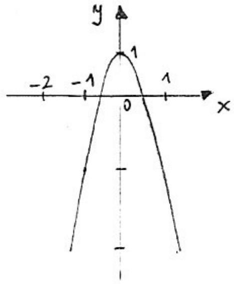
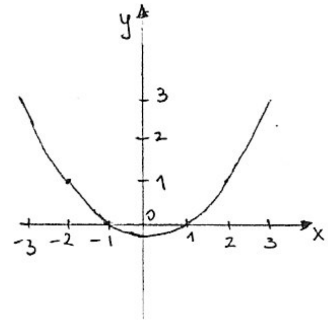
1. Verifica settimanale ANALISI MATEMATICA (18 - 22/10/2004)

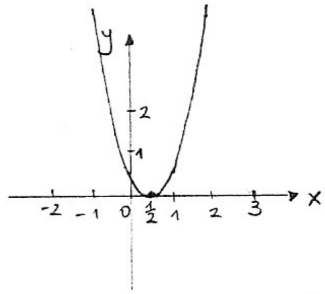
1) i) $y = -3x^2 + 1$



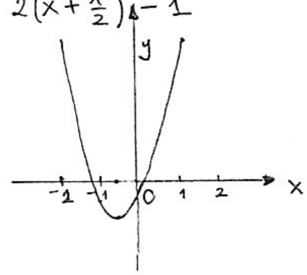
ii) $y = \frac{1}{3}x^2 - \frac{1}{3}$



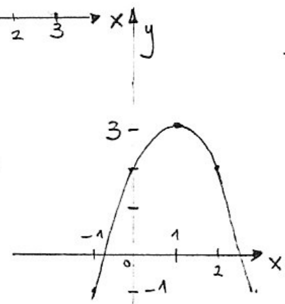
iii) $y = 2\left(x - \frac{1}{2}\right)^2$



iv) $y = 2\left(x + \frac{1}{2}\right)^2 - 1$

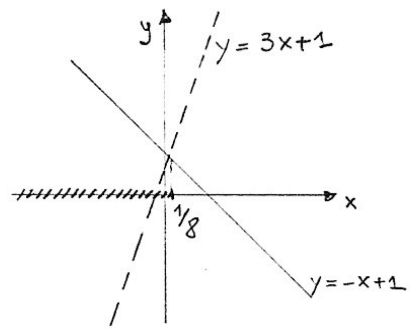


v) $y = -x^2 + 2x + 2 = -(x-1)^2 + 3$



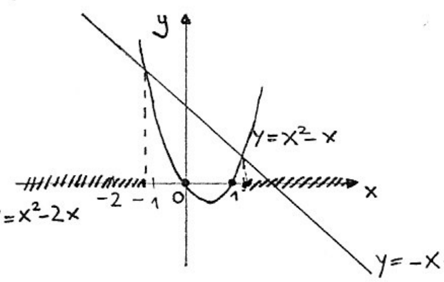
) i) $3x + \frac{1}{2} < -x + 1$

$\iff \quad 4x < \frac{1}{2} \quad \iff \quad \underline{\underline{x < \frac{1}{8}}}$



ii) $x^2 - x \geq -x + 2$

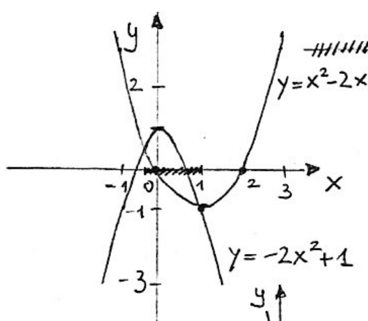
$\iff \quad x^2 \geq 2 \quad \iff \quad \underline{\underline{x \leq -\sqrt{2} \quad o \quad x \geq \sqrt{2}}}$



iii) $x^2 - 2x \leq -2x^2 + 1$

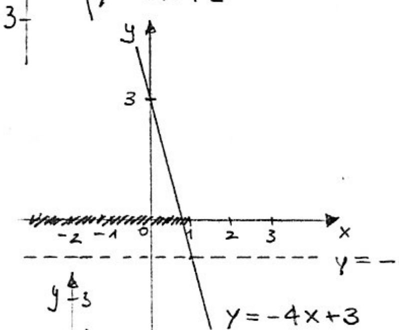
$\iff \quad 3x^2 - 2x - 1 \leq 0$

$\iff \quad -\frac{1}{3} \leq x \leq 1$



iv) $-4x + 3 \geq -1$

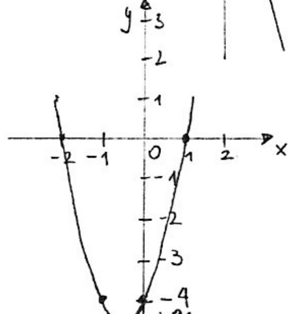
$\iff 4x \leq 4 \quad \iff \quad x \leq 1$



) i) $y = 2x^2 + 2x - 4 = 2\left(x + \frac{1}{2}\right)^2 - \frac{9}{2}$

se $k < -\frac{9}{2}$, $\nexists$ soluz. dell'eq.

