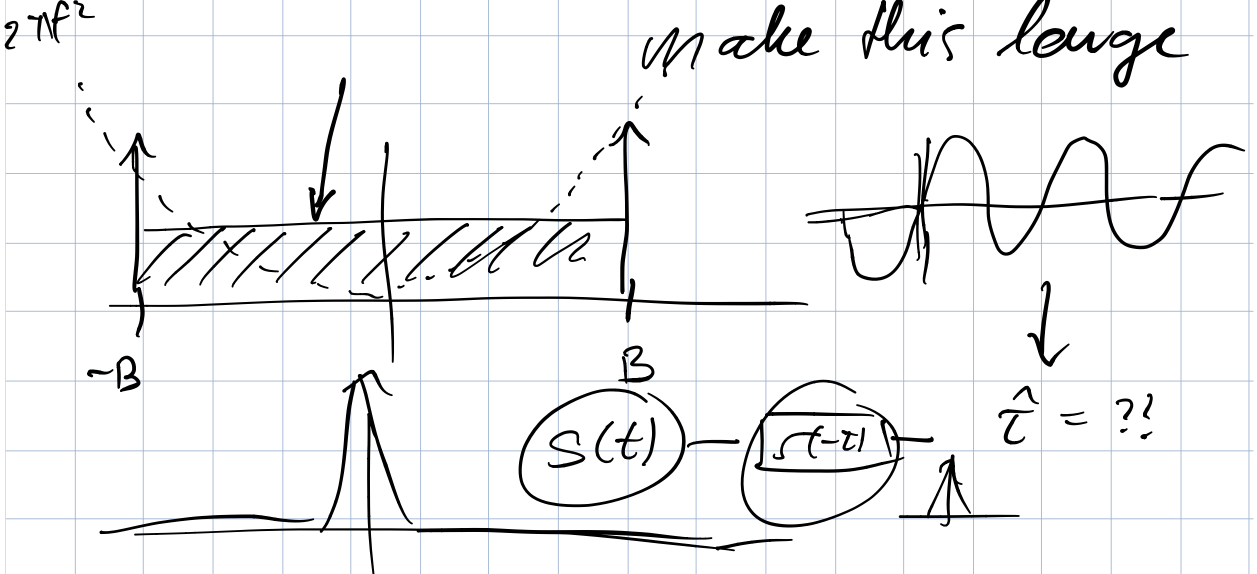
$$= \int_{-B}^{B} (2\pi f)^2 S^2(f)\, df$$

CRLB:

noise variance

$$E(|\hat{\tau} - \tau|^2) \geq \frac{\sigma^2}{\int_{-B}^{B} (2\pi f)^2 S^2(f)\, df}$$

make this large




• CRLB theoretical tool

bound — if close to bound ✓

- ideas estimator design
  identify optimal estimator ✓
- no design methodology ✗