General Bayesian Estimation

$$p(\underline{s}[n] \mid \underline{x}) \qquad \underline{x} = [x[0], \cdots x[n-1]]$$

$$= \int p(s[n], \underline{s}[n-1] \mid \underline{x})\, d s[n-1]$$

① $= \int p(s[n] \mid \underline{s}[n-1], \cancel{\underline{X}}) \cdot p(s[n-1] \mid \underline{X})\, ds_{[n-1]}$

Makov condition

Update from $n-1 \longrightarrow n$

$$P(\underline{s}[n] \mid x[n]\, \underline{X}) = \frac{p(s[n], x[n]\, \underline{X})}{p(x[n], \underline{X})}$$

$s[n] = A s[n-1] + Q w[n]$

$$= \frac{p(x[n] \mid \underline{s}[n], \underline{X})\, P(s[n], \underline{X})}{p(x[n] \mid \underline{X}) \cdot P(\underline{X})}$$

$$= \frac{p(x[n] \mid \underline{s}[n], \cancel{\underline{X}})\, p(\underline{s}[n] \mid \underline{X}) \cdot \cancel{p(\underline{X})}}{p(x[n] \mid \underline{X}) \cdot \cancel{P(\underline{X})}}$$

② $$= \frac{\overbrace{p(x[n] \mid \underline{s}[n])}^{\text{likelihood}} \cdot p(\underline{s}[n] \mid \underline{X})}{p(x[n] \mid \underline{X})}$$

all measurements up to n

measurements up to $n-1$

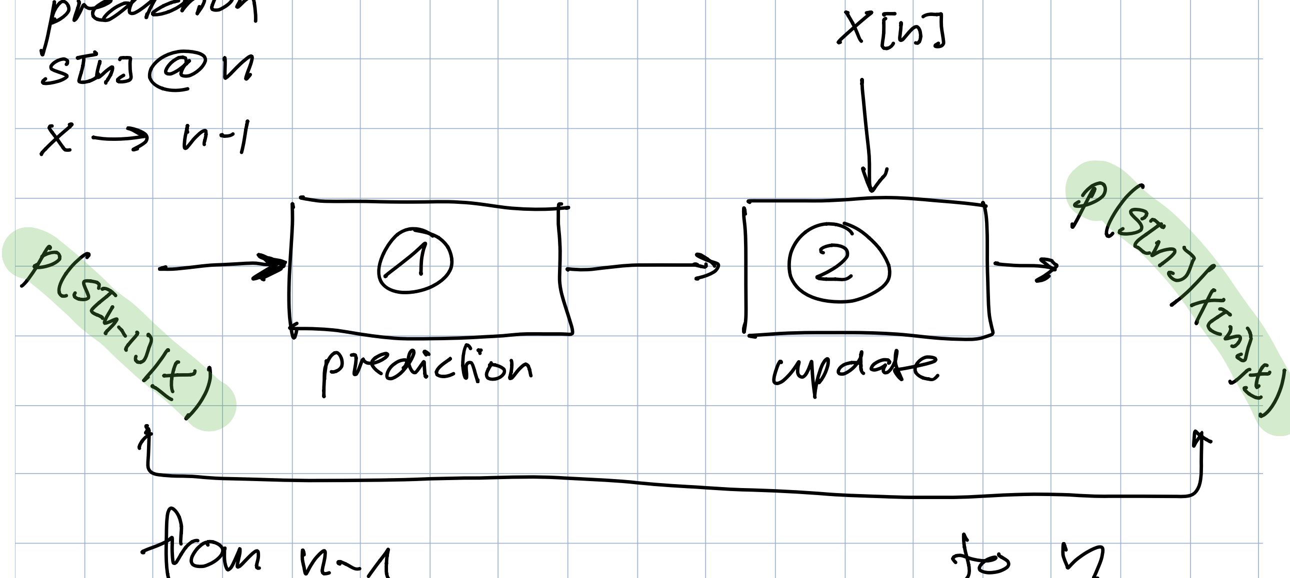
$p(x[n] \mid s[n])$

$$p(s[n] \mid x[n], \underline{X}) = \frac{p(x[n] \mid \underline{s}[n])}{p(x[n] \mid \underline{X})} \cdot p(\underline{s}[n] \mid \underline{X})$$