$2x^2 + 2x - 4 = k$;



se $k=-\frac{9}{2}$ $\exists!$ soluzione dell'eq. $2x^2+2x-4=k$;

se $k>-\frac{9}{2}$ $\exists$ono 2 soluzioni dell'eq. $2x^2+2x-4=k$. $\square$

Risolviamo ora l'eq. analiticamente: $2x^2+2x-4-k=0$; si ha

$x_{1/2}=\dfrac{-2\pm\sqrt{4+8(4+k)}}{4}$; allora $\nexists$ soluzione se $4+8(4+k)<0$, ossia

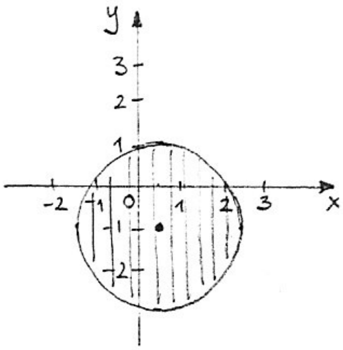
$k<-\frac{9}{2}$, $\exists!$ soluzione se $k=-\frac{9}{2}$, $\exists$ono due soluzioni se $k>-\frac{9}{2}$. $\square$

) $2x^2+2x-4=x-3 \iff 2x^2+x-1=0 \iff x_{1/2}=\dfrac{-1\pm\sqrt{1+8}}{4}=\dfrac{-1\pm3}{4}\begin{cases}\frac{1}{2}\\-1\end{cases}$

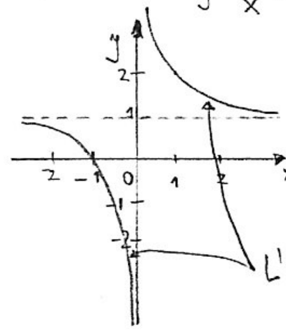
Poniamo $P=(-1,-4)$ e $Q=\left(\frac{1}{2},-\frac{5}{2}\right)$. Allora

$d(P,Q)=\sqrt{\left(-1-\frac{1}{2}\right)^2+\left(-4+\frac{5}{2}\right)^2}=\sqrt{\frac{9}{4}+\frac{9}{4}}=\underline{\underline{\frac{3}{2}\sqrt{2}}}$. $\blacksquare$

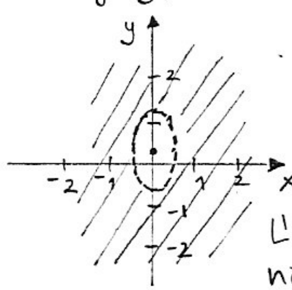
) i)



ii) $x(y-1)=1 \iff y=\frac{1}{x}+1$

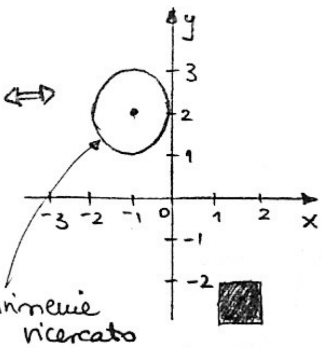


iii) $4x^2+\left(y-\frac{1}{3}\right)^2>1 \iff \dfrac{x^2}{\frac{1}{4}}+\left(y-\frac{1}{3}\right)^2>1$



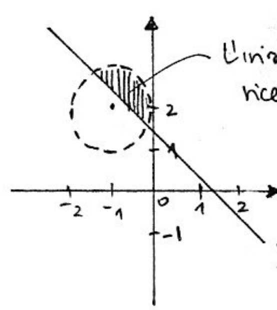
iv) $x^2+y^2+2x-4y+4=0 \iff$

$(x+1)^2+(y-2)^2=1$

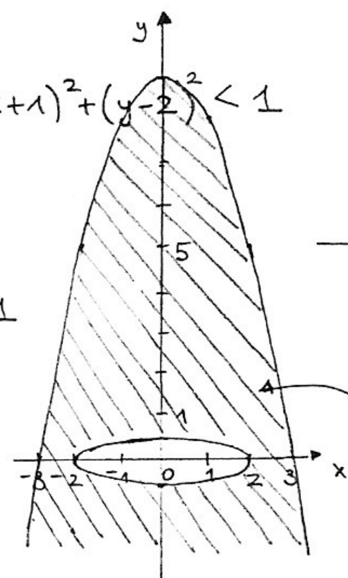


$\blacksquare$

) i) $\begin{cases}x^2+y^2+2x-4y+4<0 \iff (x+1)^2+(y-2)^2<1\\ y\geq -x+\frac{3}{2}\end{cases}$

