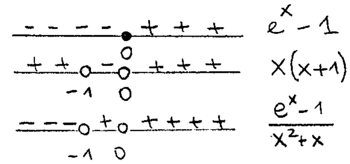
Esame Scritto di ANALISI MATEMATICA – 12/01/05.

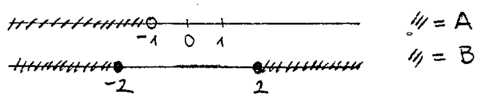
FILA (A) 1) i) $A = \left\{ x \in \mathbb{R} : \ \dfrac{e^x - 1}{x^2 + x} < 0 \right\} = \underline{\underline{\left]-\infty, -1\right[}}.$



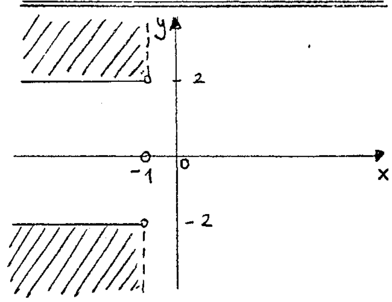
$B = \left\{ x \in \mathbb{R} : \ x^2 - |x| \geq 2 \right\} = \underline{\underline{\left]-\infty, -2\right] \cup \left[2, +\infty\right[}}.$

$$: \begin{cases} x \geq 0 \\ x^2 - x - 2 \geq 0 \end{cases} \quad \text{o} \quad \begin{cases} x < 0 \\ x^2 + x - 2 \geq 0 \end{cases}$$

$$\Leftrightarrow \left\{ x \geq 2 \right. \quad \text{o} \quad \left\{ x \leq -2 \right.$$



ii) $\underline{\underline{A \cup B = \left]-\infty, -1\right[\cup \left[2, +\infty\right[}}$; $\underline{\underline{A \cap B = \left]-\infty, 2\right]}}$; $\underline{\underline{A \setminus B = \left]-2, -1\right[}}$.

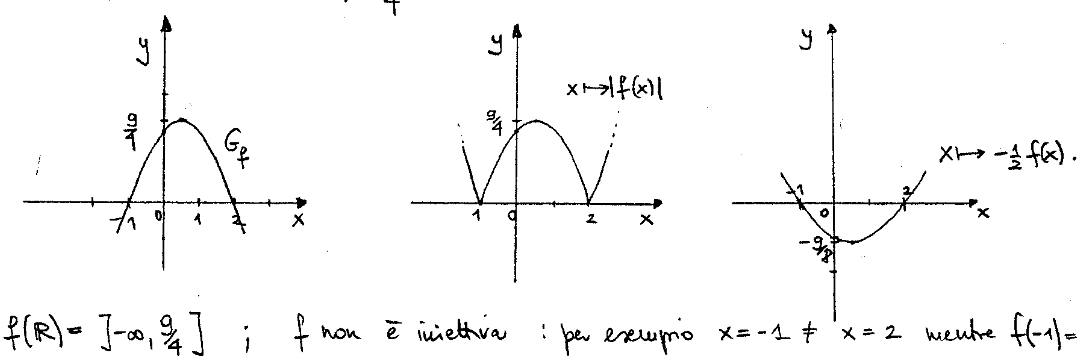


$A \times B =$ ///

$\square$

2) $f(x) = -x^2 + x + 2 = -\left(x - \frac{1}{2}\right)^2 + \frac{9}{4}$

i)



ii) $f(\mathbb{R}) = \left]-\infty, \frac{9}{4}\right]$; f non è iniettiva : per esempio $x = -1 \neq x = 2$ mentre $f(-1) = f(2) = 0$

f non è suriettiva : $f(\mathbb{R}) \neq \mathbb{R}$.

iii) $y = f(0) + f'(0)(x - 0)$ $\qquad f'(x) = -2x + 1 \qquad f'(0) = 1$

$\underline{\underline{y = 2 + x}}$.

iv) equazione retta perpendicolare alla retta r : $y = -x + b$.

Si ha $f(x) = -x + b \iff -x^2 + x + 2 = -x + b \iff x^2 - 2x + b - 2 = 0$

Allora $y=-x+b$ non interseca il grafico di f $\Longleftrightarrow$

$$4-4(b-2)<0$$

ossia $1<b-2$; $\Rightarrow$ $b>3$. Concludiamo allora che l'eq. delle rette $\perp$ a r che non intersecano mai il grafico di f è $y=-x+b$ con $b>3$. $\square$

3) i) Vedi Es. 3 3 prova intermedia 12/01/05 Fila A.

ii) se $k<0$, $f(x)=k$ ha 1 soluzione;

se $0\le k<e^3$ $f(x)=k$ non ha soluzioni;

se $k=e^3$ $f(x)=k$ ha 1 soluzione;

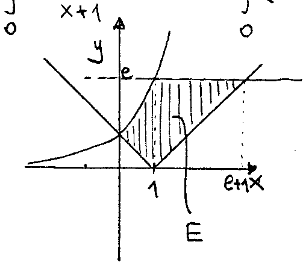
se $k>e^3$ $f(x)=k$ ha 2 soluzioni.

iii) f non è definita in $x=2\in[1,3]$. Non sono soddisfatte le ipotesi del teorema di Weierstrass. $\square$

4) i) $\displaystyle\int_0^1 \frac{x}{x^2+4}\,dx = \frac{1}{2}\Big[\log(x^2+4)\Big]_0^1 = \frac{1}{2}(\log 5-\log 4) = \frac{1}{2}\log\frac{5}{4} = \underline{\underline{\log\sqrt{\tfrac{5}{4}}}}$.

$$\int_0^1 \frac{x^2-2x-3}{x+1}\,dx = \int_0^1 \frac{\cancel{(x+1)}(x-3)}{\cancel{x+1}}\,dx = \int_0^1 (x-3)\,dx = \left[\frac{x^2}{2}-3x\right]_0^1 = \frac{1}{2}-3 = \underline{\underline{-\frac{5}{2}}}.$$

ii)



$$\text{area } E = \int_0^1 \left(e^x-(1-x)\right)dx + \frac{e^2}{2} =$$

$$= \left[e^x - x + \frac{x^2}{2}\right]_0^1 + \frac{e^2}{2} =$$

$$= \left(e-1+\frac{1}{2}\right)-(1)+\frac{e^2}{2} = \underline{\underline{\frac{e^2}{2}+e-\frac{3}{2}}}. \quad \square$$

5) Vedi Es. 5 3 prova intermedia 12/01/05 Fila A.

6) Vedi Es. 6 " . $\blacksquare$