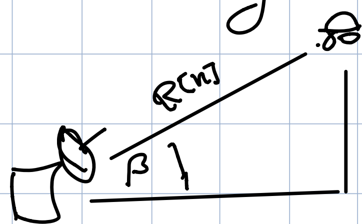
Vehicle Tracking



$$\underline{x}[n] = \begin{bmatrix} R[n] \\ \beta[n] \end{bmatrix} = \underbrace{\begin{bmatrix} \sqrt{v_x^2[n] + v_y^2[n]} \\ \tan^{-1}\left(\frac{v_y[n]}{v_x[n]}\right) \end{bmatrix}}_{\underline{h}(\underline{s}[n])} + \underline{w}[n]$$

treating non-linear dependencies

$$U\,\underline{s}[n] \longleftarrow \underline{h}(\underline{s}[n])$$

Modify the following steps:

1) Estimator Update

$A \cdot \hat{\underline{s}}[n-1]$ (reg. Kalman)

$$\hat{\underline{s}}[n] = \hat{\underline{s}}[n|n-1] + \underline{K}[n]\big(\underline{x}[n] - \underbrace{U\,\hat{\underline{s}}[n|n-1]}_{\hat{\underline{x}}[n]}\big)$$

Our case:

i) $\underline{s}[n] = A\,\underline{s}[n-1] + B\,\underline{u}[n]$

ii) $\underline{x}[n] = h(\underline{s}[n]) + \underline{w}[n]$

1) $$\hat{\underline{s}}[n] = \hat{\underline{s}}[n|n-1] + \underline{K}[n]\big(\underline{x}[n] - \underline{h}(\hat{\underline{s}}[n|n-1])\big)$$

2) Error Variance Update:

$$M[n] = E\left[(\underline{s}[n] - \hat{\underline{s}}[n])(\underline{s}[n] - \hat{\underline{s}}[n])^T\right]$$

$$= E\Big[\big(\underline{s}[n] - \hat{\underline{s}}[n|n-1] - \underline{k}[n]\big(\underline{x}[n] - \underline{h}(\hat{\underline{s}}[n|n-1])\big)\big)$$

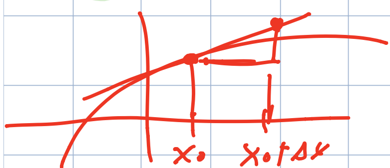
$$\times \big(\underline{s}[n] - \hat{\underline{s}}[n|n-1] - \underline{k}[n]\big(\underbrace{\underline{x}[n] - \underline{h}(\hat{\underline{s}}[n|n-1])}_{J_{\underline{h}}(\underline{s}[n]-\hat{\underline{s}}[n|n-1]) + \underline{w}[n]}\big)\big)^T$$

$$E[a+b] = E[a] + E[b]$$

Now we use

$$\underline{x}[n] - \underline{h}(\hat{\underline{s}}[n|n-1]) = \overbrace{\underline{h}(\underline{s}[n]) + \underline{w}[n]}^{\underline{x}[n]} - \underline{h}(\hat{\underline{s}}[n|n-1])$$

$$= \underline{h}(\underline{s}[n]) - \underline{h}(\underbrace{\hat{\underline{s}}[n|n-1]}_{\equiv x_0}) + \underline{w}[n]$$



$$f(x_0+\Delta x) - f(x_0) \approx \frac{d}{dx}f(x)\Big|_{x=x_0} \cdot \Delta x$$

$$= J_{\underline{h}}(\hat{\underline{s}}[n|n-1])(\underline{s}[n] - \hat{\underline{s}}[n|n-1]) + \underline{w}[n]$$

$$M[n] = M[n|n-1] - E[(\underline{s}[n]-\hat{\underline{s}}[n|n-1])(\underline{s}[n]-\hat{\underline{s}}[n|n-1])^T] J_h^T[n] \cdot \underline{k}^T[n]$$

$$- \ldots\ldots$$

$$- \ldots\ldots$$

(2)

$$M[n] = (I - \underline{k}[n] J_h[n]) M[n|n-1]$$

$$J_h(\hat{\underline{S}}[n|n-1]) = \begin{bmatrix} \frac{\partial h_1}{\partial s_1} & \cdots & \frac{\partial h_1}{\partial s_p} \\ \vdots & & \\ \frac{\partial h_M}{\partial s_1} & \cdots & \frac{\partial h_M}{\partial s_p} \end{bmatrix}$$

$$\underline{h}(\underline{s}) = \begin{bmatrix} h_1(s_1, \ldots s_p) \\ \vdots \\ h_M(s_1 \cdots s_p) \end{bmatrix}$$

