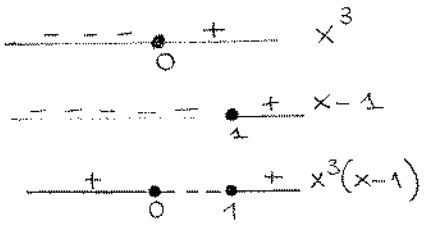
Esame scritto di ANALISI MATEMATICA , 22/09/03

1) i) $x^4 - x^3 \geq 0 \iff x^3(x-1) \geq 0$



$\iff x \leq 0$ oppure $x \geq 1$

$\Rightarrow A =]-\infty, 0] \cup [1, +\infty[$

$|x| - 1 < x + 1 \iff \begin{cases} x \geq 0 \\ x - 1 < x + 1 \end{cases}$

oppure

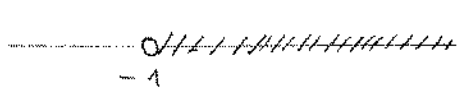
$\begin{cases} x < 0 \\ -x - 1 < x + 1 \end{cases}$

$\iff \begin{cases} x \geq 0 \\ -1 < 1 \end{cases}$ oppure $\begin{cases} x < 0 \\ 2x > -2 \end{cases}$

$\iff x \geq 0$ oppure $-1 < x < 0$

$\Rightarrow B =]-1, +\infty[$

A (zona ///)



B (")

ii) $A \cup B =]-\infty, +\infty[= \mathbb{R}$

iii) $A \cap B =]-1, 0] \cup [1, +\infty[$

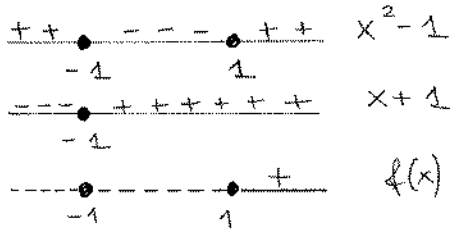
$A \setminus B =]-\infty, -1]$

} non sono insiemi limitati.

2) i) $f(x) = (x^2 - 1)(x + 1)$ insieme di def $= \mathbb{R}$

$\lim_{x \to +\infty} f(x) = +\infty \qquad \lim_{x \to -\infty} f(x) = -\infty$

Segno di f:

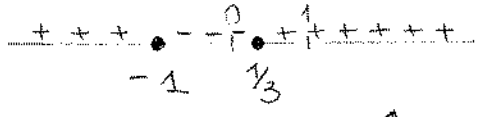


$f(x) = x^3 + x^2 - x - 1$

$f'(x) = 3x^2 + 2x - 1$

$3x^2 + 2x - 1 = 0 \iff x_{1/2} = \frac{-2 \pm \sqrt{4 + 12}}{6}$

$= \frac{-2 \pm 4}{6} \begin{matrix} \nearrow -1 \\ \searrow \frac{1}{3} \end{matrix}$

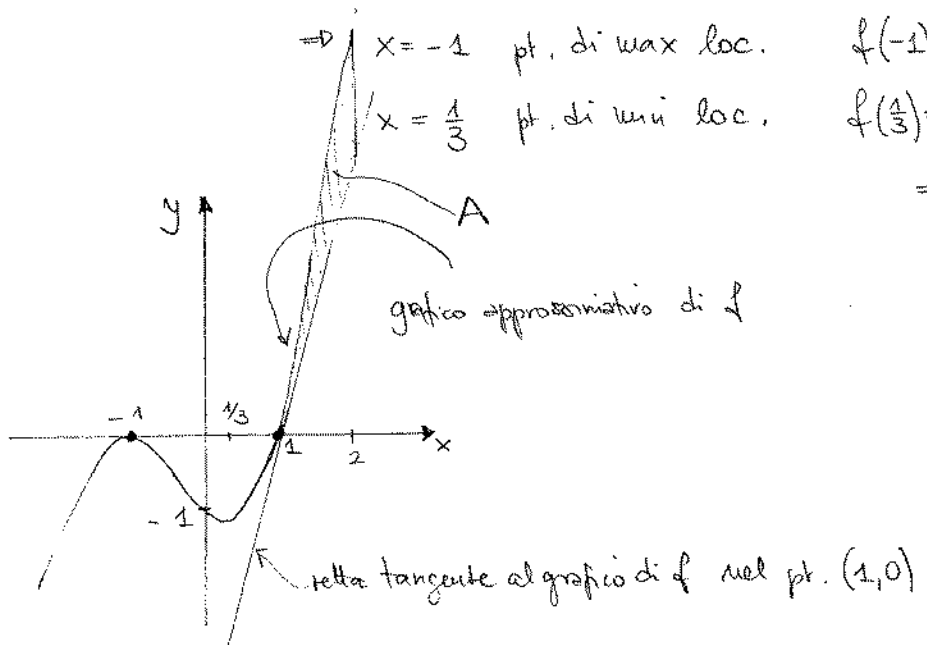


monotonia di f ↗ ↘ ↗ $f(x)$

⇒ $x=-1$ pt. di max loc. $\quad f(-1)=0$

$x=\frac{1}{3}$ pt. di min loc. $\quad f\left(\frac{1}{3}\right)=\left(\frac{1}{9}-1\right)\left(\frac{1}{3}+1\right)=$

