Channel Model

$$\underline{x} = U \cdot \underline{h} + \underline{w}$$

noise: Gaussian $\sim N(0, \sigma^2)$

Bayesian approx:

$$\underline{\hat{h}} = U^T (U U^T + \sigma^2 I)^{-1} \cdot \underline{x}$$

$$p(\underline{x}; \underline{h}) = C \cdot e^{-\frac{\|\underline{x} - U\underline{h}\|^2}{2\sigma^2}}$$

Compute CRLB: $\ln p(\underline{x}; \underline{h}) =$

$$\ln C - \|\underline{x} - U\underline{h}\|^2 \cdot \frac{1}{2\sigma^2}$$

$$= \cancel{\ln C} - \frac{1}{2\sigma^2}\left(\cancel{\underline{x}^T \underline{x}} - \underline{x}^T U \underline{h} - \underline{h}^T U^T \cdot \underline{x} + \underline{h}^T U^T U \underline{h}\right)$$

$$\frac{\partial \ln p(\underline{x}; \underline{h})}{d\underline{h}} = \frac{1}{\cancel{2}\sigma^2}\left(\cancel{2} \cdot U^T \cdot \underline{x} - \cancel{2} U^T U \cdot \underline{h}\right)$$

$(U^T U)(U^T U)^{-1}$

$$= \frac{1}{\sigma^2}\left(U^T\underline{x} - U^TU\,\underline{h}\right)$$

$$= \frac{U^TU}{\sigma^2}\left((U^TU)\cdot U^T\cdot\underline{x} - \underline{h}\right)$$

(2) (1)

$$= I(h)\cdot\left(\hat{h} - \underline{h}\right)$$

read off: 						classical

(1) $\hat{h} = (U^TU)\,U^T\cdot\underline{x}$

(2) $E\left((\hat{\underline{h}}-\underline{h})(\hat{\underline{h}}-\underline{h})^T\right) = \frac{1}{I(h)} = \sigma^2(U^TU)^{-1}$

$$\hat{h} = \underbrace{U^T\underbrace{(U^TU+\sigma^2 I)^{-1}}_{n\times n}}_{p\times n}\cdot\underline{x}$$

← matrix inversion lemma

$$\hat{h} = (\underbrace{UU^T + \sigma^2 I_{p\times p}}_{p\times p})^{-1}\cdot \underbrace{U^T\underline{x}}_{p\times n}$$

$p \times p$

measurement noise

Bayesian: $\hat{h} = (UU^T + \sigma^2 I)^{-1} U^T \underline{x}$

Classical: $\hat{h} = (UU^T)^{-1} \cdot U^T \underline{x}$

MMSE <u>equalizer</u>

zero-<u>forcing</u> equ<u>alizer</u>

<u>Least-squares estimation:</u>

$$\min_{\underline{h}} |\underline{x} - U\underline{h}|^2 \longrightarrow \hat{h}$$

LS: $$\underline{\hat{h}} = \arg\min_{\underline{h}} \underbrace{|\underline{x} - U\underline{h}|^2}_{\text{quadratic form}}$$

def. $\equiv$ $$\hat{h} = \underbrace{(U^T U)^{-1}} \cdot U^T \cdot \underline{x} \longrightarrow \underline{\text{CRLB}}$$

ocal estimator

pseudo-inverse of U $\quad U^{-1} \neq$ not exist

$$\underbrace{\overset{p \times n}{U^{+}} \cdot \overset{n \times p}{U}} \cdot X$$

