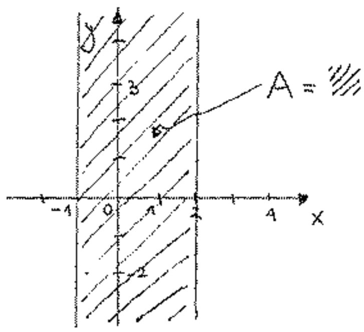
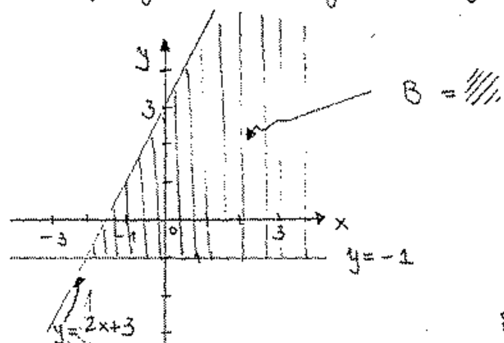
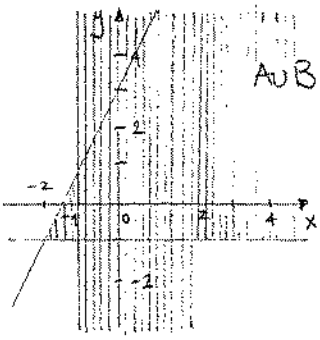
Esame scritto di ANALISI MATEMATICA – Rovereto, 6 luglio 2005

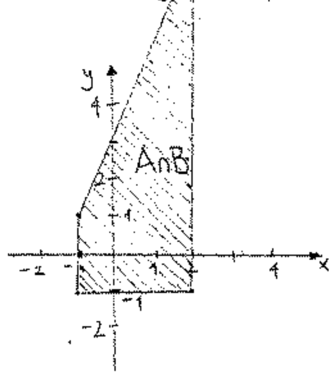
1) i) $A=\{(x,y)\in\mathbb{R}^2 : -1\le x\le 2\}$ $\qquad B=\{(x,y)\in\mathbb{R}^2 : -1\le y\le 2x+3\}$.

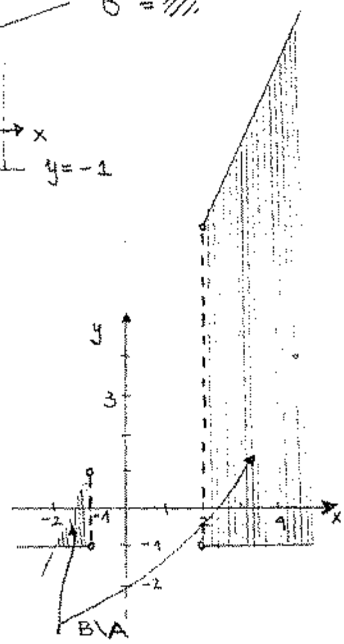




ii)

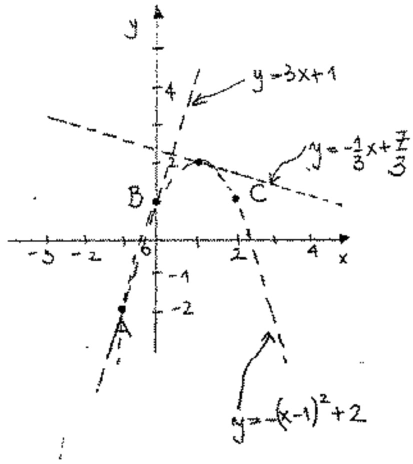






iii) $\text{area}(A\cap B)=\dfrac{2+8}{2}\cdot 3=\underline{\underline{15}}.$

2)



i) $y=ax^2+bx+c$ con a,b,c da determinare imponendo che A, B e C appartengono alla parabola. Allora si ha

$$\begin{cases} -2=a-b+c & (\text{per } A)\\ 1=c & (\text{per } B)\\ 1=4a+2b+c & (\text{per } C).\end{cases}$$

Da questo segue immediatamente $a=-1$, $b=2$, $c=1$; allora

$$\underline{\underline{y=-x^2+2x+1}}=\underline{\underline{-(x-1)^2+2}}$$

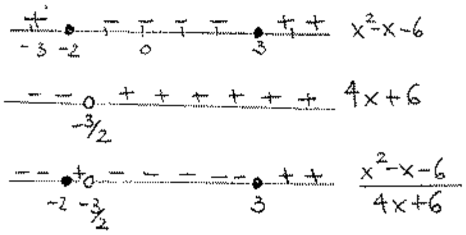
ii) Eq. della retta passante per i punti A e B : $\dfrac{y-(-2)}{x-(-1)}=\dfrac{1-(-2)}{0-(-1)}\iff$

$$\frac{y+2}{x+1}=3 \iff \underline{\underline{y=3x+1}}$$

iii) Eq. rette perpendicolari alla retta $y=3x+1$ sono $y=-\frac{1}{3}x+k$. Imponendo che tale retta passi per il pt. $(1,2)$ si ha $2=-\frac{1}{3}+k$, ossia $k=\frac{7}{3}$, e quindi $\underline{\underline{y=-\frac{1}{3}x+\frac{7}{3}}}$.