$M \times p$

Example: Vehicle tracking

$$J_h[n] = \begin{bmatrix} \frac{\partial R[n]}{\partial v_x} & \frac{\partial R[n]}{\partial v_y} & \frac{\partial R[n]}{\partial V_x} & \frac{\partial R[n]}{\partial V_y} \\ \frac{\partial \beta[n]}{\partial v_x} & \frac{\partial \beta[n]}{\partial v_y} & \frac{\partial \beta[n]}{\partial V_x} & \frac{\partial \beta[n]}{\partial V_y} \end{bmatrix} \qquad \underline{S} = \begin{bmatrix} X \\ Y \\ V_x \\ V_y \end{bmatrix}$$

$p = 4$

$M = 2$

$$= \begin{bmatrix} \frac{V_x[n]}{R[n]} & \frac{V_y[n]}{R[n]} & 0 & 0 \\ \frac{V_x[n]}{R^2[n]} & -\frac{V_y[n]}{R^2[n]} & 0 & 0 \end{bmatrix} = J_h[n]$$

$R = \sqrt{x^2+y^2}$

$\frac{dR}{dx} = \frac{1}{\cancel{2}} \frac{\cancel{2}x}{\sqrt{x^2+y^2}}$

$\beta = \tan^{-1}\left(\frac{y}{x}\right)$

$\underline{X}[n]$

$V_x[n] = R[n]\cos[\beta[n]]$

$V_y[n] = R[n]\sin[\beta[n]]$

measure

$R[n], \beta[n]$

all parameters are now known

Importance Sampling

Monte Carlo (MC) Simulation

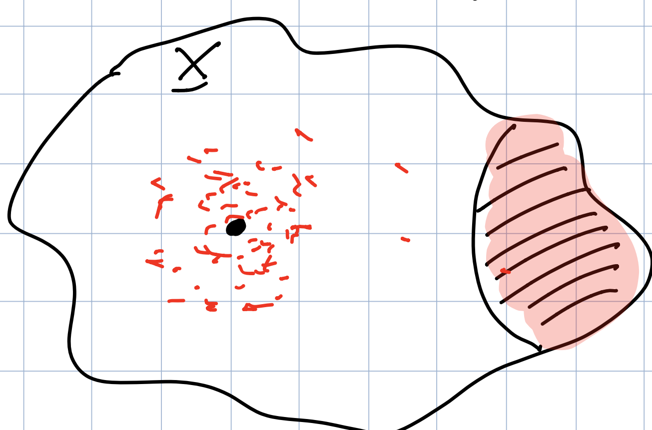
$$E[g(x)] = \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx$$

$X \sim f_X(x)$

Ex: error probabilities

$$P_e = \int_{\text{error region}} f_X(x)\, dx$$

$$= \int_{-\infty}^{\infty} I(x)\, f_X(x)\, dx$$



$$I(x) = \begin{cases} 1, & \text{if } x \text{ is in error region} \\ 0 & \text{otherwise} \end{cases}$$

$$P_e = E[I(X)] \qquad \underline{g(x) = I(x)}$$

$$E[g(X)] = \int_{-\infty} g(x) \cdot f_X(X)\, dx$$

- computationally difficult
- no closed form

$$\hat{g} \approx \frac{1}{N} \sum_{i=1}^{N} g(X_i)$$

X_i are generated $\sim f_X(x)$

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) \underbrace{\frac{f_X(x)}{p_X(x)}} \cdot p_X(x)\, dx$$

choose this as a PDF

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) \cdot w(x) \cdot p_X(x)\, dx$$

