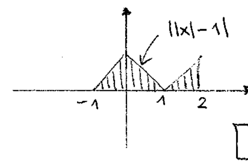
Terza Prova Intermedia di ANALISI MATEMATICA — a.a. 2004/05 — Rovereto, 23/12/04

FILA (A):

1) $\displaystyle\int_1^2 \left[(2x+1)^2 - \frac{1}{x}\right] dx = \left[\frac{1}{2}\frac{(2x+1)^3}{3} - \log x\right]_1^2 = \left(\frac{1}{2}\cdot\frac{5^3}{3} - \log 2\right) - \frac{1}{2}\frac{3^{\cancel{3}\,2}}{\cancel{3}} = \frac{125}{6} - \frac{9}{2} - \log 2 = \underline{\underline{\frac{49}{3} - \log 2}}$;

$\displaystyle\int_0^1 (e^{2x} + 3e^x)\, e^{-x}\, dx = \int_0^1 (e^x + 3)\,dx = \left[e^x + 3x\right]_0^1 = (e+3) - 1 = \underline{\underline{e+2}}$;

$\displaystyle\int_{-1}^2 \big||x|-1\big|\, dx = \underline{\underline{\frac{3}{2}}}$.



$\square$

2) i) $\operatorname{dom} f = \mathbb{R}$ $\quad \lim_{x\to-\infty} f(x) = -\infty \quad \lim_{x\to+\infty} f(x) = -\infty$

$f(x) = x^3(-x+2)$

− − − − • + + + x^3 (0)
+ + + + + + • − − $-x+2$ (2)
- - - - • + • - - - $f(x)$ (0, 2)

$\operatorname{dom} f' = \mathbb{R}$ $\quad f'(x) = -4x^3 + 6x^2 = 2x^2(-2x+3)$

$f'(x) = 0 \iff x = 0$ opp. $x = \frac{3}{2}$

+ • + x^2 (0)
+ + + + + • − − − $(-2x+3)$ ($\frac{3}{2}$)
+ + + • + • − − − $f'(x)$ (0, $\frac{3}{2}$)
↗ ↗ ↘ $f(x)$

$x = \frac{3}{2}$ pt. di max loc.

$f(0) = 0 \quad f\left(\frac{3}{2}\right) = \frac{27}{8}\left(-\frac{3}{2} + 2\right) = \frac{27}{16}$

$\operatorname{dom} f'' = \mathbb{R}$ $\quad f''(x) = -12x^2 + 12x = 12x(-x+1)$

− − − • + + + x (0)
+ + + + + • − − $-x+1$ (1)
− − − • + + • − − f'' (0, 1)
∩ ∪ ∩ f

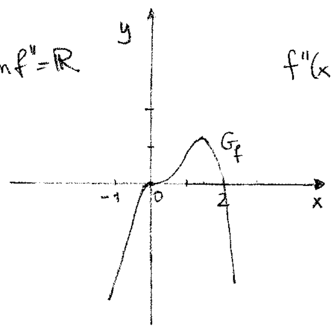
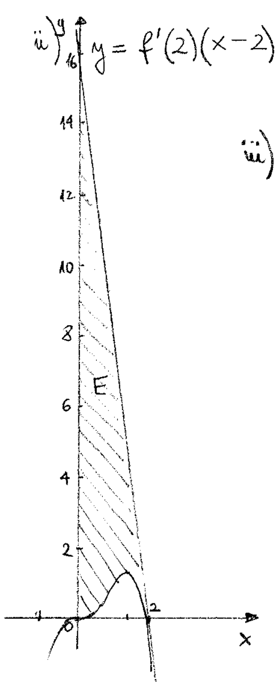


grafico approssimativo di f

ii) $y = f'(2)(x-2) = -8(x-2)$ è l'eq. della retta tg r $\quad \underline{\underline{y = -8x + 16}}$



iii) area $E = \displaystyle\int_0^2 (-8x + 16 + x^4 - 2x^3)\, dx =$

$= \left[-\cancel{8}^{\,4}\frac{x^2}{\cancel{2}} + 16x + \frac{x^5}{5} - \cancel{2}\frac{x^4}{\cancel{4}_{\,2}}\right]_0^2 = -16 + 32 + \frac{32}{5} - 8 =$

$= 8 + \frac{32}{5} = \underline{\underline{\frac{72}{5}}}$.

