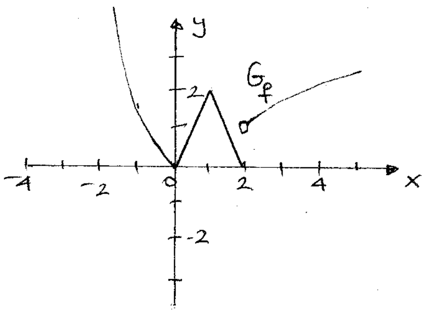
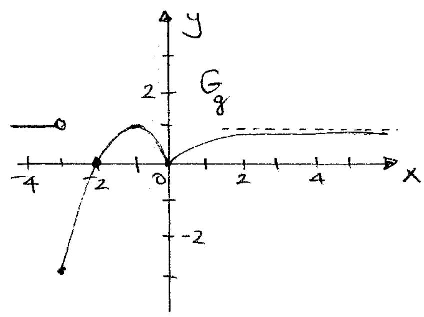
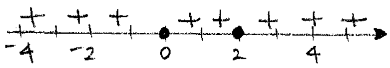
Università degli Studi di Trento – Facoltà di Scienze Cognitive
CdL in Interfacce e Tecnologie della Comunicazione
CdL in Scienze e Tecniche di Psicologia Cognitiva
Verifica Settimanale di ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2008 – 2009 – Rovereto, 9 – 12 dicembre 2008

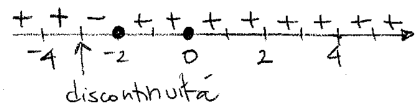
1)



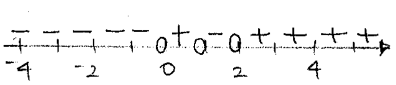


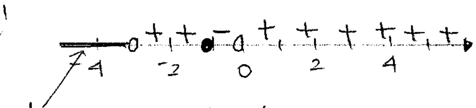
i)





ii)





$f' \nexists$ in $x=0$, $x=1$, $x=2$

$g' \nexists$ in $x=-3$, $x=0$ ☐

iii) $\min_{\mathbb{R}} f = 0$ $\quad x=0$, $x=2$ pt. di min locali (e globale)

$f(1)=2$ è massimo locale stretto ($x=1$ pt. di max loc.)

$\max_{\mathbb{R}} g = 1$ $\quad x \in]-\infty, -3[$, $x=-1$ sono pt. di min. loc. (e globale)

$\min_{\mathbb{R}} g = -3$ $\quad x=-3$ min glob. (e loc.)

$g(0)=0$ min. loc.
$x=0$ pt. di min. loc.

iv) $x=2$ pt. di discontinuità della f (infatti $\lim_{x\to 2^-} f(x) = 0 = f(2) \neq \lim_{x\to 2^+} f(x) = 1$) ☐

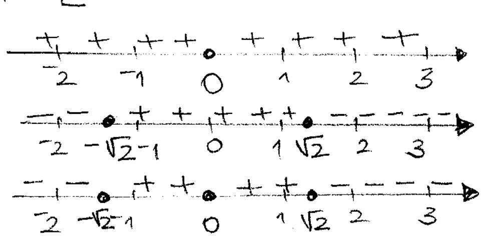
$x=-3$ pt. di discontinuità della g $\left(\lim_{x\to -3^-} g(x) = 1 \neq \lim_{x\to -3^+} g(x) = -3 = g(-3)\right)$ ■

2) • $f(x) = 2x^2 - x^4$

Nota: f è pari (e quindi il suo grafico sarà simmetrico rispetto all'asse y)

i) $\operatorname{dom} f = \mathbb{R} =]-\infty, +\infty[$

ii) $f(x) = x^2(2-x^2)$



iii) $\lim\limits_{x \to -\infty} f(x) = -\infty \qquad \Big(x^4 \big(\underbrace{\tfrac{2}{x^2}}_{\to 0} - 1 \big) \xrightarrow[x \to -\infty]{} -\infty \Big)$

$\lim\limits_{x \to +\infty} f(x) = -\infty \qquad \Big(x^4 \big(\underbrace{\tfrac{2}{x^4}}_{\to 0} - 1 \big) \xrightarrow[x \to +\infty]{} -\infty \Big)$.

iv) f è continua, poiché somma di due funzioni continue.