3) i) $\dfrac{x^2+3x}{4x+6} \geqslant 1 \iff \dfrac{x^2+3x}{4x+6} - 1 \geqslant 0 \iff \dfrac{x^2+3x-4x-6}{4x+6} \geqslant 0 \iff$

$$\frac{x^2-x-6}{4x+6} \geqslant 0$$

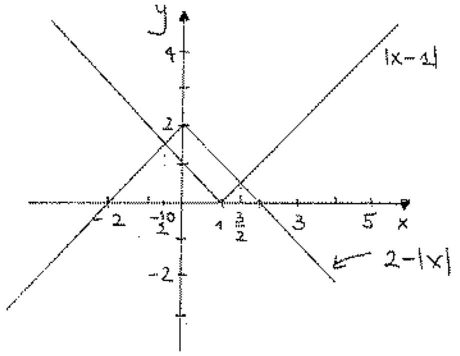


$$\underline{\underline{x \in \left[-2, -\tfrac{3}{2}\right[\cup \left[3, +\infty\right[}}$$

ii) $\log(2x+3) > \log(x^2) \iff \begin{cases} 2x+3>0 \\ x^2 > 0 \\ 2x+3 > x^2 \end{cases}$ (affinché sia def. il membro sinistro) (" il membro destro) (per monotonia della funzione log)

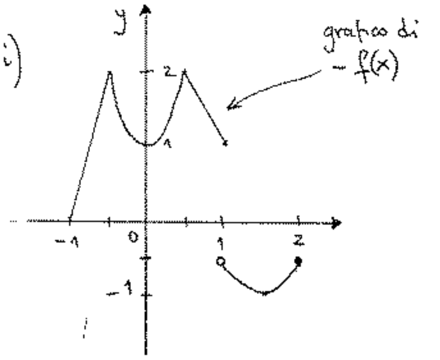
$$\iff \begin{cases} x > -3/2 \\ x \neq 0 \\ x^2-2x-3<0 \end{cases} \iff \begin{cases} x > -3/2 \\ x \neq 0 \\ -1<x<3 \end{cases} \implies \underline{\underline{x \in \left]-1, 3\right[\setminus \{0\}}}.$$

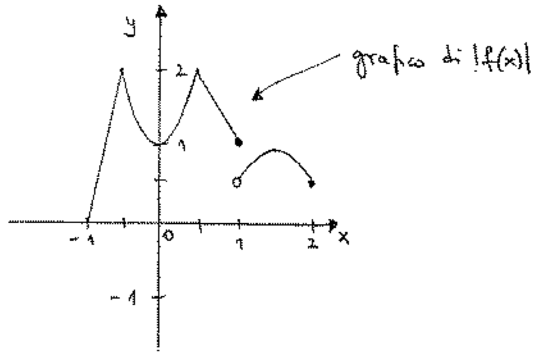
iii)



$$|x-1| \leq 2-|x| \iff \underline{\underline{x \in \left[-\tfrac{1}{2}, \tfrac{3}{2}\right]}}.$$

4) i)





ii) $\lim\limits_{x\to 1^-} f(x) = f(1) = -1 \qquad \lim\limits_{x\to 1^+} f(x) = \frac{1}{2}.$

iii) $\displaystyle\int_{-1}^{2} f(x)dx < 0$ poiché $\displaystyle\int_{-1}^{1} f(x)dx < 0$, $\displaystyle\int_{1}^{2} f(x)dx > 0$ e

$$\left|\int_{-1}^{1} f(x)dx\right| > \int_{1}^{2} f(x)dx.$$

5) $f(x) = \dfrac{x+1}{x^2+3}$: $\operatorname{dom} f = \mathbb{R}$; $f(x) > 0$ se $x > -1$
$f(-1) = 0$
$f(x) < 0$ se $x < -1$.

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to+\infty} f(x) = 0$; Inoltre f è continua su $\mathbb{R}$ (rapporto di funz. continue e $x^2+3\neq 0\ \forall x\in\mathbb{R}$)

$\operatorname{dom} f' = \mathbb{R}$; $\quad f'(x) = \dfrac{x^2+3-(x+1)2x}{(x^2+3)^2} = \dfrac{-x^2-2x+3}{(x^2+3)^2}$

pt. critici $x=-3$, $x=1$ e si ha

$- \quad -3 \quad + \quad + \quad 0 \quad + \quad 1 \quad - \quad -$ $\qquad -x^2-2x+3$

$\searrow \qquad \nearrow \qquad \searrow$ $\qquad f(x)$

$f(-3) = -\frac{1}{6} \qquad f(1) = \frac{1}{2}$

$x=-3$ pt. di min loc. forte
$x=1$ pt. di max loc. forte

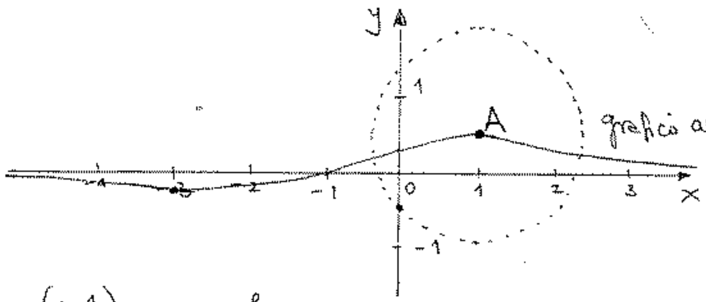


grafico approssimativo di f

ii) $A = \left(1, \frac{1}{2}\right) = \max\limits_{\mathbb{R}} f$

Sia $B = \left(0, -\frac{1}{2}\right)$. Allora $d = \operatorname{dist}(A,B) = \sqrt{(1-0)^2 + \left(\frac{1}{2}+\frac{1}{2}\right)^2} = \sqrt{2}$

iii) Eq. circonferenza centrata in A di raggio d è

$$\underline{\underline{(x-1)^2 + \left(y-\tfrac{1}{2}\right)^2 = 2}}.$$

6) $C_{35,2} = \dfrac{35!}{2!\,33!} = \dfrac{35\cdot 34}{2} = 35\cdot 17 = \underline{\underline{595}}.$