Verifica Settimanale di ANALISI MATEMATICA (29/11/04 — 3/12/04)

1) i) $\lim_{x\to 0} \frac{e^{2x}-1}{x} = \underline{\underline{2}}$ $\quad\left(\frac{0}{0}\right)$: $\frac{e^{2x}-1}{x} = \underbrace{\frac{e^{2x}-1}{2x}}_{\downarrow 1 \text{ (limite notevole)}} \cdot 2 \to 2$.

$\lim_{x\to 1^+} \frac{e^{2x}-1}{x-1} = \underline{\underline{+\infty}}$: $\frac{e^{2x}-1 \to e^2-1>0}{x-1 \to 0^+} \xrightarrow[x\to 1]{} +\infty$.

$\lim_{x\to +\infty} \frac{\log(x^2+1)}{\log(x^3+2)} = \underline{\underline{\frac{2}{3}}}$ $\quad\left(\frac{\infty}{\infty}\right)$: $\frac{\log(x^2+1)}{\log(x^3+2)} = \frac{\log\left[x^2\left(1+\frac{1}{x^2}\right)\right]}{\log\left[x^3\left(1+\frac{2}{x^3}\right)\right]} = \frac{\log x^2 + \log\left(1+\frac{1}{x^2}\right)}{\log x^3 + \log\left(1+2/x^3\right)} =$ ($\log\left(1+\frac{1}{x^2}\right) \to 0$, $\log\left(1+2/x^3\right) \to 0$)

$$= \frac{2\log x + \log\left(1+\frac{1}{x^2}\right)}{3\log x + \log\left(1+2/x^3\right)} \xrightarrow[x\to+\infty]{} \frac{2}{3} .$$

$\lim_{x\to 0} \frac{\log(1+3x)}{e^{2x}-1} = \underline{\underline{\frac{3}{2}}}$ $\quad\left(\frac{0}{0}\right)$: $\underbrace{\frac{\log(1+3x)}{3x}}_{\downarrow 1} \cdot \frac{3\cancel{x}}{2\cancel{x}} \cdot \underbrace{\frac{2x}{e^{2x}-1}}_{\downarrow 1} \xrightarrow[x\to 0]{} \frac{3}{2}$.

ii) $\lim_{x\to 1^+} \frac{\log(x-1)}{x-1} = -\infty$ $\quad\left(\frac{-\infty}{\text{infinitesimo positivo}}\right)$: non è una forma indeterminata!

$\lim_{x\to +\infty} \frac{e^{x-3x^2}+x}{x} = \underline{\underline{1}}$ $\quad\left(\frac{\infty}{\infty}\right)$: $\frac{e^{x-3x^2}+x}{x} \xrightarrow[x\to+\infty]{} 1$. ($e^{x-3x^2} \to 0$)

$\lim_{x\to -\infty} \frac{x^5+1}{e^{x-x^3}} = \underline{\underline{0}}$ $\quad\left(\frac{\infty}{\infty}\right)$: $\frac{x^5+1}{e^{x-x^3}} = \frac{x^5\left(1+1/x^5\right)}{e^{-x^3\left(1-1/x^2\right)}} \longrightarrow 0$ poiché $e^{-x^3} >> x^5$ per $x\to-\infty$

$\lim_{x\to 2^+} \frac{x+1}{x^2-4} = \underline{\underline{+\infty}}$ $\quad\left(\frac{3}{\text{infinitesimo positivo}}\right)$: non è una forma indeterminata! □

2) $f(x) = \frac{x}{x-2}$

Asintoto verticale $x=2$ $\quad\left(\lim_{x\to 2^+} f(x) = +\infty,\ \lim_{x\to 2^-} f(x) = -\infty\right)$

Asintoto orizzontale per $x\to+\infty$ $\quad \underline{y=1}$ $\quad\left(\lim_{x\to+\infty} f(x) = 1\right.$

(" per $x\to-\infty$) $\quad\left.\lim_{x\to-\infty} f(x) = 1\right)$

$g(x) = \frac{x^2+1}{x-1}$

Asintoto verticale $x=1$ $\quad\left(\lim_{x\to 1^+} g(x) = +\infty,\ \lim_{x\to 1^-} g(x) = -\infty\right)$