v) $\operatorname{dom} f' = \mathbb{R}$; $f'(x) = 4x - 4x^3 = 4x(1-x^2)$

$f'(x) = 0 \iff \left.\begin{array}{l} x = 0 \\ x = \pm 1 \end{array}\right\}$ pt. critici di f

	$(-\infty,-1)$	-1	$(-1,0)$	0	$(0,1)$	1	$(1,+\infty)$
x	$-$		$-$	0	$+$		$+$
$1-x^2$	$-$	0	$+$		$+$	0	$-$
$f'(x)$	$+$	0	$-$	0	$+$	0	$-$
$f(x)$	$\nearrow$		$\searrow$		$\nearrow$		$\searrow$

$x = -1$ pt. di max. loc. stretto $\qquad \underline{\underline{f(-1) = 1}}$

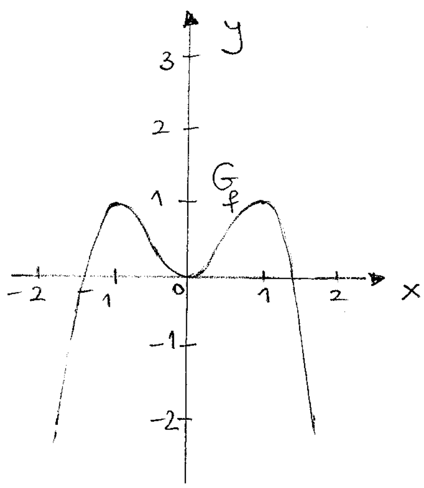
$x = 0$ pt. di min. loc. stretto $\qquad \underline{\underline{f(0) = 0}}$

$x = 1$ pt. di max. loc. stretto $\qquad \underline{\underline{f(1) = 1}}$.

vi) $\operatorname{dom} f'' = \mathbb{R}$; $f''(x) = 4 - 12x^2 = 4(1-3x^2)$

	$(-\infty,-\frac{1}{\sqrt3})$	$-\frac{1}{\sqrt3}$	$(-\frac{1}{\sqrt3},\frac{1}{\sqrt3})$	$\frac{1}{\sqrt3}$	$(\frac{1}{\sqrt3},+\infty)$
$f''(x)$	$-$	0	$+$	0	$-$
$f(x)$	$\cap$		$\cup$		$\cap$

pt. di flesso

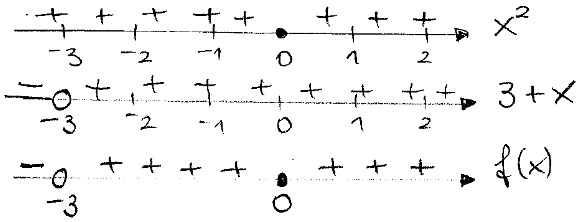


□

•• $f(x) = \dfrac{x^2}{3+x}$

i) $\mathrm{dom} f = \mathbb{R}\setminus\{-3\} =]-\infty, -3[\cup]-3, +\infty[$

ii)



iii) $\lim\limits_{x\to-\infty} f(x) = -\infty$ $\qquad \left(\dfrac{x^2}{3+x} = \dfrac{x^{\cancel{2}}}{\cancel{x}\left(\frac{3}{x}+1\right)} \xrightarrow[x\to-\infty]{} -\infty \right)$ $\quad \frac{3}{x} \to 0$

$\lim\limits_{x\to-3^-} f(x) = -\infty$ $\qquad \left(\dfrac{x^2 \to 9}{x+3} \longrightarrow -\infty \right)$ infinitesimo negativo $\Rightarrow x=-3$ asintoto verticale

$\lim\limits_{x\to-3^+} f(x) = +\infty$

$\lim\limits_{x\to+\infty} f(x) = +\infty$

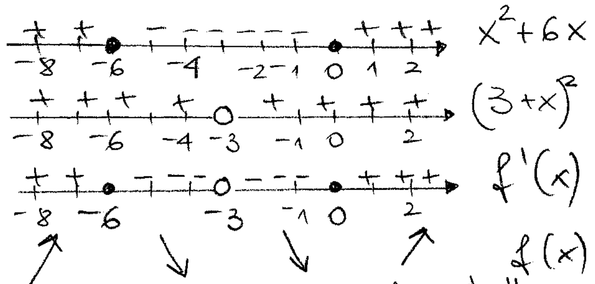
Oss. f ha asintoto obliquo: $y = x-3$ (per $x\to-\infty$, per $x\to+\infty$)