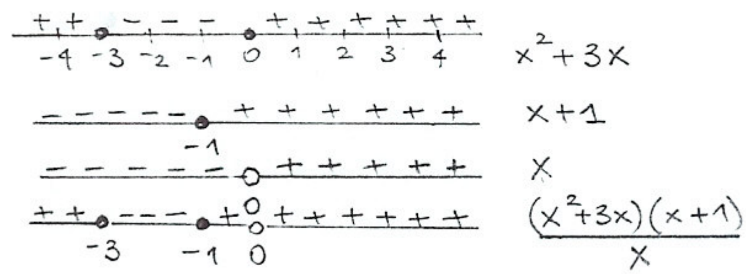


ii) $\begin{cases}\dfrac{x^2}{4}+4y^2\geq 1 \iff \dfrac{x^2}{4}+\dfrac{y^2}{\frac{1}{4}}\geq 1\\ y\leq -x^2+9\end{cases}$



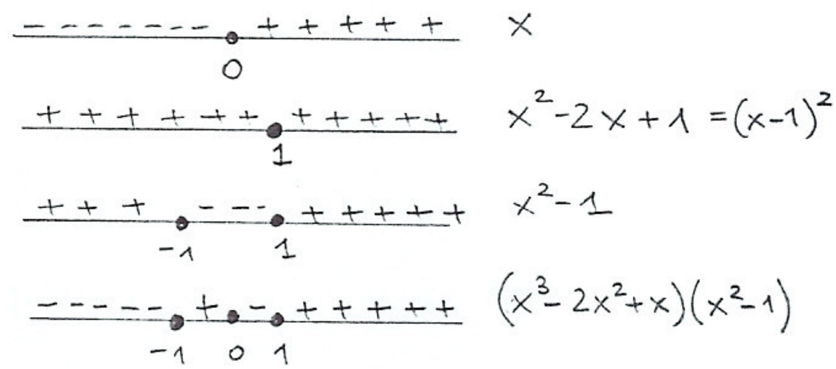
$\blacksquare$

i) $\dfrac{(x^2+3x)(x+1)}{x} < 0$:



$\underline{\underline{x \in \,]-3,-1[\,.}}$

ii) $(x^3-2x^2+x)(x^2-1) \geqslant 0$:

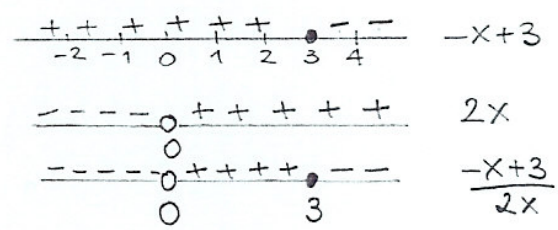


$\underline{\underline{x \in [-1,0] \cup [1,+\infty[\,.}}$

iii) $\dfrac{x+1}{x} - \dfrac{x-1}{2x} \geqslant 1$:

$$\iff \frac{2(x+1)-(x-1)-2x}{2x} \geqslant 0$$

$$\iff \frac{-x+3}{2x} \geqslant 0$$



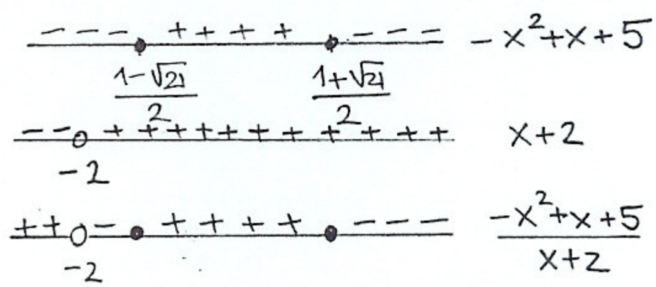
$\underline{\underline{x \in \,]0,3]\,.}}$

iv) $x+3 \geqslant \dfrac{1}{x+2} + 2x$:

$$\iff -x+3-\frac{1}{x+2} \geqslant 0$$

$$\iff \frac{-x(x+2)+3(x+2)-1}{x+2} \geqslant 0$$

$$\iff \frac{-x^2+x+5}{x+2} \geqslant 0$$



$\underline{\underline{x \in \,]-\infty,-2[\, \cup \left[\frac{1-\sqrt{21}}{2}, \frac{1+\sqrt{21}}{2}\right].}}$