$\square$

3) i) $\operatorname{dom} f = \mathbb{R}$ $\quad \lim_{x \to -\infty} f(x) = +\infty \quad \lim_{x \to +\infty} f(x) = 0$

$$\begin{array}{ccccc} + & -1 & - & 1 & + \end{array} \quad f(x)$$

$\operatorname{dom} f' = \mathbb{R} \quad f'(x) = \dfrac{2xe^x - (x^2-1)e^x}{e^{2x}} = \dfrac{2x - x^2 + 1}{e^x}$

$$-x^2+2x+1=0 \iff x_{1/2} = \frac{-2 \pm \sqrt{4+4}}{-2} = 1 \pm \sqrt{2}$$

$$\begin{array}{ccccc} - - & 1-\sqrt{2} & + + & 1+\sqrt{2} & - \end{array} \quad f'(x)$$

$$\searrow \quad \nearrow \quad \searrow \quad f(x)$$

$f'(x) = 0 \iff x_{1/2} = 1 \pm \sqrt{2}$

$x = 1-\sqrt{2}$ pt. di min loc.
$x = 1+\sqrt{2}$ pt. di max. loc.

$\operatorname{dom} f'' = \mathbb{R} \quad f''(x) = \dfrac{(-2x+2)e^x - (2x-x^2+1)e^x}{e^{2x}} =$

$$= \frac{-2x+2 - 2x + x^2 - 1}{e^x} = \frac{x^2 - 4x + 1}{e^x}$$

$$f''(x) = 0 \iff x_{1/2} = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$$

$$\begin{array}{ccccc} + & 2-\sqrt{3} & - - & 2+\sqrt{3} & + \end{array} \quad f''(x)$$

$$\cup \quad \cap \quad \cup \quad f(x)$$

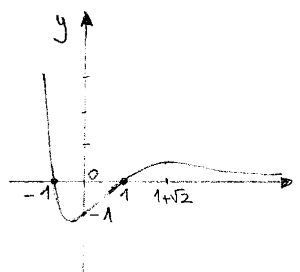


grafico approssimativo di f.

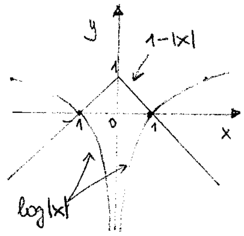
ii) $\min_{[0,+\infty[} f = f(0) = -1 \qquad x=0$ pt. di minimo

$\max_{[0,+\infty[} f = f(1+\sqrt{2}) = \dfrac{2\sqrt{2}+2}{e^{1+\sqrt{2}}} \qquad x = 1+\sqrt{2}$ pt. di massimo.

iii) $\displaystyle\int_0^1 \frac{f(x)}{(x-1)(x+1)}\,dx = \int_0^1 \frac{f(x)}{x^2-1}\,dx = \int_0^1 e^{-x}\,dx = \left[-e^{-x}\right]_0^1 = \underline{-\frac{1}{e} + 1}$

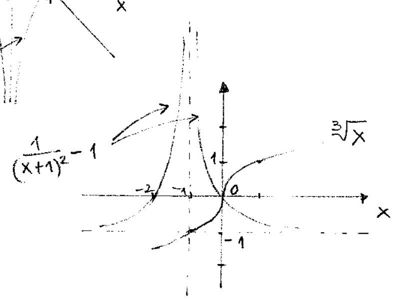
$\square$

4) i)



Soluzione: $\underline{x \in [-1,1] \setminus \{0\}}$.

ii)



Soluzione: $\underline{x \in\,]-\infty, 0] \setminus \{-1\}}$.

