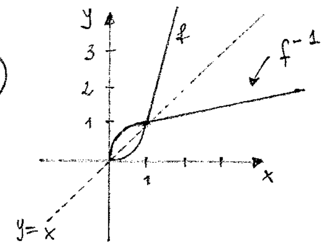
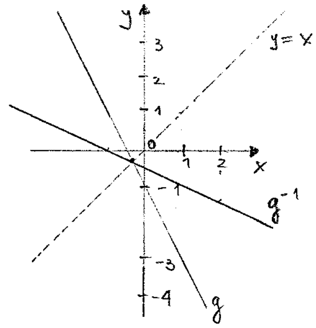
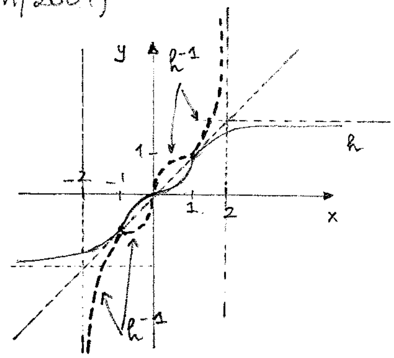
Verifica settimanale di ANALISI MATEMATICA (2-5/11/2004)

1)







$f^{-1}: [0,+\infty[\to [0,+\infty[$

$$f^{-1}(x) = \begin{cases} \sqrt{x} & \text{se } 0 \le x \le 1 \\ \frac{x}{4} + \frac{3}{4} & \text{se } x > 1 \end{cases}$$

$g^{-1}: \mathbb{R} \to \mathbb{R}$

$$g^{-1}(x) = -\frac{x}{2} - \frac{1}{2}$$

$h^{-1}:]-2,2[\to \mathbb{R}$

$$h^{-1}(x) = \begin{cases} -\frac{1}{x+2} & \text{se } x < -1 \\ \sqrt[3]{x} & \text{se } -1 \le x \le 1 \\ -\frac{1}{x-2} & \text{se } x > 1 \end{cases}$$

$\square$

2) i) $f+g: \mathbb{R} \to \mathbb{R}$ $\quad (f+g)(x) = f(x) + g(x) = \sqrt[3]{x} + 2x^2 + 1.$

$f-g: \mathbb{R} \to \mathbb{R}$ $\quad (f-g)(x) = f(x) - g(x) = \sqrt[3]{x} - 2x^2 - 1.$

$fg: \mathbb{R} \to \mathbb{R}$ $\quad (fg)(x) = f(x)\,g(x) = \sqrt[3]{x}\left(2x^2+1\right).$

$f/g: \mathbb{R} \to \mathbb{R}$ (nota: $g(x) \neq 0 \ \forall x \in \mathbb{R}$) $\quad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt[3]{x}}{2x^2+1}.$

$g \circ f: \mathbb{R} \to \mathbb{R}$ $\quad (g \circ f)(x) = g(f(x)) = 2\left(\sqrt[3]{x}\right)^2 + 1.$

$f \circ g: \mathbb{R} \to \mathbb{R}$ $\quad (f \circ g)(x) = f(g(x)) = \sqrt[3]{2x^2+1}.$

ii) $f(8) = \sqrt[3]{8} = 2$ $\quad$ (poiché $2^3 = 8$).

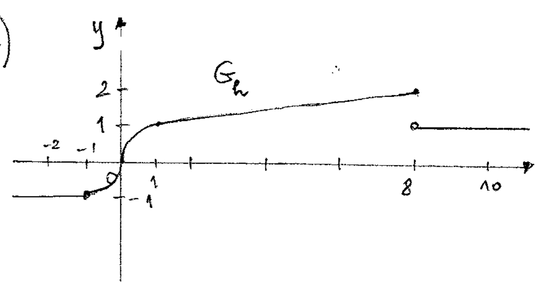
$f(-64) = \sqrt[3]{-64} = -4$ $\quad$ (poiché $(-4)^3 = -64$).