1

$$p(\underline{s}[n] \mid \underline{X}) = \int p(s[n] \mid s[n-1]) \cdot p(s[n-1] \mid \underline{X})\, ds[n-1]$$

prediction
s[n] @ n
X → n-1



Prediction Step :

$$\int p(\underline{s}[n] \mid \underline{s}[n-1])\, p(s[n-1] \mid \underline{X}) \cdot ds[n-1]$$

⇒ Particle is really a modified Monte-Carlo method

$f(s[n-1])$

$$= \int p(\underline{s}[n]|\underline{s}[n-1]) \underbrace{\frac{p(\underline{s}[n-1]|\underline{x})}{q(\underline{s}[n-1]|\underline{x})}}_{w[n]} \cdot \underbrace{q(\underline{s}[n-1]|\underline{x})}_{\text{PDF to use for drawing samples}} d\underline{s}[n-1]$$

System Model

$\underline{s}[n] = A\underline{s}[n-1] + \underline{w}[n]$

$= \underline{f}(\underline{s}[n-1]) + \underline{w}[n]$

$w[n-1]$

from the measure model

system

$$w[n] = w[n-1] \times \underbrace{\frac{p(x[n]|\underline{s}[n])\, p(s[n]|s[n-1])}{q(s[n]|s[n-1], \underline{x})}}_{\text{IS distribution}}$$

A factor $\frac{1}{p(\underline{x})}$ was moved out of integral

→ act as normalization

Algorithm steps are:

① $\sim \frac{1}{N_p} \sum_{i=1}^{N_p} f(\underline{s}_i[n-1]\, w_i[n]$

use N_p samples $\Rightarrow$ particles

$$s_i[n-1] \sim q(s[n] \mid s[n-1], \underline{x})$$

$$p(s[n] \mid \underline{x}) = \frac{1}{N_p} \sum_{i=1}^{N_p} f(s_i[n-1])\, w_i[n]$$

fix this at desired value

N_p samples enter here

② Update

measurement model

$$p(\underline{s}[n] \mid x[n], \underline{x}) = \frac{p(x[n] \mid s[n])}{\cancel{p(x[n] \mid \underline{x})}} \cdot p(\underline{s}[n] \mid \underline{x})$$