PDF

$$\hat{\ } \quad \frac{1}{\ } \sum^{N_s}$$

$$\hat{g} \approx \frac{1}{N} \sum_{i=1} g(X_i) \cdot w(X_i)$$

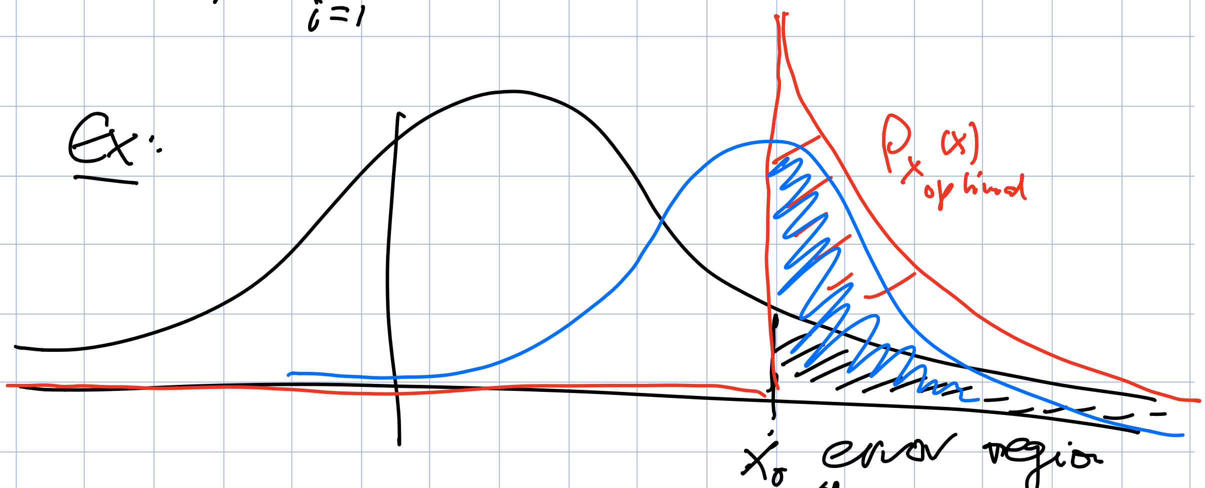
$X_i \sim$ chosen following $P_X(x)$

<u>importance sampling</u>

increase errors

$$\hat{P_e} = \frac{1}{N} \sum_{i=1}^{N_s} I(X_i) \cdot w(X_i)$$

<u>Ex:</u>



$$P_e = \int_{x_0}^{\infty} f_X(x)\,dx$$

$$P_X(x) = ?$$

1) $$\int^{\infty} P_{OPTIMA}(x) = 1$$

what

x_0

what we want?

$$f_{OPTIMA} = \frac{f_X(x)}{\int_{\text{error region}} f_X(x)\,dx}$$

$$\frac{f_X(x_i)}{p_X(x_i)} \qquad \sim p_X(x)$$

$$\hat{P}_e = \frac{1}{N_s}\sum_{i=1}^{N_s} w(X_i)\, I(X_i)$$

unbiased

$$E[\hat{P}_e] = P_e = \int_{\text{error region}} f_X(x)\,dx$$

$$\sigma_s^2 = E\left[(\hat{P}_e - P_e)^2\right]$$

$$= E\left[\hat{P}_e^2\right] + P_e^2$$

$$= \underbrace{E\left[\frac{1}{N_s^2}\sum_{i=1}^{N_s}\sum_{j=1}^{N_s} w(X_i)\,w(X_j)\,\overbrace{I(X_i)\,I(X_j)}^{I(X_i,X_j)}\right]} + P_e^2$$