$\square$

5) i) (F) f è decrescente su $[-1, 1]$, per esempio;

ii) $f'(-1) = 0 \geq f'(1)$ (V) essendo $f'(1) < 0$;

iii) $\int_{-2}^{3} f(x)dx \geq 0$ (V) $\int_{0}^{3} f(x)dx \geq 0$ (V); (interpret. geometrica dell'integrale)

iv) (V): sono soddisfatte le ipotesi del teorema di Weierstraß.

v) (V) infatti $F(x)$ è crescente su $[-2, 1]$ e decrescente su $[1, 3]$. $\square$

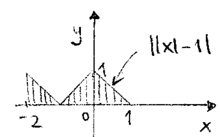
6) $D_{n,2} = \frac{n!}{(n-2)!} = n(n-1) = 20 \iff n^2 - n - 20 = 0$

$\iff n = \frac{1+\sqrt{1+80}}{2} = \underline{\underline{5}}.$ $\square$

FILA (B):

1) $\int_0^1 (e^{-2x} - 2e^{-x})e^{2x}\,dx = \int_0^1 (1 - 2e^x)\,dx = \left[x - 2e^x\right]_0^1 = (1-2e) - (-2) = \underline{\underline{3-2e}};$

$\int_{-2}^{1} ||x|-1|\,dx = \underline{\underline{\frac{3}{2}}}$;



$\int_1^2 \left[\frac{1}{2x} - (3x+1)^2\right]dx = \left[\frac{\log x}{2} - \frac{(3x+1)^3}{9}\right]_1^2 = \underline{\underline{\frac{\log 2}{2} - \frac{7^3}{9} + \frac{4^3}{9}}} = \frac{\log 2}{2} - 31.$

2) i) dom $f = \mathbb{R}$ $\lim_{x\to-\infty} f(x) = +\infty$ $\lim_{x\to+\infty} f(x) = +\infty$

$f(x) = x^3(x-3)$

- - - - - • + + + + x^3 (0)
- - - - - - - - - • + $(x-3)$ (3)
+ + + • - - - - • + $f(x)$ (0, 3)

dom $f' = \mathbb{R}$ $f'(x) = 4x^3 - 9x^2$
$= x^2(4x-9)$

+ • + + x^2 (0)
- - - - • + $4x-9$ ($\frac{9}{4}$)
- - • - • + $f'(x)$ (0, $\frac{9}{4}$)
↓ ↓ ↗ $f(x)$

$f'(x) = 0 \iff x = 0$ opp $x = \frac{9}{4}$

$f(0) = 0$ $f\left(\frac{9}{4}\right) = \frac{9^3}{4^3}\left(\frac{9}{4} - 3\right) = \frac{9^3}{4^3}\left(-\frac{3}{4}\right).$

$x = \frac{9}{4}$ pt. di min. locale.

dom $f'' = \mathbb{R}$ $f''(x) = 12x^2 - 18x = 6x(2x-3)$

- - - • + + + x (0)
- - - - - • + $2x-3$ ($\frac{3}{2}$)
+ + + • - • + $f''(x)$ (0, $\frac{3}{2}$)
∪ ∩ ∪ f

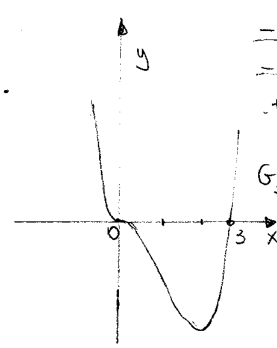
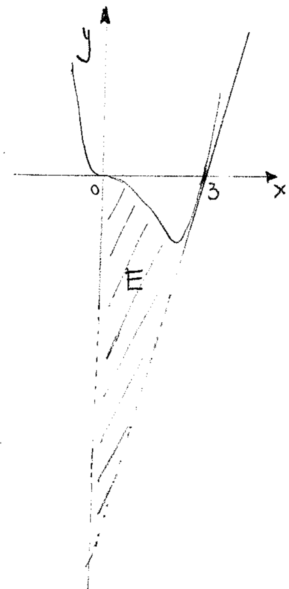


grafico approssimativo di f (non in scala!)

