

Esame scritto di ANALISI MATEMATICA - Rovereto, 28 gennaio 2005

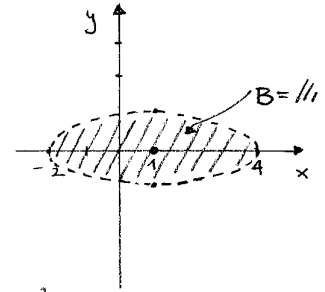
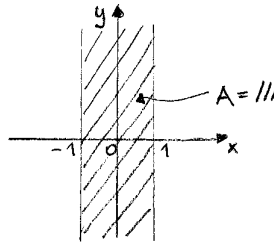
FILA (A) 1) i) $\forall x \in \mathbb{R} \exists y \in \mathbb{N} : y-1 < x \iff \forall x \in \mathbb{R} \exists y \in \mathbb{N} : y < x+1$ (F)

basta prendere $x = -2$; $\exists y \in \mathbb{N} : y < -1$.

ii) $\forall x \in \mathbb{N}, \forall y \in \mathbb{R}, -x \leq |y|$: (V) infatti $\forall x \in \mathbb{N} -x \leq 0$, mentre

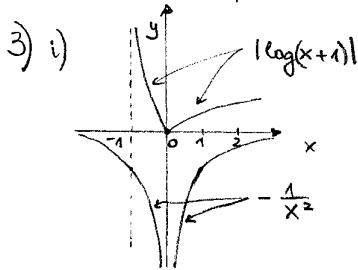
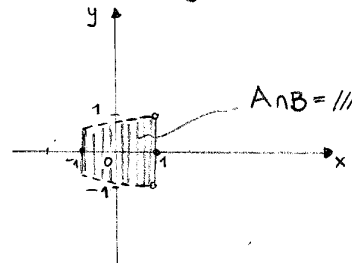
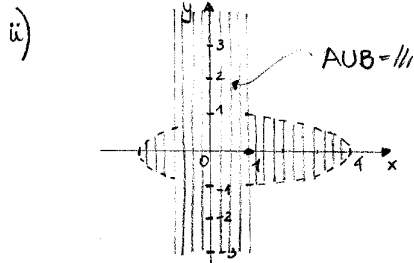
$\forall y \in \mathbb{R}, |y| \geq 0$ e quindi $-x \leq |y|$. □

2) i) $A = \{(x,y) \in \mathbb{R}^2 : |x| \leq 1\}$

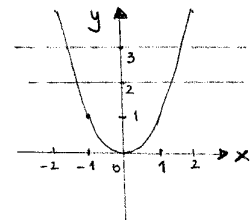


$B = \{(x,y) \in \mathbb{R}^2 : x^2 - 2x + 9y^2 < 8\} =$

$= \{(x,y) \in \mathbb{R}^2 : (x-1)^2 + 9y^2 < 9\} = \{(x,y) \in \mathbb{R}^2 : \frac{(x-1)^2}{9} + y^2 < 1\}$



$|\log(x+1)| \geq -\frac{1}{x^2} \iff \forall x \in]-1, +\infty[\setminus \{0\}$.



ii) $\log_2 4 \leq x^2 \leq \log_3 27 \iff 2 \leq x^2 \leq 3$

$\iff x \in [-\sqrt{3}, -\sqrt{2}] \cup [\sqrt{2}, \sqrt{3}]$.

iii) $\int_0^5 \frac{2}{\sqrt{x+4}} dx = 2 \left[\frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^5 = 4(3-2) = 4$

$\int_1^2 \frac{3xe^x + x^3}{x} dx = \int_1^2 (3e^x + x^2) dx = \left[3e^x + \frac{x^3}{3} \right]_1^2 = (3e^2 + \frac{8}{3}) - (3e + \frac{1}{3}) = 3(e^2 - e) + \frac{7}{3}$.

* non $(\forall x \in \mathbb{R} \exists y \in \mathbb{N} : y-1 < x) = \exists x \in \mathbb{R} : \forall y \in \mathbb{N}, y-1 \geq x$.
 non $(\forall x \in \mathbb{N}, \forall y \in \mathbb{R}, -x \leq |y|) = \exists x \in \mathbb{N}, \exists y \in \mathbb{R} : -x > |y|$. □

4) i) vedi Es. 3) i) Terza Prova Intermedia, 28/1/05 Fila (A).

$$\text{ii) } f(-4) = \frac{16-4-2}{-4} = -\frac{10}{4} = -\frac{5}{2} \quad f(-1) = \frac{1-1-2}{-1} = 2 \quad f(1) = 0 \quad f(2) = \frac{4+2-2}{2} = 2.$$

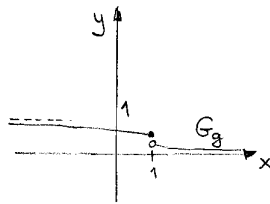
Essendo f crescente su $[-4, -1]$, e crescente su $[1, 2]$, possiamo concludere che

$$\min_A f = -\frac{5}{2} \quad \text{e pr. di min. } x = -4 \quad (A \equiv [-4, -1] \cup [1, 2])$$

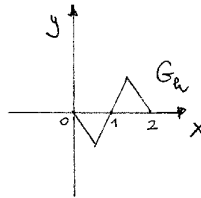
$$\max_A f = 2 \quad \text{e pr. di massimo } x = -1, x = 2.$$

iii) iv) vedi Es. 3) ii) Terza Prova Intermedia, 28/1/05 Fila (A). □

5) i) Esempio:



ii) Esempio:



$$6) 9 \cdot D_{9,4} = 9 \cdot \frac{9!}{5!} = \underline{\underline{9 \cdot 9 \cdot 8 \cdot 7 \cdot 6}}$$