normalize

Summary

Bootstrap prior

$$q(s[n] \mid s[n-1], \underline{x})$$

$$= \underbrace{P(s[n] \mid s[n-1])}_{\text{system model}}$$

Step 1 sample

$$S_i[n] \sim p(S[n] \mid S[n-1])$$

Step 2 $w_i[n]$ calculate

Step 3 $$\tilde{w}_i[n] = \frac{w_i[n]}{\sum_{i=1}^{N_p} w_i[n]}$$

Step 4 $\hat{Pr}(S[n] \mid \underline{X}, X[n])$ = prediction & update

Start out sample at $n=0$

$$S_i[0] \sim Pr(S[0] \mid X[0]$$

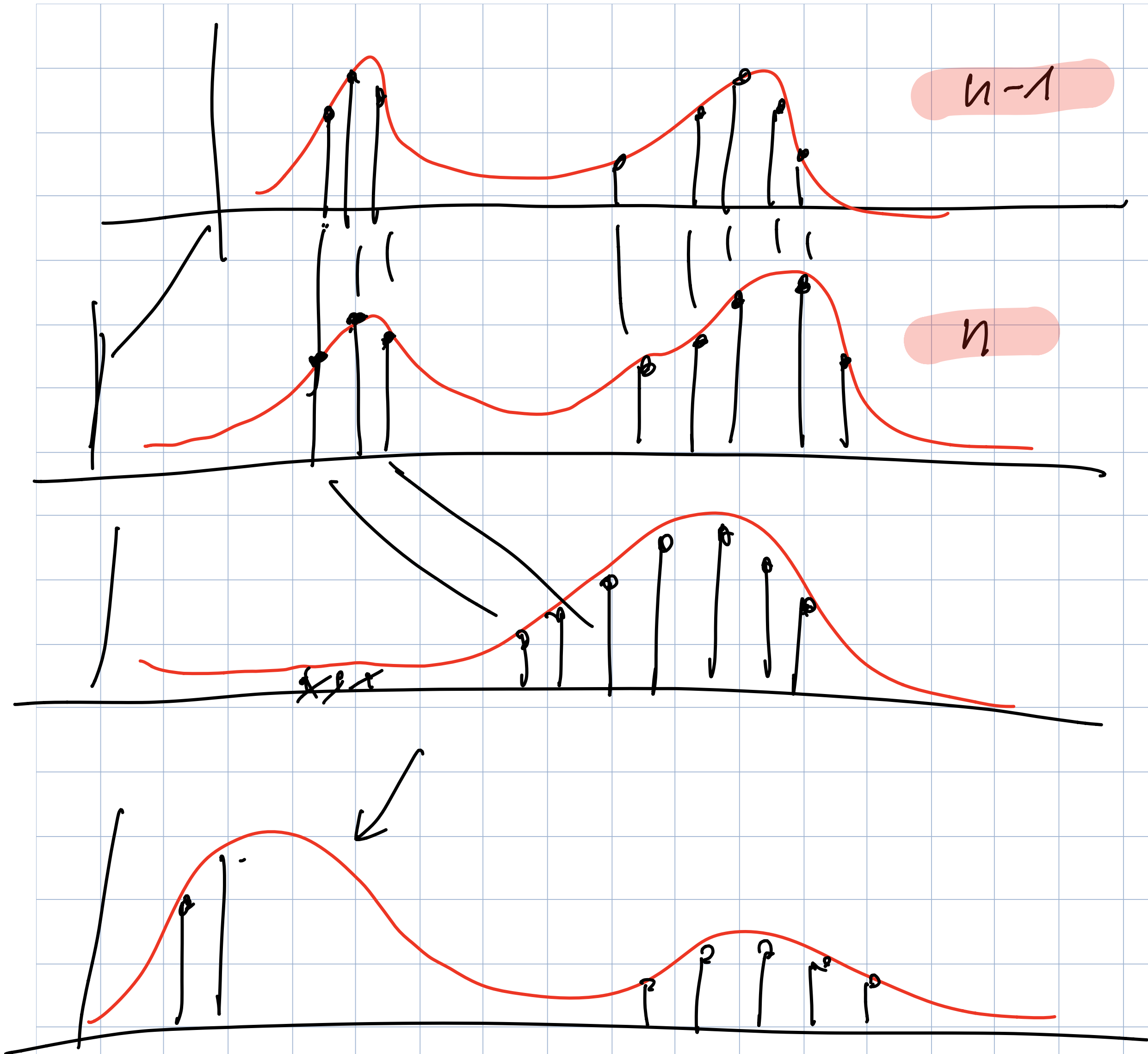
$$\downarrow$$

$$S_i[1]$$

$$\downarrow$$

$$\vdots$$

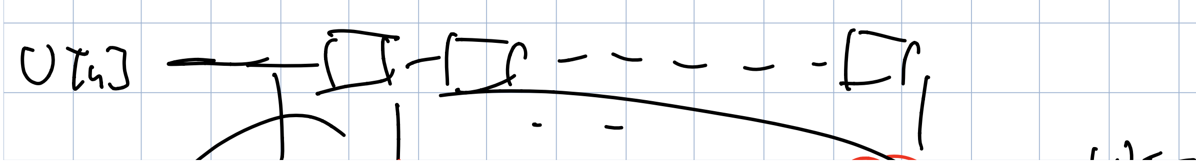
$$S_i[n] \sim Pr(S[n] \mid X[n], \underline{X})$$