ii) $y = f'(3)(x-3) = 9\cdot 3(x-3) = 27(x-3)$ è l'eq. della retta tg. r $\underline{y = 27x - 81}$



iii) area $E = \int_0^3 \left(x^4 - 3x^3 - 27x + 81\right) dx =$

$$= \left[\frac{x^5}{5} - 3\frac{x^4}{4} - 27\frac{x^2}{2} + 81x\right]_0^3 =$$

$$= \left(\frac{3^5}{5} - \frac{3^5}{4} - \frac{3^5}{2} + 3^5\right) = \frac{4\cdot 3^5 - 5\cdot 3^5 - 10\cdot 3^5 + 20\cdot 3^5}{20}$$

$$= \underline{\underline{\frac{9\cdot 3^5}{20}}}$$

□

3) i) $\mathrm{dom} f = \mathbb{R}$ $\quad \lim_{x\to -\infty} f(x) = -\infty \quad \lim_{x\to+\infty} f(x) = 0$

$- - \circ + \circ - - -$ $f(x)$ (-2, 2)

$\mathrm{dom} f' = \mathbb{R}$

$$f'(x) = \frac{-2xe^x - (4-x^2)e^x}{e^{2x}} = \frac{x^2-2x-4}{e^x}$$

$$x^2-2x-4 = 0 \iff x_{1/2} = \frac{2\pm\sqrt{4+16}}{2} = 1\pm\sqrt{5}$$

$$f'(x) = 0 \iff x_{1/2} = 1\pm\sqrt{5}$$

$+ + \circ - - - \circ + +$ $f'(x)$ ($1-\sqrt5$, $1+\sqrt5$)

$\nearrow \quad \searrow \quad \nearrow$

$x = 1-\sqrt5$ pt. di max loc.

$x = 1+\sqrt5$ pt. di min loc.

$\mathrm{dom} f'' = \mathbb{R}$

$$f''(x) = \frac{(2x-2)e^x - (x^2-2x-4)e^x}{e^{2x}}$$

$$= \frac{-x^2+4x+2}{e^x}$$

$$f''(x) = 0 \iff x_{1/2} = \frac{-4\pm\sqrt{16+8}}{2} = -2\pm\sqrt{6}$$

$- \circ + + + \circ - - -$ f'' ($-2-\sqrt6$, $-2+\sqrt6$)

$\cap \quad \cup \quad \cap \quad f$

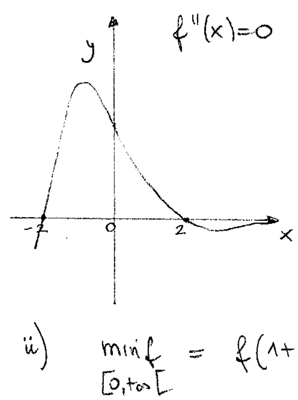


grafico approssimativo di f

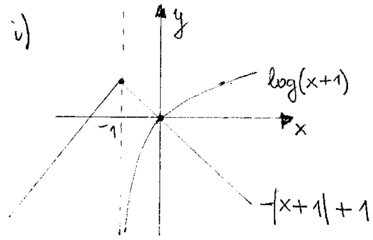
ii) $\min_{[0,+\infty[} f = f(1+\sqrt5) = \dfrac{4-6-2\sqrt5}{e^{1+\sqrt5}} = \dfrac{-2-2\sqrt5}{e^{1+\sqrt5}}$ $\qquad x = 1+\sqrt5$ pt. di minimo

$\max_{[0,+\infty[} f = f(0) = 4 \qquad x = 0$ pt. di massimo.

iii) $\displaystyle\int_{-1}^{0} \frac{f(x)}{x^2-4}\,dx = -\int_{-1}^{0} e^{-x}\,dx = \left[e^{-x}\right]_{-1}^{0} = \underline{\underline{1-\frac{1}{e}}}.$

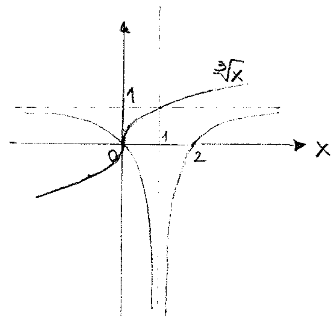
□

4) i)



Soluzione: $\underline{\underline{x \in \left]-1,0\right]}}$.

ii)



Soluzione: $\underline{\underline{x \in \left]0,+\infty\right[\setminus \{1\}}}$.