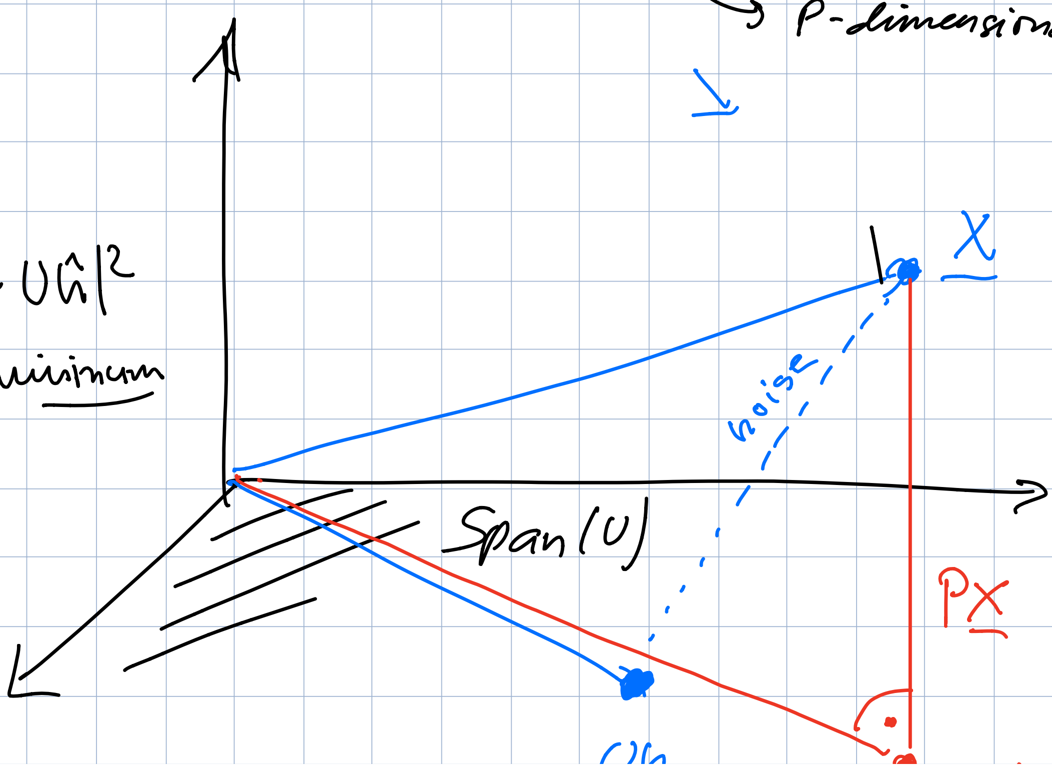
$$(U^T U)^{-1} \cdot U^T (Uh + \cancel{n})$$

$$\underbrace{(U^T U^{-1})(U^T U)} \cdot \underline{h} \longrightarrow \underline{h}$$

Geometry:

$$\underset{n\text{-dimensional}}{\underline{x}} = U\underset{p\text{-dimensional}}{\underline{h}} + n$$

$|X - U\hat{h}|^2$ is minimum



$U\underline{h}$ $\quad$ $U\hat{h}$

$$\hat{h} \longrightarrow U\hat{h} = \underbrace{U(U^TU)^{-1}U^T}_{P}\underline{x} \in \mathrm{Span}(U)$$

$$\underline{U\hat{h}} \perp (\underline{x} - \underline{U\hat{h}})$$

$$\underline{\hat{h}}^T U^T \cdot (\underline{x} - U\hat{h}) \overset{?}{=} 0$$

$$\hat{h}^T U^T \underline{x} = \underline{\hat{h}}^T U^T U \underline{\hat{h}}$$

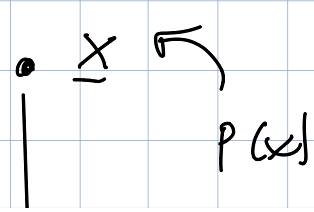
$$\underline{x}^T U (U^TU^{-1}) U^T x \qquad = \underline{x}^T U \underbrace{(U^TU)^{-1}(U^TU)}(U^TU)^{-1}U^T\underline{x}$$

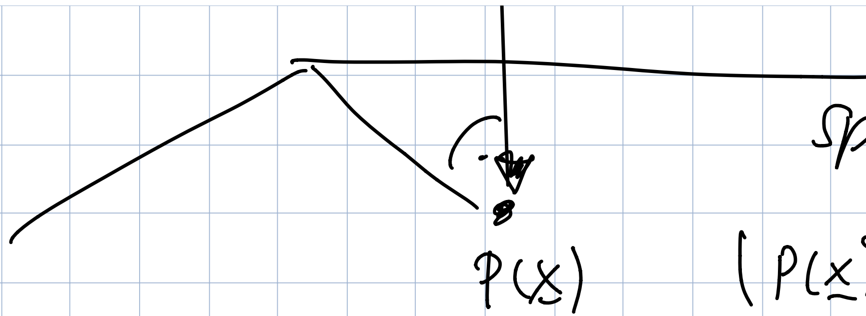
① $\quad \|$

$$\underline{x}^T U (U^TU)^{-1} U^T \underline{x}$$

②

$$P = U(U^TU)^{-1}U \quad : \text{projection matrix}$$





span(C)

$|P(\underline{x}) - \underline{x}|^2$ is minimized