Terza Prova Intermedia di ANALISI MATEMATICA – Rovereto, 28 gennaio 2005.

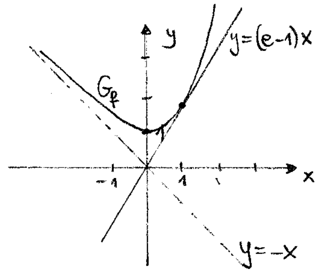
FILA (A) 1) $\int_0^5 \frac{2}{\sqrt{x+4}}\,dx = 2\left[\frac{(x+4)^{1/2}}{1/2}\right]_0^5 = 4(3-2) = \underline{\underline{4}}$;

$\int_1^2 \left(\frac{3xe^x+x^3}{x}\right)dx = \int_1^2 (3e^x+x^2)\,dx = \left[3e^x+\frac{x^3}{3}\right]_1^2 = \underline{\underline{3(e^2-e)+\frac{7}{3}}}$;

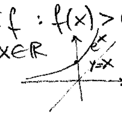
$\int_{-1}^2 (|x|-|x-1|)\,dx = \int_{-1}^2 |x|\,dx - \int_{-1}^2 |x-1|\,dx = \int_{-1}^0 (-x)\,dx + \int_0^2 x\,dx - \int_{-1}^1 (-x+1)\,dx - \int_1^2 (x-1)\,dx$

$= \left[-\frac{x^2}{2}\right]_{-1}^0 + \left[\frac{x^2}{2}\right]_0^2 - \left[-\frac{x^2}{2}+x\right]_{-1}^1 - \left[\frac{x^2}{2}-x\right]_1^2 = \frac{1}{2} + \cancel{2} - \left[\cancel{-\frac{1}{2}}+\cancel{1}+\cancel{\frac{1}{2}}+\cancel{1}\right] -$

$- \left[\cancel{2}-\cancel{2}-\frac{1}{2}+1\right] = \underline{\underline{0}}$. □



2) i) $f(x) = e^x - x$ • dom $f = \mathbb{R}$ $\lim_{x\to-\infty} f(x) = +\infty$ $\lim_{x\to+\infty} f(x) = +\infty$. Segno di f : $f(x) > 0$ $\forall x \in \mathbb{R}$



• dom $f' = \mathbb{R}$ $f'(x) = e^x - 1$ $f'(x) = 0 \Leftrightarrow x = 0$,

$f(0) = 1$

− − −	•	+ + +	$e^x - 1$
↘	0	↗	$f(x)$

$x=0$ è pt. di min locale forte (anzi, globale)

• dom $f'' = \mathbb{R}$ $f''(x) = e^x > 0 \Rightarrow f$ è convessa su $\mathbb{R}$.

ii) Ovviamente $\lim_{x\to-\infty} \frac{f(x)}{x} = -1$; inoltre $\lim_{x\to-\infty} (f(x)+x) = \lim_{x\to-\infty} e^x = 0$;

ne segue che $\underline{\underline{y=-x}}$ è asintoto obliquo, per $x\to-\infty$, della funzione f.

(si poteva dedurre subito che $f(x)$ ha come asintoto obliquo $y=-x$ essendo $\lim_{x\to-\infty} e^x = 0$).

iii) Eq. retta tg. al grafico di f nel pt $(1, e-1)$ è :

$y = (e-1) + (e-1)(x-1) \Rightarrow \underline{\underline{y = (e-1)x}}$.

Punto di intersezione delle due rette $\underline{\underline{P=(0,0)}}$. □

3) i) $f(x) = \frac{x^2+x-2}{x} = \frac{(x+2)(x-1)}{x}$ • dom $f = \mathbb{R}\setminus\{0\}$

• segno di f

	−2		0		1		
+ +	•	− −	−	−	•	+ + +	x^2+x-2
− −		− −	0	+ +		+ +	x
− −	•	+ +	0	−	•	+ + + +	$f(x)$

• $\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to0^-} f(x) = +\infty$ $\lim_{x\to0^+} f(x) = -\infty$ $\lim_{x\to+\infty} f(x) = +\infty$.

• $\mathrm{dom}\, f' = \mathbb{R}\setminus\{0\}$

$$f'(x) = \frac{(2x+1)x - (x^2+x-2)}{x^2} = \frac{2x^2+\cancel{x}-x^2-\cancel{x}+2}{x^2} = \frac{x^2+2}{x^2} > 0 \quad \forall x \in \mathbb{R}\setminus\{0\}$$

• $\mathrm{dom}\, f'' = \mathbb{R}\setminus\{0\}$

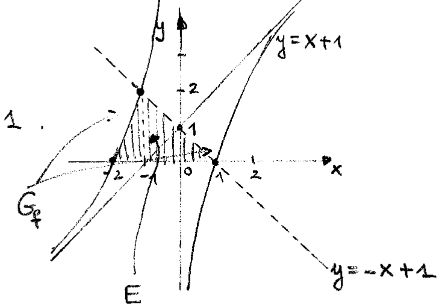
$$f''(x) = \frac{2\cancel{x}\cdot x^2 - (x^2+2)2\cancel{x}}{x^{\cancel{4}3}} = \frac{-4}{x^3}$$