$(f-g)(\pi) = f(\pi) - g(\pi) = \sqrt[3]{\pi} - 2\pi^2 - 1.$

$(fg)(3) = f(3)\,g(3) = \sqrt[3]{3}\left(2 \cdot 3^2 + 1\right) = 19\sqrt[3]{3}.$

$(g \circ f)(-1) = g(f(-1)) = g(-1) = 3.$

iii)



$-1 \le h(x) \le 2 \quad \forall x \in \mathbb{R}$

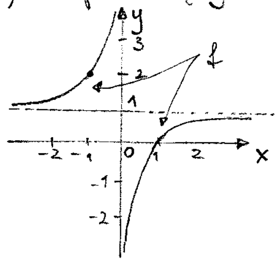
(nota: $f(x)$ è crescente e $-1 = f(-1) \le f(x) \le f(8) = 3 \quad \forall x \in [-1,8]$)

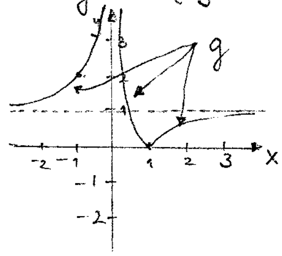
quindi h è limitata.

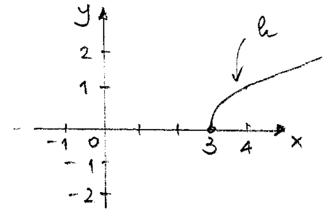
$\max_{\mathbb{R}} h = 2$ $\quad x=8$ è pt. di massimo

$\min_{\mathbb{R}} h = -1$ $\quad$ tutti i pt. $x \le -1$ sono pt. di minimo per h. $\square$

3) i) $\mathrm{dom}\, f = \mathbb{R}\setminus\{0\}$ $\qquad$ $\mathrm{dom}\, g = \mathbb{R}\setminus\{0\}$ $\qquad$ $\mathrm{dom}\, h = \{x\in\mathbb{R} : x \geq 3\} = [3, +\infty[$







ii) f non ha massimo nè minimo.

$\nexists \max\limits_{\mathbb{R}\setminus\{0\}} g \qquad \min\limits_{\mathbb{R}\setminus\{0\}} g = 0$ pt di minimo $x = 1$

$\nexists \max\limits_{[3,+\infty[} h \qquad \min\limits_{[3,+\infty[} h = 0$ pt. di minimo $x = 3$.

iii) f non è monotona su $\mathbb{R}\setminus\{0\}$. $f|_{]-\infty,0[}$, $f|_{]0,+\infty[}$ sono funzioni monotone.

g non è monotona su $\mathbb{R}\setminus\{0\}$. $g|_{]-\infty,0[}$, $g|_{]0,1]}$, $g|_{[1,+\infty[}$ sono funzioni monotone.

h è una funzione monotona su $[3,+\infty[$. $\square$

4) i) $2x + |x| \leq 1 \iff \begin{cases} 2x + x \leq 1 \\ x \geq 0 \end{cases}$ o $\begin{cases} 2x - x \leq 1 \\ x < 0 \end{cases}$

$\iff \begin{cases} x \leq 1/3 \\ x \geq 0 \end{cases}$ o $\begin{cases} x \leq 1 \\ x < 0 \end{cases}$

Soluzione: $x \in]-\infty, 1/3]$.

ii) $|x + |x|| > 3 \iff x + |x| < -3$ o $x + |x| > 3$

$\iff \begin{cases} x + x < -3 \\ x \geq 0 \end{cases}$ o $\begin{cases} x - x < -3 \\ x < 0 \end{cases}$ o $\begin{cases} x + x > 3 \\ x \geq 0 \end{cases}$ o $\begin{cases} x - x > 3 \\ x < 0 \end{cases}$

$\iff \begin{cases} x < -3/2 \\ x \geq 0 \end{cases}$ o $\begin{cases} 0 < -3 \\ x < 0 \end{cases}$ o $\begin{cases} x > 3/2 \\ x \geq 0 \end{cases}$ o $\begin{cases} 0 > 3 \\ x < 0 \end{cases}$

impossibile $\qquad$ impossibile $\qquad\qquad\qquad$ impossibile

Soluzione: $x \in]3/2, +\infty[$.

iii) $|3x + 4| \leq 2 \iff -2 \leq 3x + 4 \leq 2 \iff -6 \leq 3x \leq -2$

$\iff -2 \leq x \leq -2/3$

Soluzione: $x \in [-2, -2/3]$.

