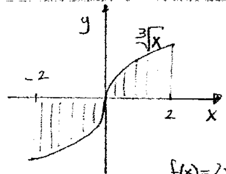
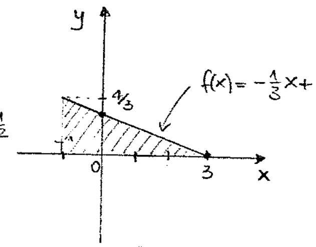
Verifica Settimanale di ANALISI MATEMATICA (13-17/12/04)

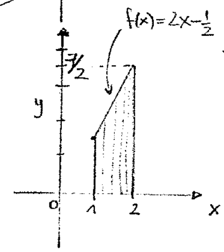
1) i) $\displaystyle\int_{-2}^{2} \sqrt[3]{x}\,dx = \underline{\underline{0}}$;



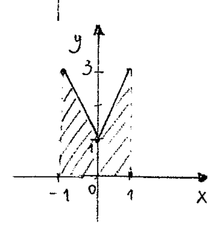
$\displaystyle\int_{-1}^{3} \left(-\tfrac{1}{3}x+1\right)dx = \frac{4}{3}\cdot\frac{\cancel{4}^{\,2}}{\cancel{2}} = \underline{\underline{\frac{8}{3}}}$;



$\displaystyle\int_{1}^{2} \left(2x-\tfrac{1}{2}\right)dx = \frac{\frac{3}{2}+\frac{7}{2}}{2}\cdot 1 = \underline{\underline{\frac{5}{2}}}$;

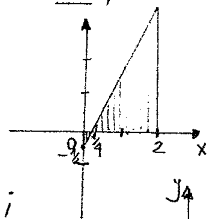


ii) $\displaystyle\int_{-1}^{1} (2|x|+1)\,dx = 2\int_{0}^{1}(2x+1)\,dx = \cancel{2}\left(\frac{1+3}{\cancel{2}}\right)\cdot 1 = \underline{\underline{4}}$;

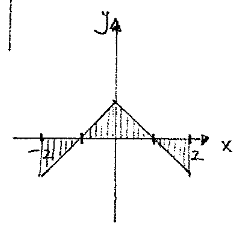


$\displaystyle\int_{0}^{2} \left(2x-\tfrac{1}{2}\right)dx = -\left(\frac{1}{4}\cdot\frac{1}{2}\cdot\frac{1}{2}\right) + \left(2-\frac{1}{4}\right)\cdot\frac{7}{2}\cdot\frac{1}{2}$

$\displaystyle = -\frac{1}{16} + \frac{49}{16} = \frac{48}{16} = \underline{\underline{3}}$;



$\displaystyle\int_{-2}^{2} (-|x|+1)\,dx = \underline{\underline{0}}$.



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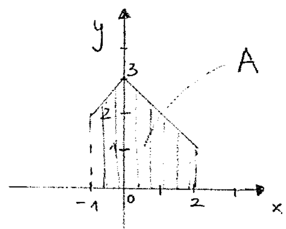
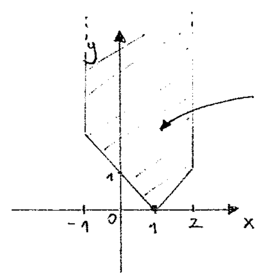
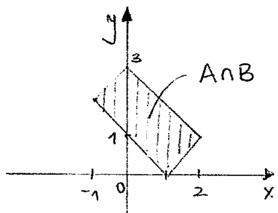
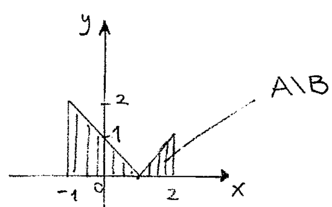
2) i)





ii)





iii) area A $= \displaystyle\int_{-1}^{2}(-|x|+3)\,dx$; area $(A\cap B) = \displaystyle\int_{-1}^{2}(-|x|+3)\,dx - \int_{-1}^{2}|x-1|\,dx$;

area $(A\setminus B) = \displaystyle\int_{-1}^{2}|x-1|\,dx$.

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3) $\displaystyle\int_{-1}^{2}(x^3+2x^2+1)\,dx = \left[\frac{x^4}{4}+2\frac{x^3}{3}+x\right]_{-1}^{2} = \left(4+\frac{16}{3}+2\right)-\left(\frac{1}{4}-\frac{2}{3}-1\right) = \underline{\underline{\frac{153}{12}}}\ \left(=\frac{51}{4}\right)$

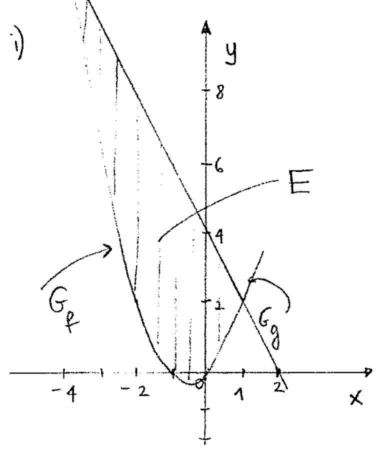
$\displaystyle\int_{1}^{2}(3e^x+x^{-3})\,dx = \left[3e^x - \frac{x^{-2}}{2}\right]_{1}^{2} = 3(e^2-e) - \left(\frac{1}{8}-\frac{1}{2}\right) = \underline{\underline{3(e^2-e)+\frac{3}{8}}}$.