$+\ +\ +\ 0\ -\ -\ -\ -\quad -x^3$

$\cup\quad 0\quad \cap\quad f(x)$

• <u>asintoti</u> : asintoto verticale $x = 0$;

asintoto obliquo per $x \to +\infty$ (per $x \to -\infty$) $\quad y = x+1$.



ii) ✓

iii) $\dfrac{x^2+x-2}{x} = -x+1 \iff \dfrac{x^2+\cancel{x}-2+x^2-\cancel{x}}{x} = 0$

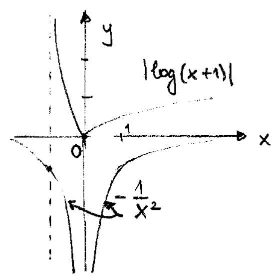
$$\iff \frac{2x^2-2}{x} = 0 \iff x=-1,\ x=1.$$

Allora

$$\text{area } E = \int_{-2}^{-1} f(x)\,dx + \frac{2\cdot 2}{2} = \int_{-2}^{-1}\left(x+1-\frac{2}{x}\right)dx + 2 =$$

$$= \left[\frac{x^2}{2} + x - 2\log|x|\right]_{-2}^{-1} + 2 = \left(\frac{1}{2}-1\right) - \cancel{2} + \cancel{2} + 2\log 2 + 2 = \underline{\underline{\frac{3}{2}+\log 4}}.$$

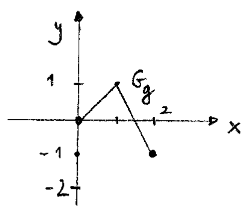
4) i)



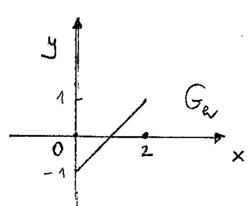
$|\log(x+1)| \geq -\dfrac{1}{x^2} \quad \underline{\underline{\forall x \in]-1,+\infty[\setminus\{0\}}}.$

ii) $\log_2 4 \leq x^2 \leq \log_3 27 \iff 2 \leq x^2 \leq 3 \iff \underline{\underline{x \in [-\sqrt{3}, -\sqrt{2}] \cup [\sqrt{2}, \sqrt{3}]}}$.

5) i) Esempio :



ii) Esempio :



6) $9 \cdot D_{9,4} = 9 \cdot \dfrac{9!}{5!} = \dfrac{9\cdot 9\cdot 8\cdot 7\cdot 6\cdot \cancel{5!}}{\cancel{5!}} = \underline{\underline{9\cdot 9\cdot 8\cdot 7\cdot 6}}$.

<u>FILA (B)</u> 1) $\displaystyle\int_0^3 \frac{3}{\sqrt{x+1}}\,dx = 3\left[\frac{(x+1)^{1/2}}{1/2}\right]_0^3 = 6\,(2-1) = \underline{6};$

$$\int_1^2\left(\frac{x^4-4xe^x}{x}\right)dx = \int_1^2 \left(x^3-4e^x\right)dx = \left[\frac{x^4}{4}-4e^x\right]_1^2 = \left(4-4e^2\right)-\left(\frac{1}{4}-4e\right) = \underline{4(e-e^2)+\frac{15}{4}};$$

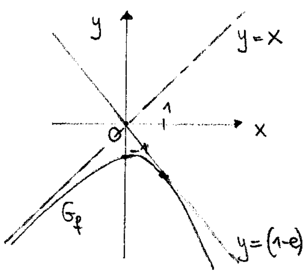
$$\int_{-2}^3\left(|x-2|-|x|\right)dx = \int_{-2}^0\left((2-x)+x\right)dx + \int_0^2\left((2-x)-x\right)dx + \int_2^3\left((x-2)-x\right)dx =$$

$$= \int_{-2}^0 2\,dx + \int_0^2 (2-2x)\,dx - \int_2^3 2\,dx = \left[2x\right]_{-2}^0 + \left[2x-x^2\right]_0^2 - \left[2x\right]_2^3 =$$

$$= 4 + (\cancel{4}-\cancel{4}) - (6-4) = \underline{2}.$$

□

2) i) $f(x) = x - e^x$ • dom $f = \mathbb{R}$ $\quad \lim_{x\to-\infty} f(x) = -\infty \quad \lim_{x\to+\infty} f(x) = -\infty$.



• dom $f' = \mathbb{R}$ $\quad f'(x) = 1-e^x \quad f'(x)=0 \iff x=0$;

$f(0) = -1$

+ + +	0	− − − −		$1-e^x$
↗	0	↘		$f(x)$

$x=0$ pt. di max loc forte (anzi, globale)

• dom $f'' = \mathbb{R}$ $\quad f''(x) = -e^x < 0 \Rightarrow f$ è concava su $\mathbb{R}$.

ii) Ovviamente $\lim_{x\to-\infty}\frac{f(x)}{x} = 1$, inoltre $\lim_{x\to-\infty}\left(f(x)-x\right) = \lim_{x\to-\infty}\left(-e^x\right) = 0$;

ne segue che $\underline{y=x}$ è asintoto obliquo, per $x\to-\infty$, della funzione f.

iii) Eq. retta tg. al grafico di f nel pt. $(1, 1-e)$ è:

$$y = (1-e) + (1-e)(x-1) \quad\Rightarrow\quad \underline{y=(1-e)x}.$$

Punto di intersezione delle due rette è l'origine.

