

ANALISI MATEMATICA : Ancora qualche esercizio sull'esponenziale e logaritmo 30.11.04

1. Risolvete

$$i) \log_5 x = 2 \quad \log_{\frac{1}{3}} x = -1 \quad \log_{\sqrt{2}} x = 2 \quad \log_{0.1} x = -1 ;$$

$$ii) \log_x \sqrt{3} = -\frac{1}{2} \quad \log_x \frac{1}{16} = -2 \quad \log_x 0.001 = 3 .$$

2. Riducete (usando le proprietà dei logaritmi) ad un unico logaritmo le seguenti espressioni :

$$i) \frac{1}{5} \left(\log x - \frac{1}{3} \log x + \frac{1}{2} \left(\log x - \frac{1}{3} \log y \right) \right)$$

$$ii) 2 \log 5 - 3 \log 2 + \log 4$$

$$iii) \frac{1}{2} \left(\log x + \frac{1}{2} \log x - \log y - 3 \log z \right)$$

3. Individuate l'intervallo di definizione delle seq. equazioni logaritmiche :

$$i) \log(2x+3) = 4 - \log(x+5)$$

$$ii) 3 \log(1-4x) = \log\left(-\frac{x}{2} - 1\right)$$

$$iii) \log \sqrt{x} = \log(-x).$$

4. Risolvete le seguenti eq. logaritmiche :

$$i) \log \sqrt{x-1} + \log \sqrt{x+3} = 0$$

$$ii) \log x^4 + \log x^3 + \log x^2 + \log x = 5$$

$$iii) \log(x+1) + \log(x-1) = 1$$

5. Risolvete le seguenti disequazioni logaritmiche :

$$i) \log \log(2x-3) > 0$$

$$ii) \sqrt{\log_{\frac{1}{3}}(x^2-8)} > 0$$

$$iii) 2 \log(2x+1) - \log(x+2) < \log(5-2x)$$

6. Risolvete le seguenti equazioni :

$$i) 10^{x(x+5)} \cdot 10^4 = 1$$

$$ii) 2^{-2x+1} = 2^{\frac{1}{-2x+1}}$$

$$iii) \frac{3}{4} \cdot 2^x - \frac{1}{5} 3^x = \frac{1}{4} \cdot 2^x$$

Soluzioni

$$1.) i) x = 5^2 = \underline{25} \quad x = \frac{1}{3}^{-1} = \underline{3} \quad x = \sqrt{2}^2 = \underline{2} \quad \left(\frac{1}{10}\right)^{-1} = x \Rightarrow x = \underline{10}$$

$$ii) x^{-\frac{1}{2}} = \sqrt{3} \Leftrightarrow x^{-\frac{1}{2}} = \left(\frac{1}{3}\right)^{-\frac{1}{2}} \Rightarrow x = \underline{\frac{1}{3}}$$

$$x^{-2} = \frac{1}{16} \Leftrightarrow x^{-2} = 4^{-2} \Rightarrow x = \underline{4}$$

$$x^3 = \frac{1}{1000} \Leftrightarrow x^3 = \left(\frac{1}{10}\right)^3 \Rightarrow x = \underline{\frac{1}{10}}$$

□

$$2.) i) \frac{1}{5} (\log x - \log x^{\frac{3}{5}} + \log x^{\frac{2}{5}} - \log y^{\frac{1}{5}}) = \frac{1}{5} \log \frac{x\sqrt{x}}{\sqrt[3]{x^6}\sqrt{y}} = \log \sqrt[30]{\frac{x^7}{y}}$$

$$ii) \log 25 - \log 8 + \log 4 = \log \left(\frac{100}{8}\right) = \log \frac{25}{2}$$

$$iii) \frac{1}{2} (\log x + \log x^{\frac{1}{2}} - \log y - \log z^3) = \frac{1}{2} \log \left(\frac{x^{\frac{3}{2}}}{y z^3}\right) = \log \sqrt{\frac{x^{\frac{3}{2}}}{y z^3}}$$