5) i) (V); (vedi grafico!)

ii) $f'(0) > 0 = f'(2)$ (V);

iii) $\int_{-2}^{3} f(x)\,dx \le 0$ (F); $\int_{0}^{3} f(x)\,dx \ge 0$ (V); (interpret. geometrica della derivata)

iv) (V): sono soddisfatte le ipotesi del teorema di Weierstraß.

v) (V) (vale $F(-2)=0$; poi è crescente fino al pt. di intersezione del grafico di f con l'asse x, e poi decresce, ma $F(3)>0$).

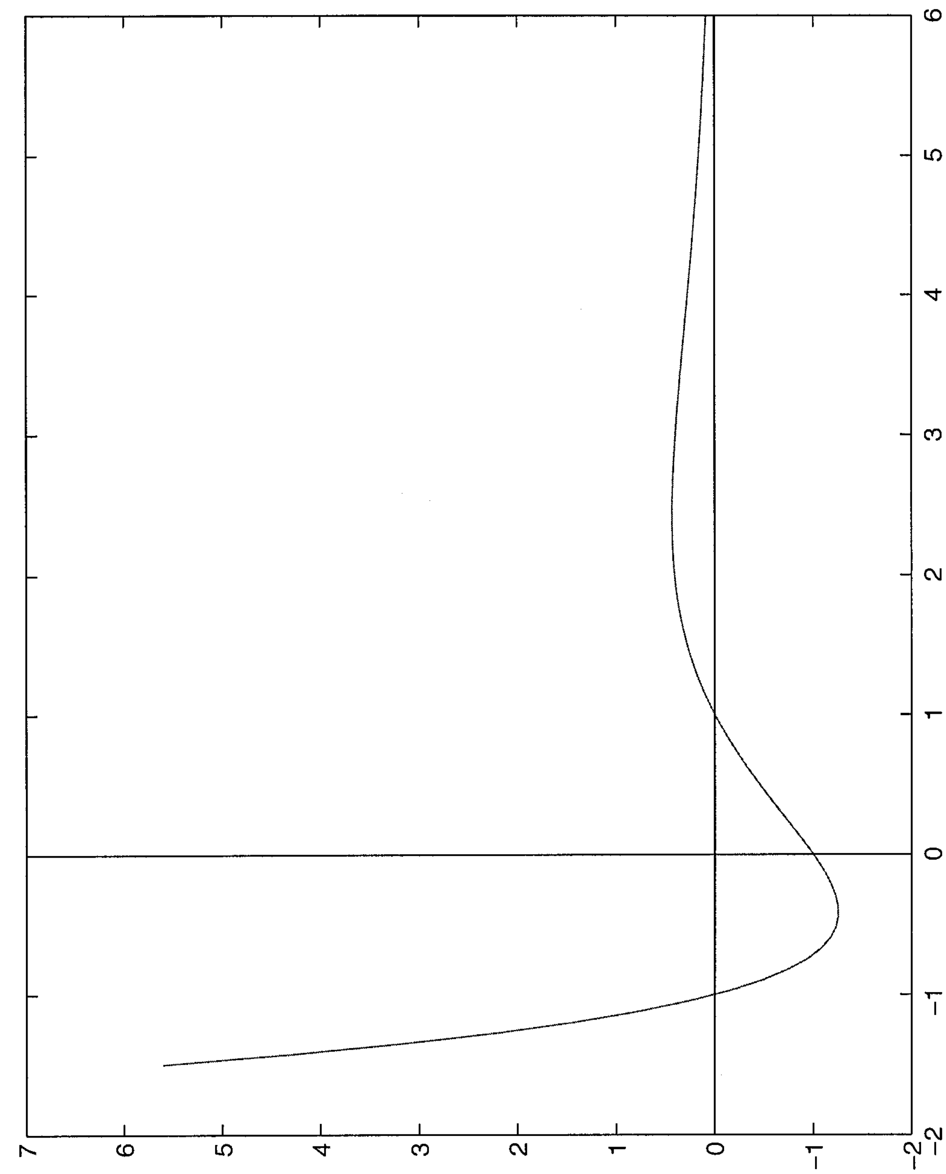
6) $D_{n,2} = \frac{n!}{(n-2)!} = n(n-1) = 30 \Leftrightarrow n^2-n-30=0$