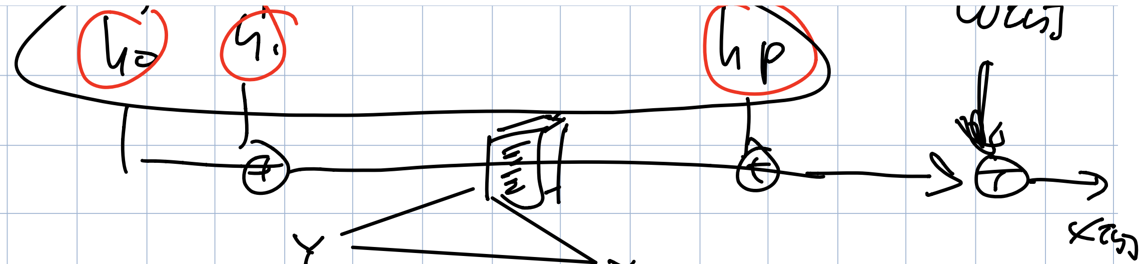


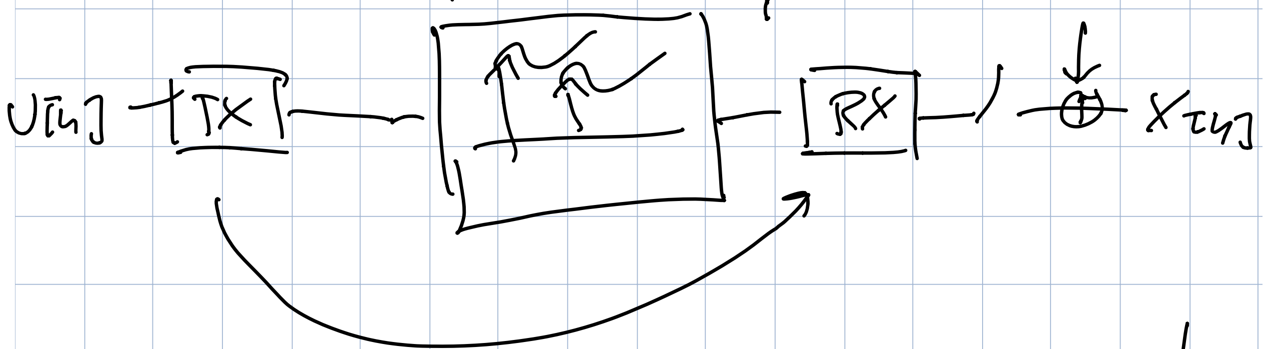
Channel estimator

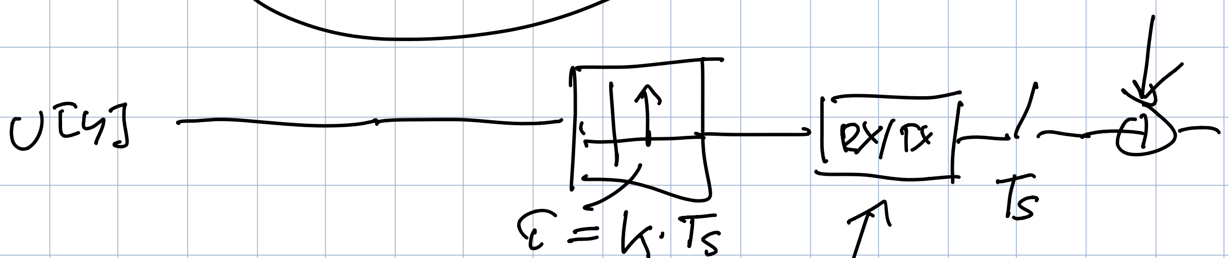
ext. Kalman
wiener filter

