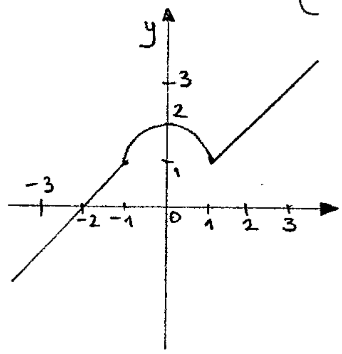
Verifica settimanale di ANALISI MATEMATICA (25-29/10/2004)

1. $f:\mathbb{R}\to\mathbb{R}$ $\quad f(x)=\begin{cases} x+2 & \text{se } x<-1 \\ -x^2+2 & \text{se } -1\le x\le 1 \\ x & \text{se } x>1. \end{cases}$

i)



ii) • f non è iniettiva: basta prendere $x_1=-1$, $x_2=1$;

si ha $x_1\neq x_2$ mentre $f(x_1)=f(x_2)=1$.

•• f è suriettiva; infatti $f(\mathbb{R})=\mathbb{R}$.

Infatti, per $y<1$ basta prendere $x=y-2$ e si ha $f(x)=y$.

Se $y=1$, basta prendere $x=-1$ (opp. $x=1$) e si ha $f(x)=y$.

Se $y>1$, basta prendere $x=y$ e si ha $f(x)=y$.

iii)

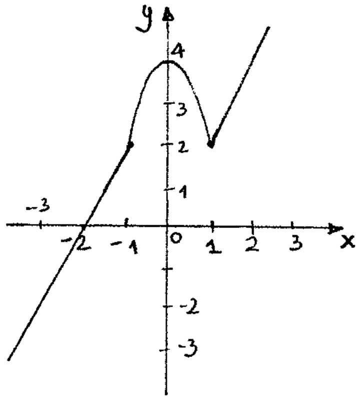


grafico di $2f(x)$

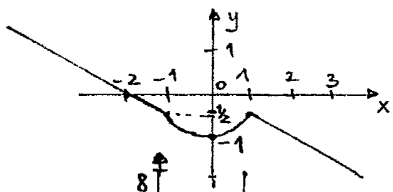
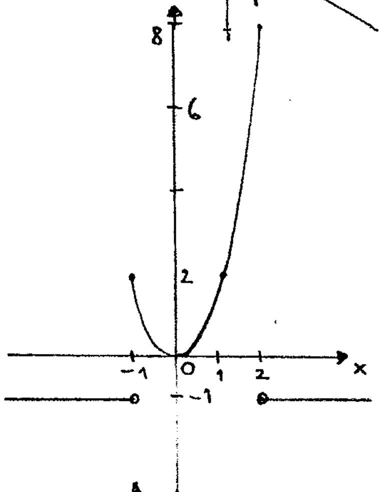


grafico di $-\frac{1}{2}f(x)$

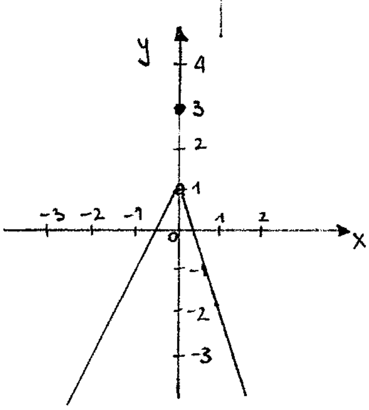
□

2) $f(x)=\begin{cases} 2x^2 & \text{se } -1\le x\le 2 \\ -1 & \text{se } x<-1 \text{ o } x>2 \end{cases}$



i) $\underline{\underline{f(\mathbb{R})=\{-1\}\cup[0,8]}}$.

$g(x)=\begin{cases} 2x+1 & \text{se } x<0 \\ 3 & \text{se } x=0 \\ -3x+1 & \text{se } x>0 \end{cases}$



$\underline{\underline{g(\mathbb{R})=]-\infty,1[\cup\{3\}}}$.

ii) $f(\mathbb{R})$ è un sottoinsieme limitato di $\mathbb{R}$: infatti $-1\le x\le 8$ $\forall x\in f(\mathbb{R})$.

$\min f(\mathbb{R})=-1$, $\max f(\mathbb{R})=8$.

iii)

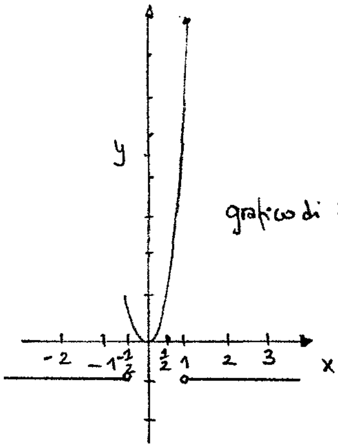


grafico di $x \mapsto f(2x)$.

