Verifica Settimanale di ANALISI MATEMATICA (15 – 19 novembre 2004)

1) i) $f(x) = \dfrac{x}{\sqrt{x^2-1}}$ $\quad$ $\mathrm{dom} f = \{x \in \mathbb{R} : x^2 - 1 > 0\} = \underline{\underline{]-\infty, -1[\cup]1, +\infty[}}$;

$g(x) = (1+x)\sqrt[3]{x^2-1}$ $\quad$ $\mathrm{dom}\, g = \underline{\underline{\mathbb{R}}}$ (poichè $\sqrt[3]{\ }$ è definita su tutto $\mathbb{R}$)

$h(x) = \log(x^2+3)$ $\quad$ $\mathrm{dom}\, h = \{x \in \mathbb{R} : x^2 + 3 > 0\} = \underline{\underline{\mathbb{R}}}$.

ii) $f(x) = \dfrac{x}{\log(x+1)}$ $\quad$ $\mathrm{dom} f = \{x \in \mathbb{R} : x+1 > 0,\ x+1 \neq 1\} = \underline{\underline{]-1, +\infty[\setminus \{0\}}}$.

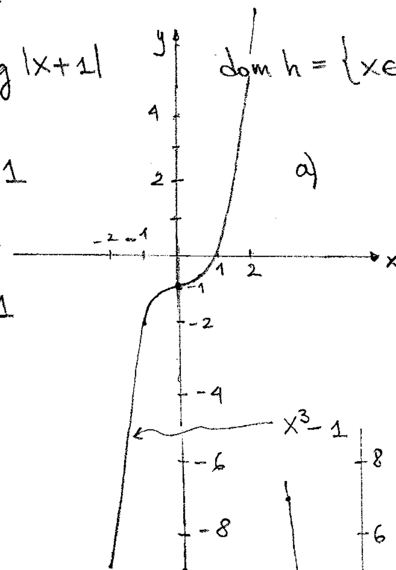
$g(x) = \dfrac{1+x}{e^{x^2-1}}$ $\quad$ $\mathrm{dom}\, g = \underline{\underline{\mathbb{R}}}$ (poichè e^x è definita $\forall x \in \mathbb{R}$ e è sempre $\neq 0$)

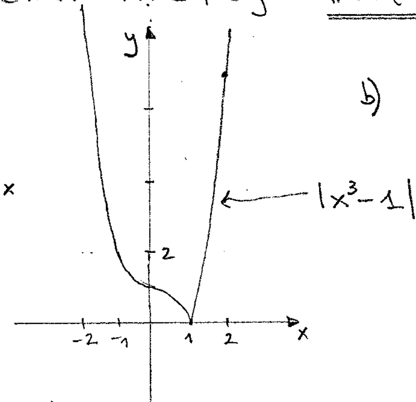
$h(x) = \log|x+1|$ $\quad$ $\mathrm{dom}\, h = \{x \in \mathbb{R} : x+1 \neq 0\} = \underline{\underline{\mathbb{R} \setminus \{-1\}}}$. $\square$