Asintoto obliquo: $\underline{y = x+1}$ per $x\to+\infty$ (per $x\to-\infty$)

Infatti: $\frac{g(x)}{x} = \frac{x^2+1}{x^2-x} \longrightarrow 1$ per $x\to+\infty$ (per $x\to-\infty$)

$$g(x) - x = \frac{x^2+1}{x-1} - x = \frac{\cancel{x^2}+1-\cancel{x^2}+x}{x-1} = \frac{x+1}{x-1} \longrightarrow 1 \text{ per } x\to+\infty \ (\text{per } x\to-\infty)$$

$h(x) = \frac{x}{x^2+1}$

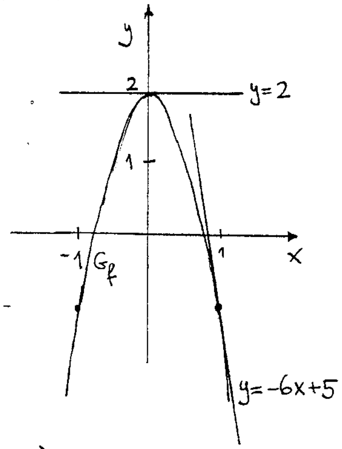
Asintoto orizzontale $y=0$ per $x\to+\infty$

" (per $x\to-\infty$)

$$\left(\lim_{x\to-\infty} h(x) = 0\ ,\ \lim_{x\to+\infty} h(x) = 0\right).$$

□

3) i) $f(x) = -3x^2+2$



$f'(x) = -6x \qquad f'(0)=0$

$\underline{y = 2 \text{ eq. retta tg al grafico di } f \text{ in } (0,2)}$,

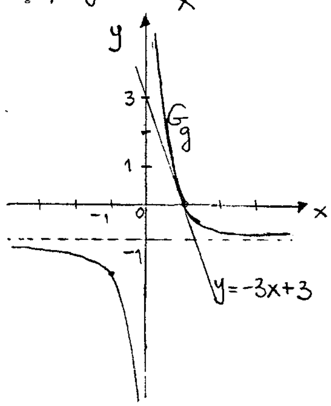
$f'(1) = -6$

$y = -1 - 6(x-1)$ quindi

$\underline{y = -6x+5 \text{ eq. retta tg. al grafico di } f \text{ in } (1,-1)}$.

$\blacksquare$

ii) $g(x) = \frac{1}{x^3} - 1$

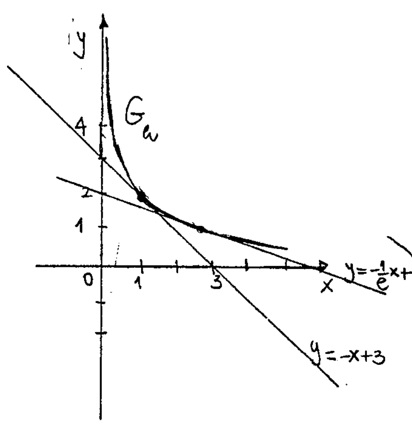


$g'(x) = -\frac{3}{x^4} \qquad g'(1) = -3$

$y = -3(x-1)$

$\underline{y = -3x+3 \text{ eq. retta tg. al grafico di } g \text{ in } (1,0)}$.

iii) $h(x) = -\log x + 2$



$h'(x) = -\frac{1}{x} \qquad h'(1) = -1$

$y = 2 - 1(x-1)$

$\underline{y = -x+3 \text{ eq. retta tg. al grafico di } h \text{ in } (1,2)}$

$h'(e) = -\frac{1}{e}$

$y = 1 - \frac{1}{e}(x-e)$

$\underline{y = -\frac{x}{e} + 2 \text{ eq. retta tg. al grafico di } h \text{ in } (e,1)}$.