$\Leftrightarrow n = \frac{1+\sqrt{1+120}}{2} = \underline{\underline{6}}$.

grafico di

$f(x) = \frac{x^2-1}{e^x}$

FILA A Es. 3



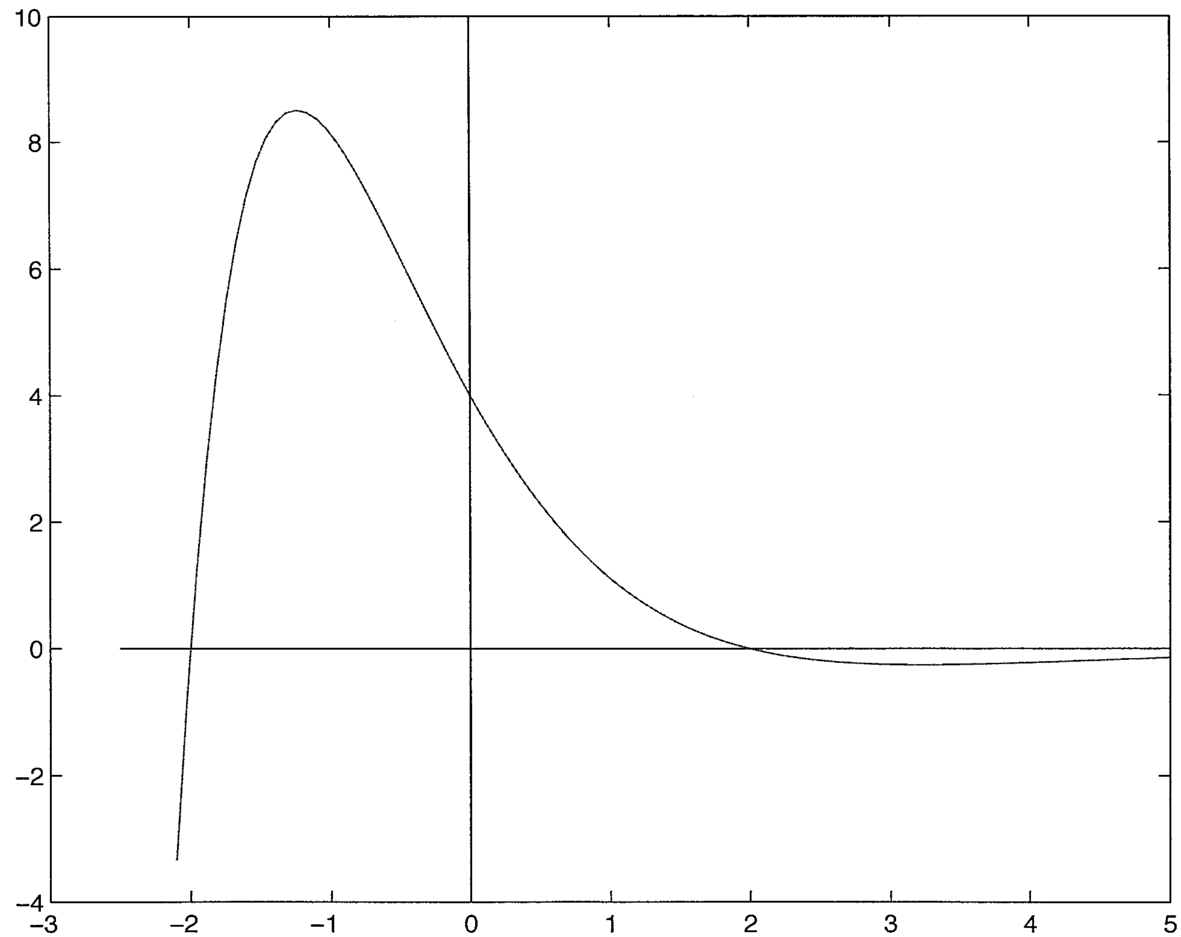


grafico di

$f(x) = \frac{4-x^2}{e^x}$

FILA B Es.3