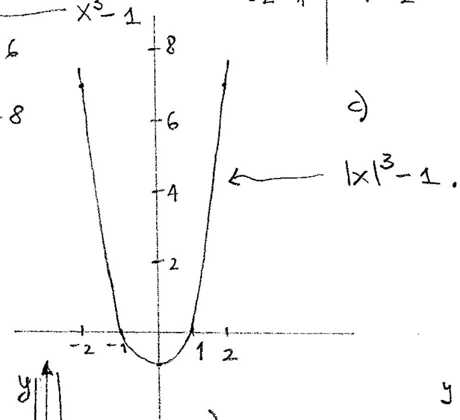
2) i) a) $x^3 - 1$

b) $|x^3-1|$

c) $|x|^3 - 1$





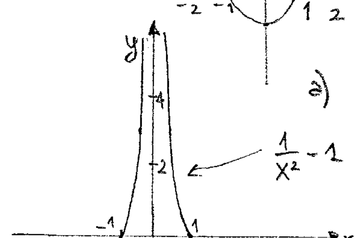


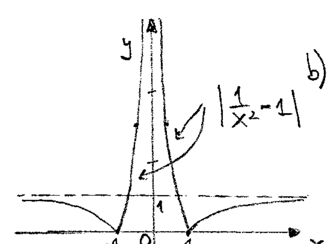
ii) a) $\dfrac{1}{x^2} - 1$

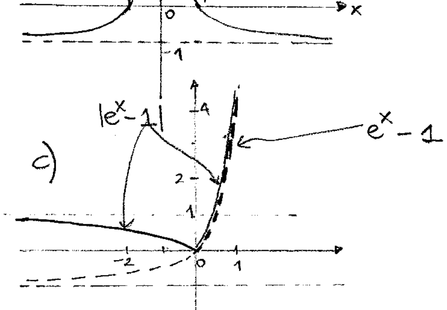
b) $\left|\dfrac{1}{x^2} - 1\right|$

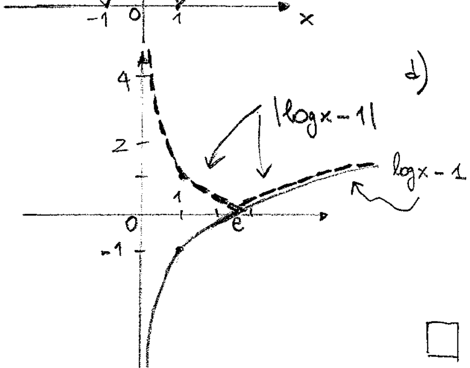
c) $|e^x - 1|$

d) $|\log x - 1|$









$\square$

3) i) $\dfrac{(x^2)^{3/2}\,x^{-1}\,\frac{1}{y}+yx^2}{(1+y^2)\,x^{2/5}} = \dfrac{x^3\,x^{-1}\,\frac{1}{y}+yx^2}{(1+y^2)\,x^{2/5}} = \dfrac{x^2\,\frac{1}{y}+yx^2}{(1+y^2)\,x^{2/5}} =$

$$= \frac{x^2\,\cancel{(1+y^2)}}{y\,\cancel{(1+y^2)}\,x^{2/5}} = \frac{x^{8/5}}{y} = \underline{\underline{x^{8/5}\,y^{-1}}}.$$

ii) $\dfrac{x\,z^{3/2}\,z^{-1/2}+x^{-1}}{x^2z+1} = \dfrac{xz+\frac{1}{x}}{x^2z+1} = \dfrac{1}{x}\dfrac{\cancel{(x^2z+1)}}{\cancel{(x^2z+1)}} = \dfrac{1}{x} = \underline{\underline{x^{-1}}}.$

iii) $\dfrac{\left((x^2)^{1/5}\,x^{-3/15}-x^{1/5}+z\right)y^4}{y^{-2}} = \left(x^{2/5-3/15}-x^{1/5}+z\right)y^6 = \left(x^{3/15}-x^{1/5}+z\right)y^6 =$

$$= \underline{\underline{zy^6}}.$$

$\square$

4) $f(-3) = 2^3 = 8$ $\qquad g\left(\frac{1}{8}\right) = \log_2 \frac{1}{8} = -3$ $\qquad \left(2^{-3} = \frac{1}{2^3} = \frac{1}{8}\right)$

$f(-2) = 2^2 = 4$ $\qquad g\left(\frac{1}{4}\right) = \log_2 \frac{1}{4} = -2$ $\qquad \left(2^{-2} = \frac{1}{2^2} = \frac{1}{4}\right)$