4) $\displaystyle\int_1^2 \frac{3x^2+4x^3}{x}\,dx = \int_1^2 (3x+4x^2)\,dx = \left[3\frac{x^2}{2}+4\frac{x^3}{3}\right]_1^2 = \left(6+\frac{32}{3}\right)-\left(\frac{3}{2}+\frac{4}{3}\right) = \underline{\underline{\frac{83}{6}}}$.

$\displaystyle\int_0^1 e^{3x}\,dx = \left[\frac{e^{3x}}{3}\right]_0^1 = \underline{\underline{\frac{1}{3}\left(e^3-1\right)}}$.

$\displaystyle\int_1^2 \left[(x+1)^2-\frac{1}{x^2}\right]dx = \left[\frac{(x+1)^3}{3}+x^{-1}\right]_1^2 = \left(9+\frac{1}{2}\right)-\left(\frac{8}{3}+1\right) = \underline{\underline{\frac{35}{6}}}$. □

5) i)



$f(x)=g(x) \iff x^2+x=-2x+4$
$\iff x^2+3x-4=0 \implies x_{1/2} = \frac{-3\pm\sqrt{9+16}}{2}$
$= \frac{-3\pm 5}{2} \; \begin{smallmatrix} -4 \\ 1 \end{smallmatrix}$

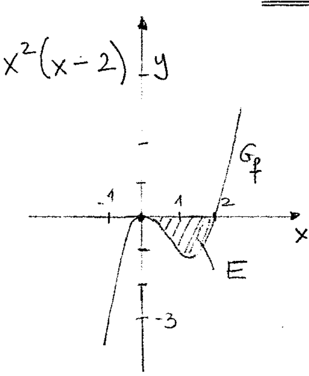
$$\text{area } E = \int_{-4}^{1} g(x)\,dx - \int_{-4}^{1} f(x)\,dx = \int_{-4}^{1} \left(g(x)-f(x)\right)dx =$$

$$= \int_{-4}^{1} \left(-2x+4-x^2-x\right)dx = \int_{-4}^{1}\left(-3x+4-x^2\right)dx$$

$$= \left[-3\frac{x^2}{2}+4x-\frac{x^3}{3}\right]_{-4}^{1} = \left(-\frac{3}{2}+4-\frac{1}{3}\right)-\left(-\frac{3\cdot\cancel{16}^{\,8}}{\cancel{2}}-16+\frac{64}{3}\right)$$

$$= \underline{\underline{21-\frac{1}{6}}}\left(=\frac{125}{6}\right).$$

ii) $f(x) = x^3-2x^2 = x^2(x-2)$



+ +	0	+ +			x^2
- - - -		- - - -	2	+ +	$x-2$
- - - -	0	- - -	2	+ +	$f(x)$

$f'(x) = 3x^2-4x = x(3x-4)$

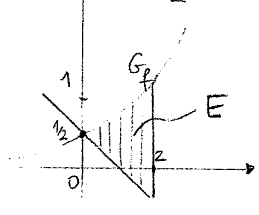
- - - -	0	+ + +		+ + +	x
- - - -		- - -	$\frac{4}{3}$	+ +	$3x-4$
+ + +	0	- - -	$\frac{4}{3}$	+ +	$f'(x)$
↗		↘		↗	

$f(0)=0 \quad f\left(\frac{4}{3}\right) = \frac{16}{9}\left(\frac{4}{3}-2\right) = -\frac{16}{9}\cdot\frac{2}{3}$
$= -\frac{32}{27}$

$$\text{area } E = -\int_0^2 (x^3-2x^2)\,dx = -\left[\frac{x^4}{4}-2\frac{x^3}{3}\right]_0^2 =$$

$$= -\left[4-2\cdot\frac{8}{3}\right] = -\left[\frac{12-16}{3}\right] = \underline{\underline{\frac{4}{3}}}.$$

iii)



$$\text{area } E = \int_0^2 \left(f(x)-g(x)\right)dx = \int_0^2 \left(\frac{e^x}{2}+x-\frac{1}{2}\right)dx = \left[\frac{e^x}{2}+\frac{x^2}{2}-\frac{1}{2}x\right]_0^2 =$$

6) i) $\sum_{k=0}^{11} (2k+1)$; $\sum_{k=1}^{15} \frac{1}{k(k+1)}$;

ii) $\sum_{k=1}^{6} \frac{2k}{3^k}$; $\sum_{k=1}^{6} \left(\frac{2}{3}\right)^k$.

iii) $\sum_{k=2}^{5} \frac{3k}{1+k^2} = \frac{6}{5} + \frac{9}{10} + \frac{12}{17} + \frac{15}{26}$;

$\sum_{i=0}^{6} \left(\frac{1}{2}\right)^i = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64}$;

$\sum_{m=2}^{4} (m+1)^m = 3^2 + 4^3 + 5^4 = 9 + 64 + 625 = 698.$ $\square$