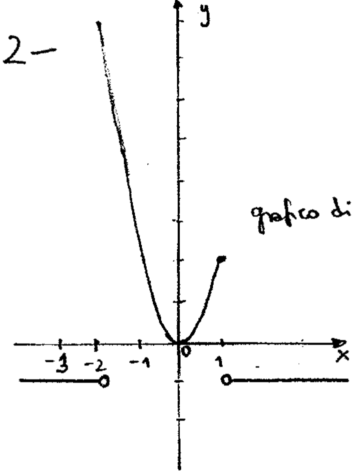


grafico di $x \mapsto f(-x)$.

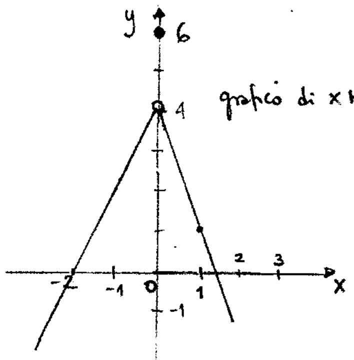


grafico di $x \mapsto g(x)+3$

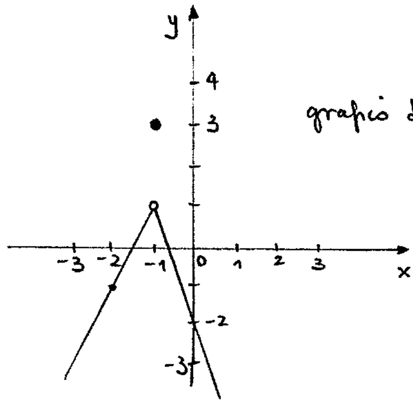
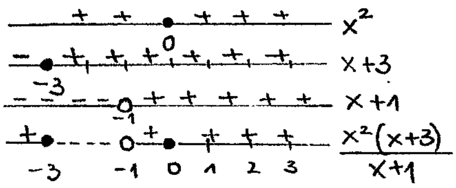


grafico di $x \mapsto g(x+1)$.

3) $A = \{x \in \mathbb{R} : x^2 - x - 2 < 0\} = \underline{]-1, 2[}$ $B = \left\{x \in \mathbb{R} : \dfrac{x^2(x+3)}{x+1} \le 0\right\} = [-3, -1[\cup \{0\}$

i) A è un insieme limitato, poiché

$$-1 < x \le 2 \quad \forall x \in A$$



B è limitato, poiché

$$-3 \le x \le 0 \quad \forall x \in B.$$

ii) $\nexists \min A$, $\nexists \max A$; $\min B = -3$, $\max B = 0$.

4) i) $f(x) = x-2$ $g(x) = x^2+3x$: $\underline{\mathrm{dom}(g \circ f) = \mathbb{R}}$ $(g \circ f)(x) = g(f(x)) = (x-2)^2 + 3(x-2)$

ossia $\underline{(g \circ f)(x) = x^2 - x - 2}$;

$\underline{\mathrm{dom}(f \circ g) = \mathbb{R}}$ $\underline{(f \circ g)(x) = f(g(x)) = x^2 + 3x - 2}$.

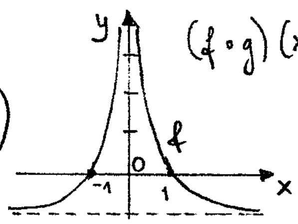
ii) $f(x) = \sqrt{x-2}$ $g(x) = x^2+3$ $\underline{\mathrm{dom}(g \circ f) = [2, +\infty[}$ $(g \circ f)(x) = g(f(x)) = (\sqrt{x-2})^2 + 3$

ossia $(g \circ f)(x) = \underline{x+1}$.

$\underline{\mathrm{dom}(f \circ g) = \mathbb{R}}$ (nota: $g(x) \ge 3$ e quindi $f(g(x))$ ha sempre senso)

$(f \circ g)(x) = f(g(x)) = \sqrt{x^2+3-2} = \underline{\sqrt{x^2+1}}$.

5)



$f(x) = \frac{1}{x^2} - 1$ $E = \{x \in \mathbb{R} : -1 < x < 0 \text{ o } 3 < x \le 5\}$, allora

$$f(E) = \left[\tfrac{1}{25} - 1, \tfrac{1}{9} - 1\right[\cup \,]0, +\infty[= \left[-\tfrac{24}{25}, -\tfrac{8}{9}\right[\cup \,]0, +\infty[.$$