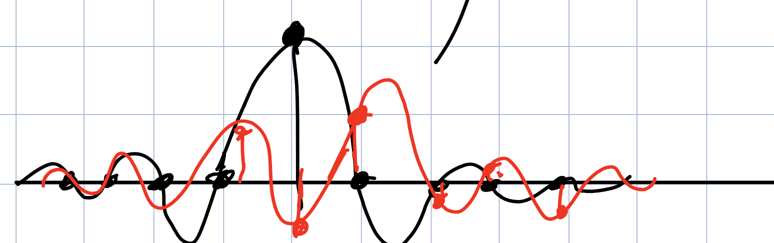
+ include correlation in the channel taps

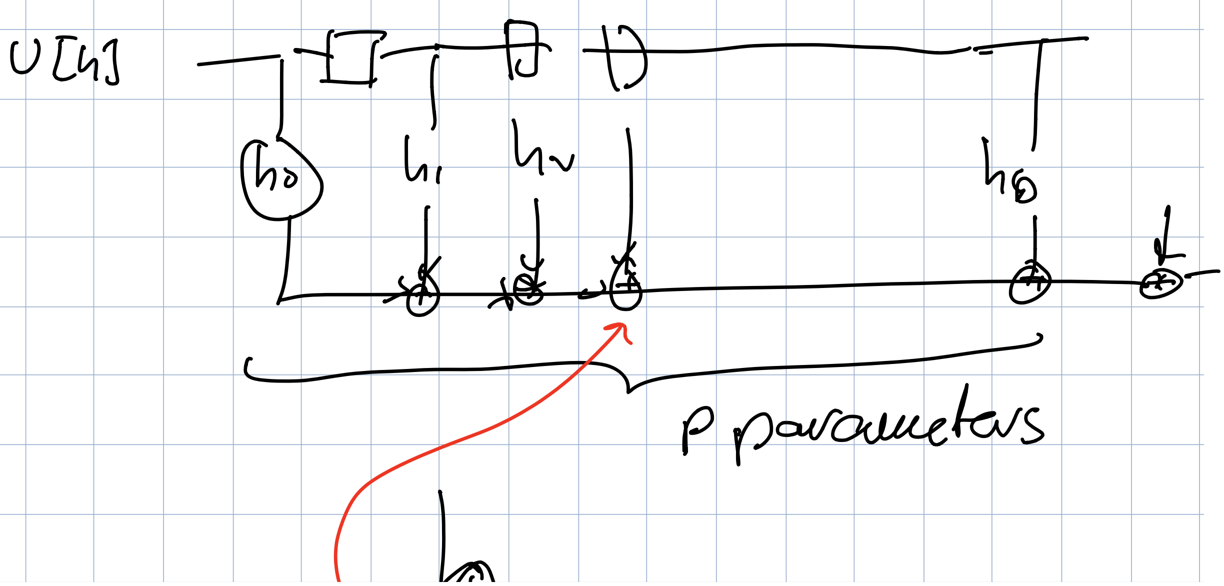


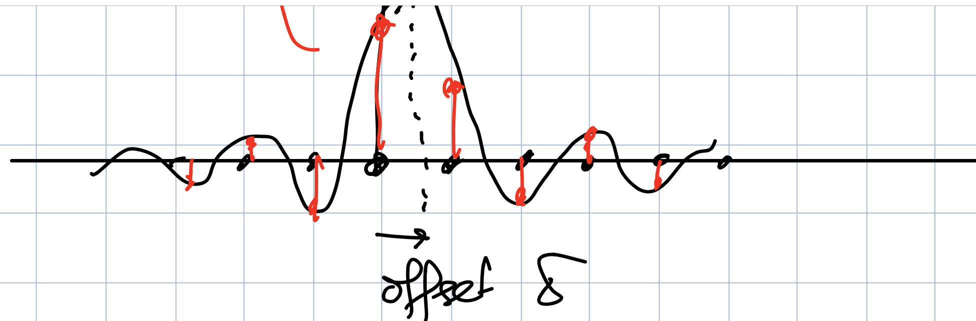

h0
h1
hp
w[n]
x[n]