$=-\frac{8}{9}\cdot\frac{4}{3}=-\frac{32}{27}$



ii) $y=f(1)+f'(1)(x-1) \quad \Rightarrow \quad \underline{y=4(x-1)}$

iii) $\text{Area}(A)=\int_1^2\left[f(x)-4(x-1)\right]dx=\int_1^2\left(x^3+x^2-x-1-4x+4\right)dx=$

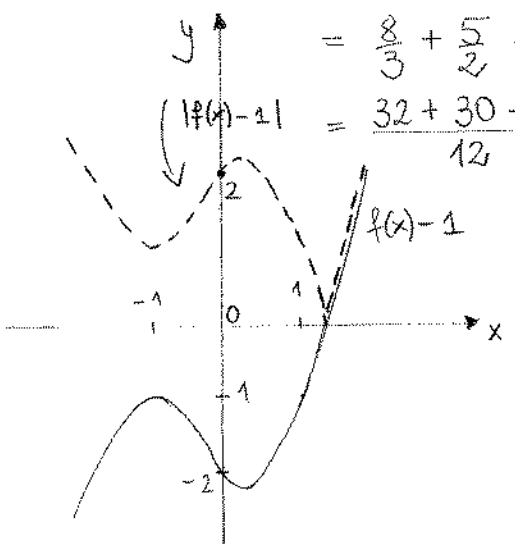
$=\int_1^2\left(x^3+x^2-5x+3\right)dx=\left[\frac{x^4}{4}+\frac{x^3}{3}-5\frac{x^2}{2}+3x\right]_1^2$

$=\cancel{4}+\frac{8}{3}-\cancel{10}+\cancel{6}-\frac{1}{4}-\frac{1}{3}+\frac{5}{2}-3$

$=\frac{8}{3}+\frac{5}{2}-\frac{7}{12}-3$

$=\frac{32+30-7}{12}-3=\frac{55}{12}-3=\underline{\frac{19}{12}}$

iv)



i) f è definita su $[-1,2]$, che è un intervallo.

ii) $x=0$, $x=2$ sono pt. di max. locali su A

$x=-1$, $x=b$ sono pt. di min. locali su A.

max loc. $f(0)$ in $x=0$

$f(2)$ in $x=2$,

min loc. 0 in $x=-1$

$f(b)$ in $x=b$.

iii) in $[-1,0]$, $[b,2]$ funzione crescente

in $[0,-1[$, $]-1,b]$ funzione decrescente. ▨

4) i) $(f\circ g)(1) = f(g(1)) = f(2)$ ma non si sa quanto vale $f(2)$!

$\underline{(g\circ f)(0)} = g(f(0)) = g(1) = \underline{2}$!

ii) $f'(x) = e^x + xe^x$ $\underline{f'(3)} = e^3 + 3e^3 = \underline{4e^3}$.

iii) $\int_1^3 \left(\frac{x^2-1}{x}\right) dx = \int_1^3 \left(x - \frac{1}{x}\right) dx = \left[\frac{x^2}{2} - \log x\right]_1^3 =$

$= \frac{9}{2} - \log 3 - \frac{1}{2} = \underline{4 - \log 3}.$

$\int_1^3 \left(\frac{x^2-1}{x+1}\right) dx = \int_1^3 (x-1)\,dx = \left[\frac{x^2}{2} - x\right]_1^3 = \frac{9}{2} - 3 - \frac{1}{2} + 1 =$

$= 4 - 2 = \underline{2}$.

iv) $\frac{10}{100} \cdot \frac{8}{100} = \underline{0{,}8\,\%}$

v) X = costo $\qquad X + X\cdot\frac{4}{100} = X\cdot\frac{104}{100} = Y$

$Y - Y\cdot\frac{2}{100} = Y\cdot\frac{98}{100} = X\cdot\frac{104}{100}\cdot\frac{98}{100} = X\cdot\frac{10192}{100^2}$

variazione $= \underline{1{,}92\,\%}$ ■

5) $C_{10,4} = \frac{10!}{4!\,6!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6!}{4!\,6!} = \frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2} = \underline{210}$ ■