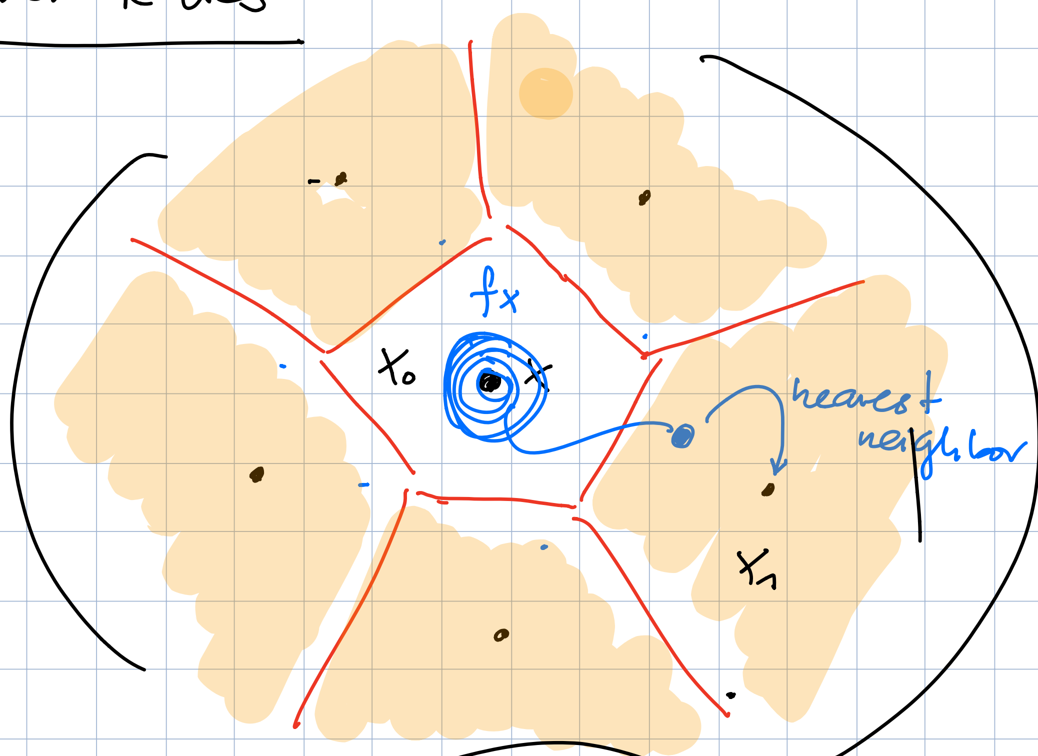
$$\hat{\sigma}_{IS}^2 \sim \frac{1}{N_s^2}\Big(\cancel{\sum_{i=1}^{N} w(X_i)\,I(X_i)}\ ??\Big)^2 + \left(\frac{1}{N_s}\sum_{i=1}^{N} w(x_i)\,I(X_i)\right)^2$$

$$\sigma^2_{MC}$$

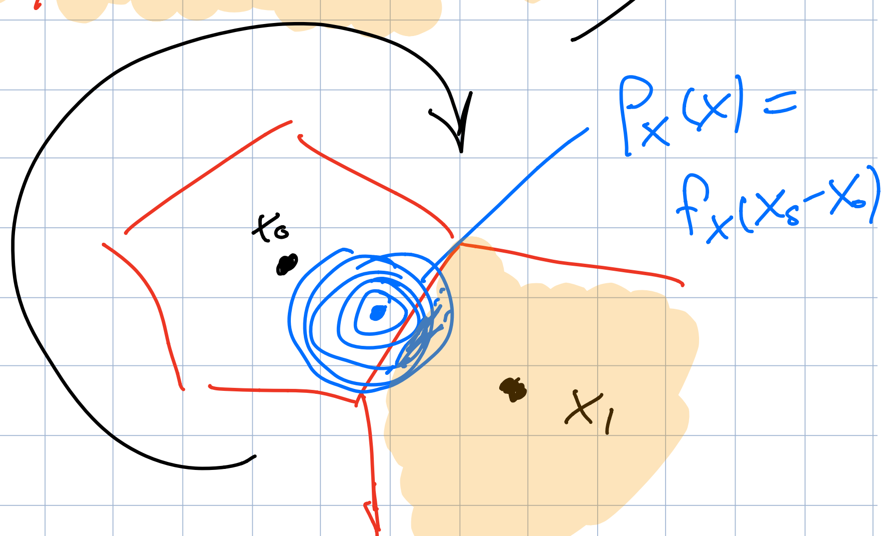
Gain of IS: $G_S = \frac{1}{\delta^2_{IS}} \gg 1$