U[n]
TX
RX
X[n]


U[n]
RX/TX
τ = k · Ts
Ts

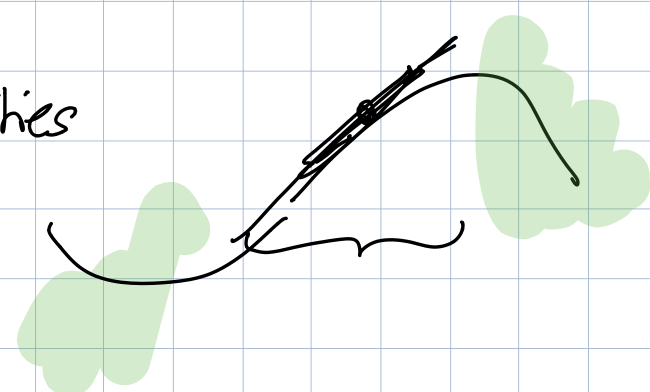


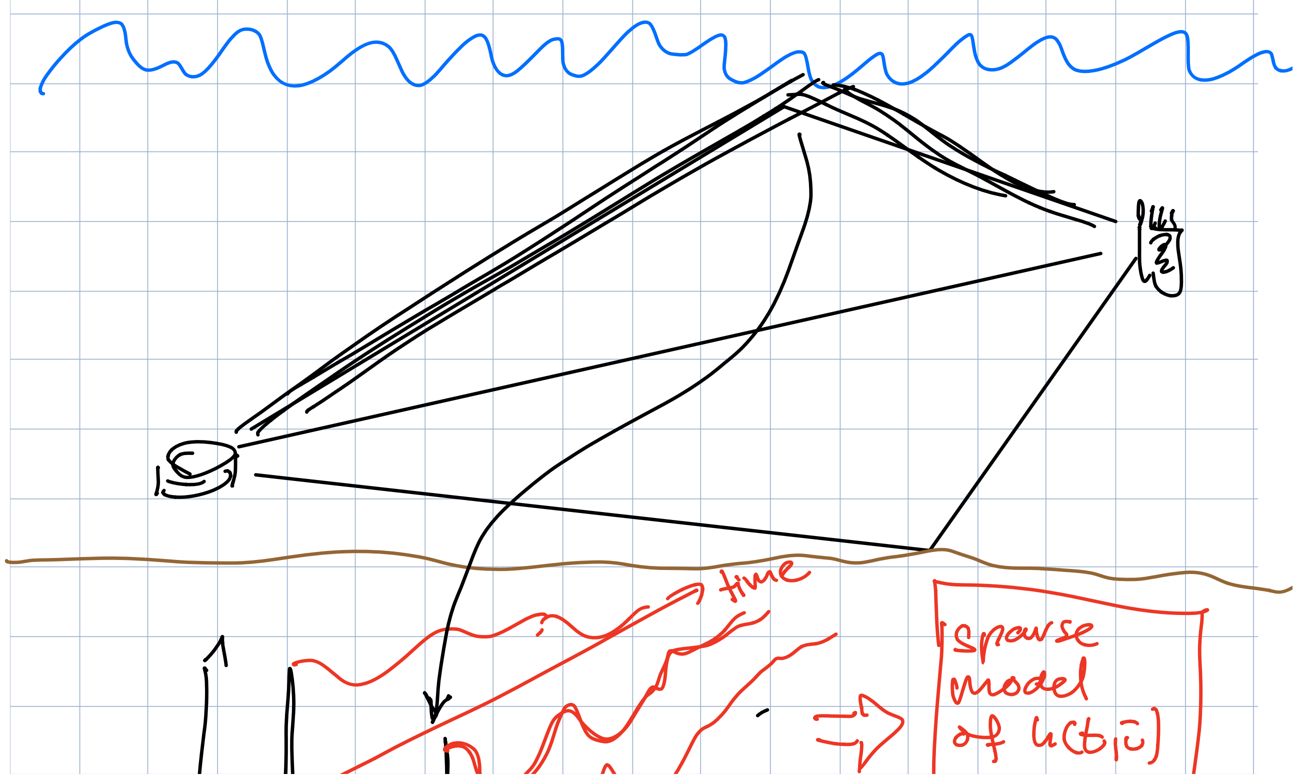

U[n]
h0
h1
h2
hp
p parameters