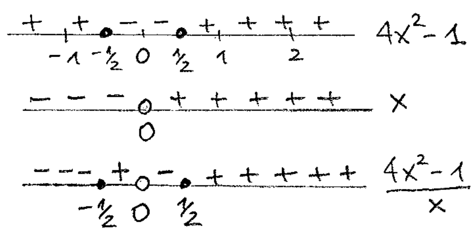
iv) $|x+x^2| \leq 0 \iff x+x^2=0 \iff x=0$ o $x=-1$.

Soluzione : $\underline{\underline{x \in \{-1, 0\}}}$.

$\square$

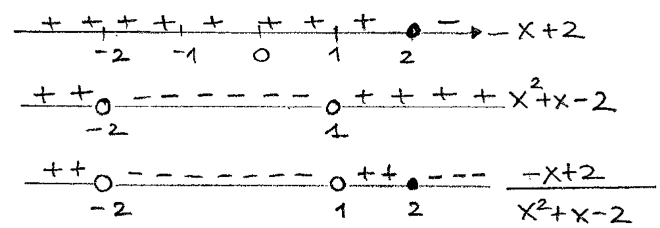
5) i) $A = \left\{ x \in \mathbb{R} : \frac{x^2-1}{x} + 3x < 0 \right\} = \underline{\underline{\left]-\infty, -\frac{1}{2}\right[\cup \left]0, \frac{1}{2}\right[}}$.

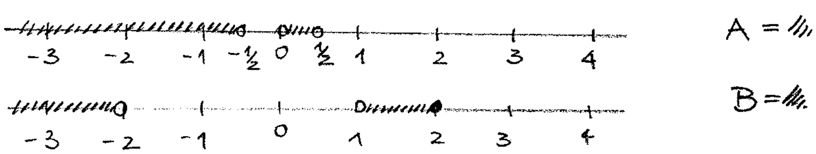
: $\frac{x^2-1+3x^2}{x} < 0 \iff \frac{4x^2-1}{x} < 0$



$B = \left\{ x \in \mathbb{R} : \frac{x^2}{x^2+x-2} - 1 \geq 0 \right\} = \underline{\underline{\left]-\infty, -2\right[\cup \left]1, 2\right]}}$.

: $\frac{x^2}{x^2+x-2} - 1 = \frac{\cancel{x^2} - \cancel{x^2} - x + 2}{x^2+x-2}$

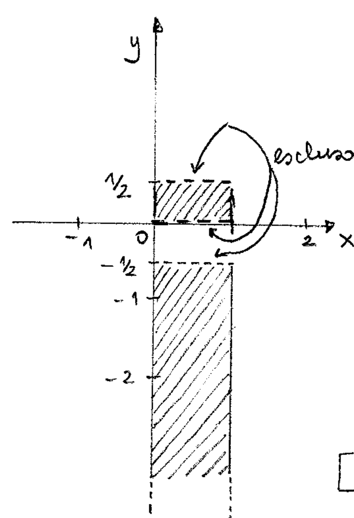




ii) $A \cup B = \underline{\underline{\left]-\infty, -\frac{1}{2}\right[\cup \left]0, \frac{1}{2}\right[\cup \left]1, 2\right]}}$.

$A \cap B = \underline{\underline{\left]-\infty, -2\right[}}$

$[0,1] \times A = \{ (x,y) \in \mathbb{R}^2 : x \in [0,1], y \in A \}$



$\square$