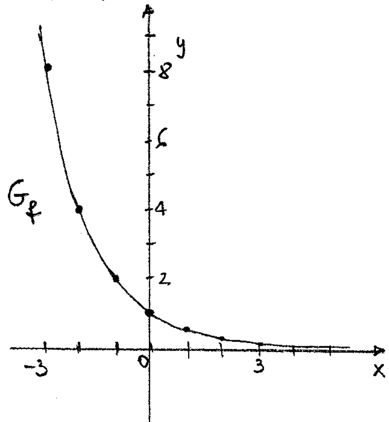
$f(-1) = 2^1 = 2$ $\qquad g\left(\frac{1}{2}\right) = \log_2 \frac{1}{2} = -1$

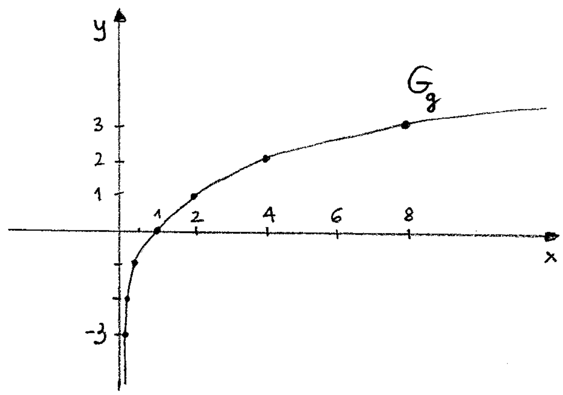
$f(0) = 2^0 = 1$ $\qquad g(1) = \log_2 1 = 0$

$f(1) = 2^{-1} = 1/2$ $\qquad g(2) = \log_2 2 = 1$

$f(2) = 2^{-2} = 1/4$ $\qquad g(4) = \log_2 4 = 2$

$f(3) = 2^{-3} = 1/8$ $\qquad g(8) = \log_2 8 = 3$.





$\square$

5) i) $\log_3 9 = \underline{\underline{2}}$ $\quad$ (infatti $3^2 = 9$) $\quad$ $\log_2 16 = \underline{\underline{4}}$ $\quad$ (infatti $2^4 = 16$) $\quad$ $\log e = \log_e e = \underline{\underline{1}}$.

ii) $10^x = 1000 \iff 10^x = 10^3 \iff \underline{\underline{x=3}}$.

$3^x = 1 \iff 3^x = 3^0 \iff \underline{\underline{x=0}}$.

$4^x = 2\cdot 3^x \iff \left(\frac{4}{3}\right)^x = 2 \iff x\log\frac{4}{3} = \log 2 \iff \underline{\underline{x = \frac{\log 2}{\log 4/3}}}$.

iii) $\log_3 x = 3 \quad \Rightarrow \quad x = 3^3 = \underline{\underline{27}}.$

$$\log_3 x = \log_3 2 - \log_3 (x+1) \quad \Rightarrow \begin{cases} x > 0 \\ x+1 > 0 \\ \log_3 x(x+1) = \log_3 2 \end{cases} \Leftrightarrow \begin{cases} x > 0 \\ x(x+1) = 2 \end{cases}$$

$$\Leftrightarrow \begin{cases} x > 0 \\ x^2 + x - 2 = 0 \end{cases} \Leftrightarrow \begin{cases} x > 0 \\ x_{1/2} = \frac{-1 \pm \sqrt{1+8}}{2} \end{cases}$$

$$\Rightarrow \underline{\underline{x = 1}}$$

$$\log x^2 \leq 1 \quad \Leftrightarrow \quad 0 < x^2 \leq e \quad \Leftrightarrow \quad \underline{\underline{x \in [-\sqrt{e}, \sqrt{e}] \setminus \{0\}}}.$$

iv) $\log e^x \geq 2 \quad \Leftrightarrow \quad \underline{\underline{x \geq 2}} \qquad$ (infatti $\log e^x = x \ \forall x \in \mathbb{R}$)

$$e^{\log x} \leq 2 \quad \Leftrightarrow \quad \begin{cases} x > 0 \\ x \leq 2 \end{cases} \qquad (\text{infatti } e^{\log x} = x \ \ \forall x > 0)$$

quindi $\forall x \in \underline{\underline{]0, 2]}}$.

$\square$