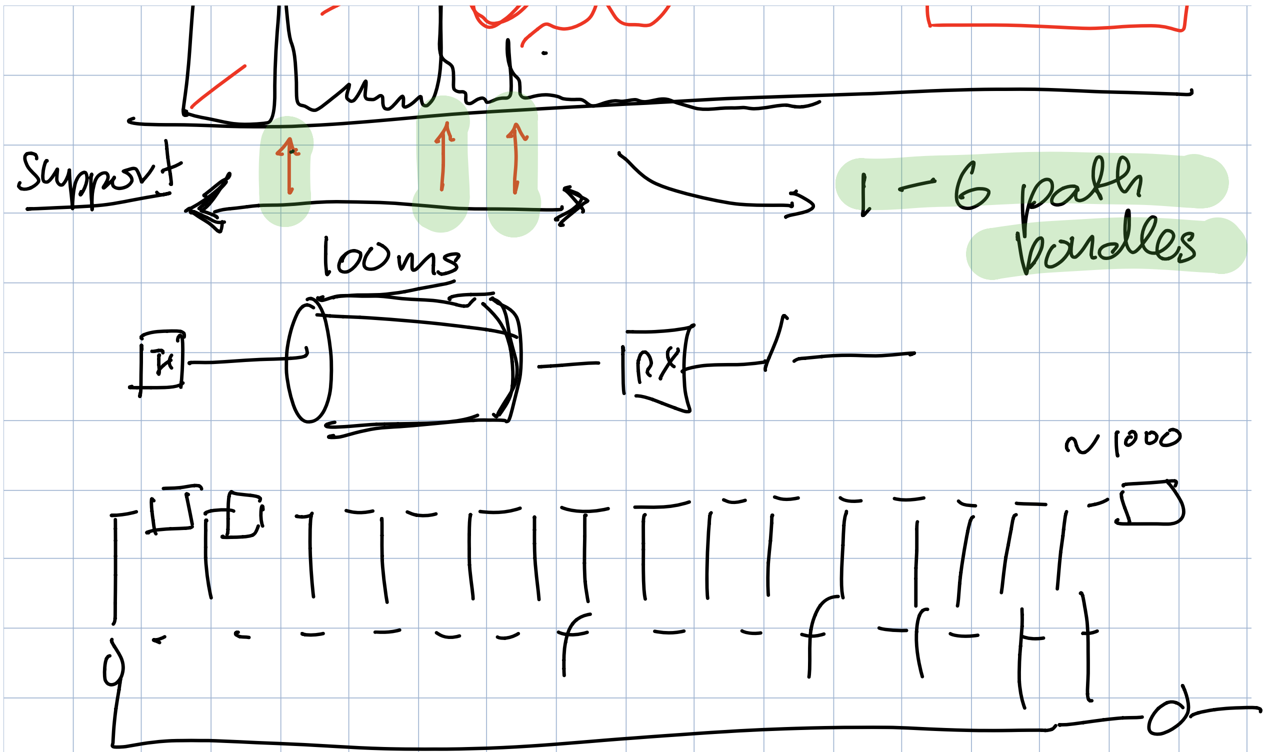


unscented Kalman $\gg$ ext. Kalman

non-linearities







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