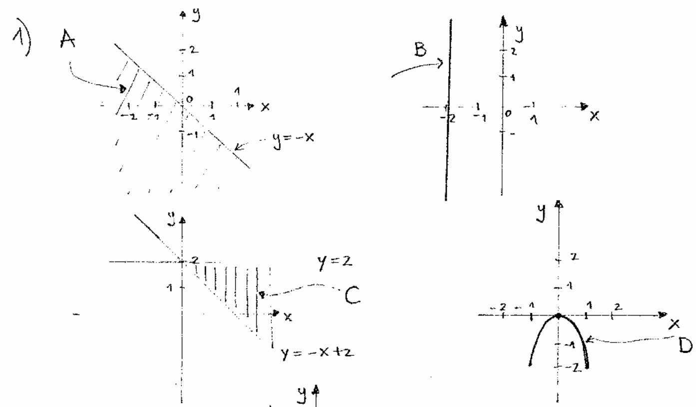
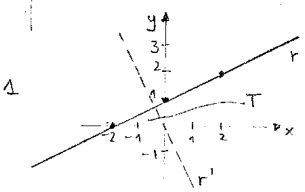
Verifica Settimanale del Percorso di Analisi Matematica (11–15/10/2004)

1)



□

2) a) $y = \frac{1}{2}x + 1$



b) pendenza di r' è -2 : allora $y = -2x$ è la retta r' cercata.

c) La regione T è un triangolo : se $(\bar{x}, \bar{y})$ è il pt. di intersezione di r e r' allora area $T = \frac{\text{base} \times \text{altezza}}{2} = \frac{\cancel{2} \cdot \bar{y}}{\cancel{2}} = \bar{y}$.

Abbiamo $\frac{1}{2}x + 1 = -2x \iff \frac{5}{2}x = -1 \iff \bar{x} = -\frac{2}{5}$

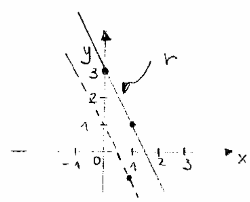
$$\Longrightarrow \bar{y} = -2\left(-\frac{2}{5}\right) = \frac{4}{5}$$

Quindi area $T = \frac{4}{5}$. □

3) a) $P_1 = (x_1, y_1) = (0, 3)$

$P_2 = (x_2, y_2) = (1, 1)$ $\rightsquigarrow$ $\frac{y-1}{3-1} = \frac{x-1}{0-1}$

$\rightsquigarrow$ $\frac{y-1}{2} = -(x-1) \Longrightarrow \underline{\underline{y = -2x + 3}}$



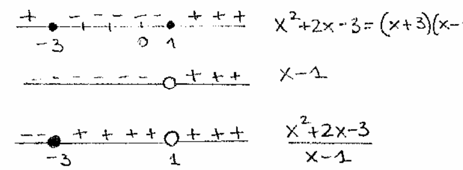
b) r' $y = -2x + q$: imponendo che $(1, -1)$ appartenga alla retta r' si ha $-1 = -2 + q$ ossia $q = 1$

c) Le equazioni delle rette perpendicolari alla retta r' sono

$$y=\frac{1}{2}x+q \quad , \quad q\in\mathbb{R}.$$

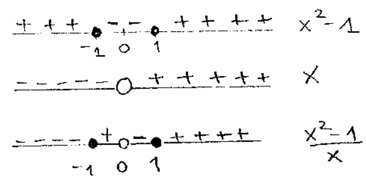
□

4) $A=\left\{x\in\mathbb{R}:\ \frac{x^2+2x-3}{x-1}>0\right\} = \,]-3,1[\,\cup\,]1,+\infty[$

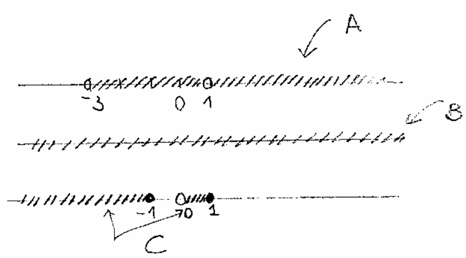


$B=\{x\in\mathbb{R}:\ 2x^2-3x+3>x^2-1\}=\{x\in\mathbb{R}:\ x^2-3x+4>0\}=\mathbb{R}$

$C=\left\{x\in\mathbb{R}:\ \frac{x^2-1}{x}\leq 0\right\}=\,]-\infty,-1]\cup\,]0,1]$



i)

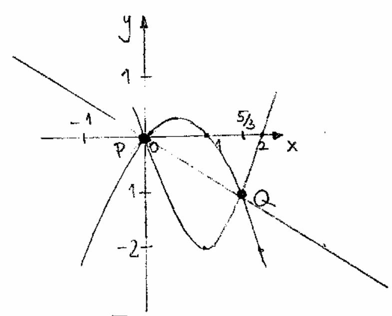


ii) $A\cup B=\mathbb{R}$; $A\cap B=A$; $B\cap C=C$; $B\setminus C=\,]-1,0]\cup\,]1,+\infty[$.

□

5) $y=-x^2+x=-x(x-1)$

$y=2x^2-4x=2x(x-2)$



$-x^2+x=2x^2-4x$

$\iff 3x^2-5x=0$

$\iff x(3x-5)=0 \iff x=0$ opp. $x=\frac{5}{3}$

$y=0$ $\qquad y=-\frac{25}{9}+\frac{5}{3}=\frac{-25+15}{9}=-\frac{10}{9}$

L'eq. cercata è della forma $y=mx$ (poiché $q=0$ dovendo passare la retta per l'origine). Imponendo che $(x,y)=\left(\frac{5}{3},-\frac{10}{9}\right)$ appartenga alla retta si ottiene $m=-\frac{2}{3}$. Dunque $y=-\frac{2}{3}x$. ■