$\square$

4) i) $\lim_{x\to-\infty} f(x) = 1 \qquad \lim_{x\to+\infty} f(x) = \frac{1}{2} \qquad \lim_{x\to 2^-} f(x) = \frac{1}{2} \qquad \lim_{x\to 2^+} f(x) = -\infty$

$\lim_{x\to 3^-} f(x) = 0 \qquad \lim_{x\to 3^+} f(x) = 2.$

ii) f è continua in $]-\infty, 2[\; \cup \;]2,3[\; \cup \;]3,+\infty[$.

iii) $]-\infty, -\frac{3}{2}]$, $[0,2]$, $]2,3]$ f è crescente

$[-\frac{3}{2}, 0]$ $\quad]3,+\infty[$ $\quad f$ è decrescente.

iv) $f' > 0$ su $]-\infty, -\frac{3}{2}[$, su $]0, 2[$, su $]2, 3[$

$f' < 0$ su $]-\frac{3}{2}, 0[$, su $]3, +\infty[$

$f' = 0$ in $x = -\frac{3}{2}$, $x = 0$. $\blacksquare$

i) $\lim_{x\to-\infty} g(x) = +\infty$ $\lim_{x\to+\infty} g(x) = \frac{1}{2}$ $\lim_{x\to 0^-} g(x) = +\infty$ $\lim_{x\to 0^+} g(x) = -1$

$\lim_{x\to 2^-} g(x) = 1$ $\lim_{x\to 2^+} g(x) = \frac{1}{2}$.

ii) g è continua in $]-\infty, -1[\cup]-1, 0[\cup]0, 2[\cup]2, +\infty[$.

iii) $]-\infty, -1[$ $[2, 3]$ g è decrescente

$]-1, 0[$ $]0, 2]$ $[3, +\infty[$ g è crescente.

iv) $g' < 0$ su $]-\infty, -1[$, su $]2, 3[$

$g' > 0$ su $]-1, 0[$, su $]0, 2[$, su $]3, +\infty[$

$g' = 0$ in $x = 3$. $\square$

5) i) $(2xe^x + x^2)' = 2e^x + 2xe^x + 2x = 2e^x(1+x) + 2x \quad \forall x \in \mathbb{R};$

$(x^3 \log x + \sqrt{x})' = 3x^2 \log x + x^3 \cdot \frac{1}{x} + \frac{1}{2\sqrt{x}} \quad \forall x > 0;$

$$\left(\frac{x^2e^x - 1}{e^x}\right)' = \frac{(x^2e^x-1)'e^x - (x^2e^x-1)e^x}{e^{2x}} = \frac{(2xe^x + x^2e^x)e^x - (x^2e^x-1)e^x}{e^{2x}} =$$
$$= \frac{2xe^x + x^2e^x - x^2e^x + 1}{e^x} \quad \forall x \in \mathbb{R};$$

ii) $$\left(\frac{x^3 - 2x + \log x}{x^2+1}\right)' = \frac{(x^3-2x+\log x)'(x^2+1) - (x^3-2x+\log x)(x^2+1)'}{(x^2+1)^2} =$$
$$= \frac{(3x^2 - 2 + \frac{1}{x})(x^2+1) - (x^3 - 2x + \log x)(2x)}{(x^2+1)^2} \quad \forall x > 0;$$

$$\left[\left(\frac{1}{x} + 3x\right)(e^x + x)\right]' = \left(-\frac{1}{x^2} + 3\right)(e^x + x) + \left(\frac{1}{x} + 3x\right)(e^x + 1) \quad \forall x \neq 0;$$

$$\left[\frac{(5x^4-3)2^x}{x+1}\right]' = \frac{[(5x^4-3)2^x]'(x+1) - (5x^4-3)2^x}{(x+1)^2} = \frac{[20x^3 \cdot 2^x + (5x^4-3)2^x \log 2](x+1) - (5x^4-3)2^x}{(x+1)^2}$$
$\forall x \neq -1;$

iii) $$\left(\frac{x\log x + 3}{x^2}\right)' = \frac{(x\log x + 3)'x^2 - (x\log x + 3)2x}{x^4} = \frac{(\log x + x \cdot \frac{1}{x})x^2 - 2x(x\log x + 3)}{x^4} =$$
$$= \frac{(\log x + 1)x^2 - 2x(x\log x + 3)}{x^4} \quad \forall x > 0;$$

$\left(xe^{x+1} - \frac{1}{x}\right)' = e^{x+1} + xe^{x+1} + \frac{1}{x^2} \quad \forall x \neq 0;$

$(e^2x^2\log x)' = e^2 2x \log x + e^2x^2 \cdot \frac{1}{x} = e^2x(2\log x + 1) \quad \forall x > 0.$ $\square$