□

3) i) $f(x) = \dfrac{x^2-x-2}{x} = \dfrac{(x-2)(x+1)}{x}$ • dom $f = \mathbb{R}\setminus\{0\}$

• segno di f

	−1		0		2		
+	•	− −		− −	•	+ + +	x^2-x-2
− − −		−	o	+ +		+ +	x
− −	•	+	o	− −	•	+ + +	$f(x)$

• $\lim_{x\to-\infty} f(x) = -\infty \quad \lim_{x\to0^-} f(x) = +\infty \quad \lim_{x\to0^+} f(x) = -\infty \quad \lim_{x\to+\infty} f(x) = +\infty$.

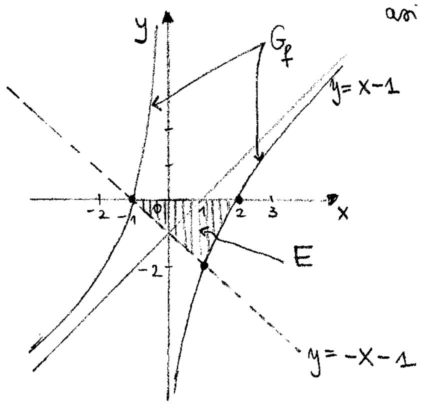
- dom $f' = \mathbb{R}\setminus\{0\}$

$$f'(x) = \frac{(2x-1)x - (x^2-x-2)}{x^2} = \frac{2x^2 \cancel{-x} - x^2 \cancel{+x} + 2}{x^2} = \frac{x^2+2}{x^2} > 0 \quad \forall x \in \mathbb{R}\setminus\{0\}$$

- dom $f'' = \mathbb{R}\setminus\{0\}$ $\quad f''(x) = -\frac{4}{x^3}$ $\qquad + + + \underset{0}{\circ} - - - \quad -x^3$ $\qquad \cup \quad \cap \quad f$

- asintoti: asintoto verticale $x = 0$;

asintoto obliquo per $x \to +\infty$ (per $x \to -\infty$) $y = x - 1$



ii) ✓

iii) $\frac{x^2-x-2}{x} = -x-1 \iff \frac{x^2 \cancel{-x} - 2 + x^2 \cancel{+x}}{x} = 0 \iff \frac{2(x^2-1)}{x} = 0$

$\iff x = -1,\ x = 1.$

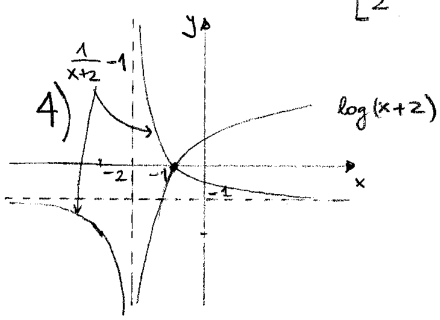
Allora

$$\text{area } E = \frac{2 \cdot 2}{2} + \int_1^2 (0 - f(x))\,dx = 2 - \int_1^2 \left(x - 1 - \frac{2}{x}\right) dx =$$

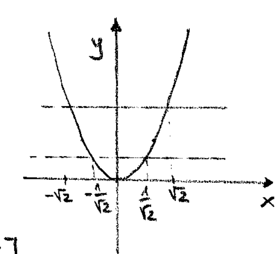
$$= 2 - \left[\frac{x^2}{2} - x - 2\log x\right]_1^2 = 2 - \left[\cancel{2} - \cancel{2} - 2\log 2 - \frac{1}{2} + 1\right] = \underline{\underline{\frac{3}{2} + \log 4.}}$$

□

4)



i) $\log(x+2) \le \frac{1}{x+2} - 1 \iff \underline{\underline{-2 < x \le -1}}.$

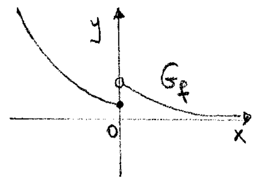


ii) $\log_3 \sqrt{3} \le x^2 \le \log_4 16 \iff \frac{1}{2} \le x^2 \le 2$

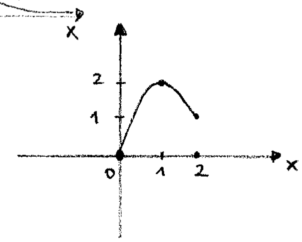
$\iff \underline{\underline{x \in \left[-\sqrt{2}, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \sqrt{2}\right]}}.$

□

5) i) Esempio:



ii) Esempio:



6) $6\cdot D_{9,5} = 6\cdot\frac{9!}{4!} = \frac{6\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot \cancel{4!}}{\cancel{4!}} = \underline{\underline{6\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5}}$.