$$\delta^2_{MC} = \frac{P_e(1-P_e)}{N_S}$$

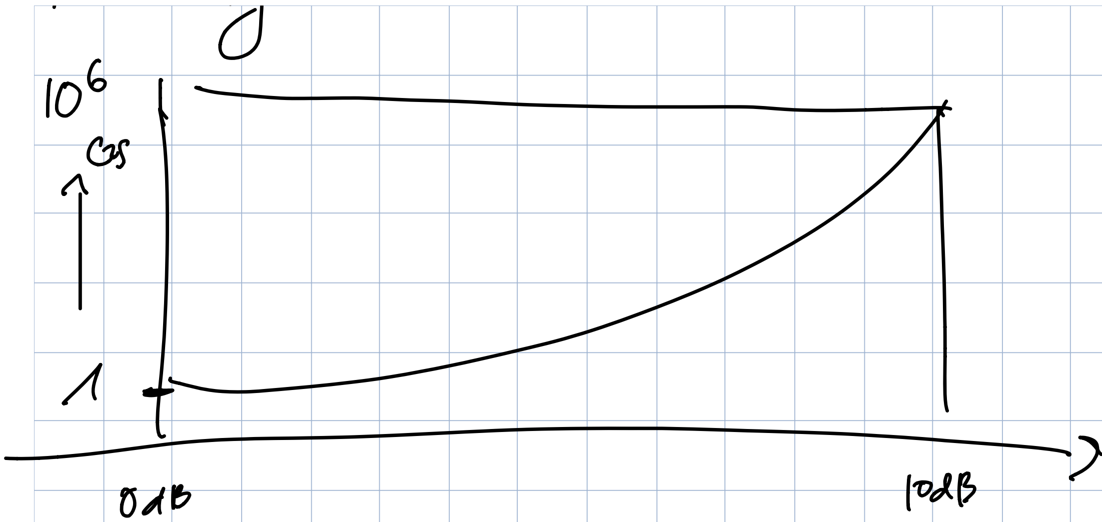
Error Rates

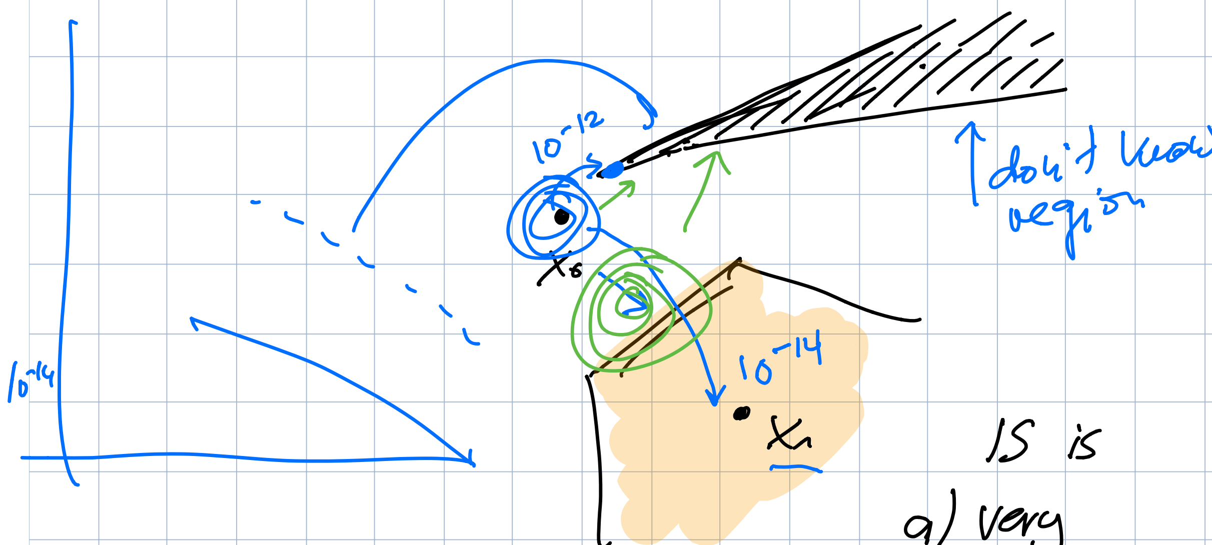


5(?) cases



[7,4]
Hamming





IS is

a) very powerful

b) very treacherous