□

$$3.) i) \begin{cases} 2x+3 > 0 \\ x+5 > 0 \end{cases} \Leftrightarrow \begin{cases} x > -\frac{3}{2} \\ x > -5 \end{cases} \Rightarrow \underline{x > -\frac{3}{2}}$$

$$ii) \begin{cases} 1-4x > 0 \\ -\frac{x}{2}-1 > 0 \end{cases} \Leftrightarrow \begin{cases} x < \frac{1}{4} \\ x < -2 \end{cases} \Rightarrow \underline{x < -2}$$

$$iii) \begin{cases} x > 0 \\ -x > 0 \end{cases} \text{ mai! L'insieme di definizione \u00e9 vuoto!}$$

□

$$4.) i) \begin{cases} x-1 > 0 \\ x+3 > 0 \\ \log \sqrt{x-1} \sqrt{x+3} = 0 \end{cases} \Leftrightarrow \begin{cases} x > 1 \\ (x-1)(x+3) = 1 \end{cases} \Leftrightarrow \begin{cases} x > 1 \\ x_{\frac{1}{2}} = -1 \pm \sqrt{5} \end{cases} \Rightarrow \underline{x = -1 + \sqrt{5}}$$

$$ii) \begin{cases} x > 0 \\ 4 \log x + 3 \log x + 2 \log x + \log x = 5 \end{cases} \Leftrightarrow \begin{cases} x > 0 \\ 10 \log x = 5 \end{cases} \Leftrightarrow \begin{cases} x > 0 \\ \log x = \frac{1}{2} \end{cases} \Leftrightarrow \underline{x = \sqrt{e}}$$

$$iii) \begin{cases} x+1 > 0 \\ x-1 > 0 \\ \log(x+1)(x-1) = 1 \end{cases} \Leftrightarrow \begin{cases} x > 1 \\ (x+1)(x-1) = e \end{cases} \Leftrightarrow \begin{cases} x > 1 \\ x^2 = 1+e \end{cases} \Rightarrow \underline{x = \sqrt{1+e}}$$

□

$$5.) i) \begin{cases} 2x-3 > 0 \\ \log(2x-3) > 1 \end{cases} \Leftrightarrow \begin{cases} x > \frac{3}{2} \\ 2x-3 > e \end{cases} \Leftrightarrow \begin{cases} x > \frac{3}{2} \\ x > \frac{e+3}{2} \end{cases} \Rightarrow \underline{x > \frac{e+3}{2}}$$

$$ii) \begin{cases} x^2-8 > 0 \\ x^2-8 < 1 \end{cases} \Leftrightarrow \begin{cases} x^2 > 8 \\ x^2 < 9 \end{cases} \Rightarrow \underline{\{-3 < x < -2\sqrt{2}\} \cup \{2\sqrt{2} < x < 3\}}$$

$$iii) \begin{cases} 2x+1 > 0 \\ x+2 > 0 \\ 5-2x > 0 \\ \log \frac{(2x+1)^2}{x+2} < \log(5-2x) \end{cases} \Leftrightarrow \begin{cases} x > -\frac{1}{2} \\ x > -2 \\ x < \frac{5}{2} \\ \frac{(2x+1)^2}{x+2} < 5-2x \end{cases} \Leftrightarrow \begin{cases} x > -\frac{1}{2} \\ x < \frac{5}{2} \\ x < -2 \text{ oppure } -\frac{3}{2} < x < 1 \end{cases} \Rightarrow \underline{-\frac{1}{2} < x < 1}$$

$$6.) i) 10^{x^2+5x+4} = 10^0 \Leftrightarrow x^2+5x+4=0 \Rightarrow \underline{x=-1}, \underline{x=-4}$$

$$ii) \begin{cases} -2x+1 \neq 0 \\ -2x+1 = \frac{1}{-2x+1} \end{cases} \Leftrightarrow \begin{cases} x \neq \frac{1}{2} \\ (-2x+1)^2 = 1 \end{cases} \Leftrightarrow \underline{x=0}, \underline{x=1}$$

$$iii) \frac{1}{2} \cdot 2^x - \frac{1}{5} 3^x = 0 \Leftrightarrow \frac{1}{2} \cdot 2^x = \frac{1}{5} 3^x \Leftrightarrow \left(\frac{2}{3}\right)^x = \frac{2}{5} \Leftrightarrow x \log\left(\frac{2}{3}\right) = \log\left(\frac{2}{5}\right) \Leftrightarrow x = \frac{\log 2 - \log 5}{\log 2 - \log 3}$$

□