$$\begin{cases} \lim\limits_{\substack{x\to-\infty \\ (+\infty)}} \dfrac{f(x)}{x} = \lim\limits_{\substack{x\to-\infty \\ (+\infty)}} \dfrac{x^2}{3x+x^2} = 1 \ (=a) \\ \lim\limits_{\substack{x\to-\infty \\ (+\infty)}} \left(f(x)-x\right) = \lim\limits_{\substack{x\to-\infty \\ +\infty}} \left(\dfrac{\cancel{x^2}-3x-\cancel{x^2}}{3+x} \right) = -3 \ (=b) \end{cases}$$

iv) f è continua in tutti i pt. del suo insieme di def. (rapporto di fun. continue);

v) $\mathrm{dom} f' = \mathbb{R}\setminus\{-3\}$ $\qquad f'(x) = \dfrac{2x(3+x) - x^2}{(3+x)^2} = \dfrac{x^2+6x}{(3+x)^2} = \dfrac{x(x+6)}{(3+x)^2}$

$f'(x) = 0 \Leftrightarrow \left.\begin{matrix} x=0 \\ x=-6 \end{matrix}\right\}$ pt. critici di f



$x=-6$ pt. di max. loc. stretto $\qquad f(-6) = \frac{36}{-3} = -12$

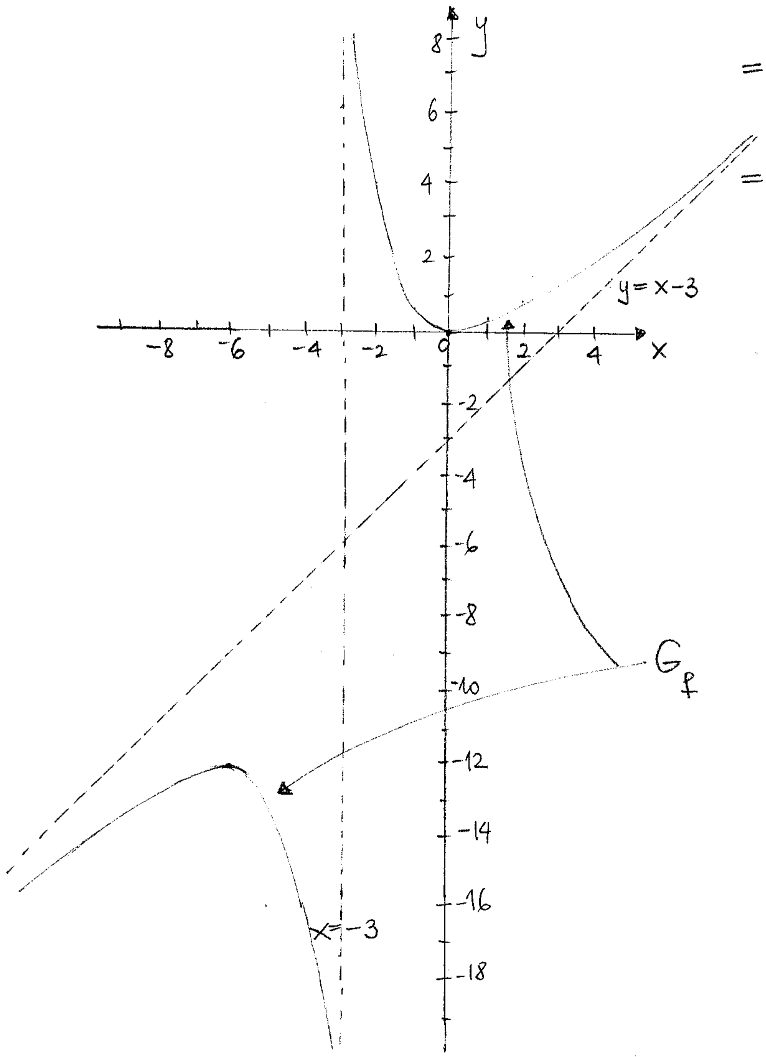
$x=0$ pt. di min. loc. stretto $\qquad f(0) = 0$

vi) $\mathrm{dom}\, f'' = \mathbb{R}\setminus\{-3\}$

$$f''(x) = \frac{(2x+6)(3+x)^{\cancel{2}} - (x^2+6x)\,2\,\cancel{(3+x)}}{(3+x)^{\cancel{4}\,3}}$$

$$= \frac{\cancel{6x} + \cancel{2x^2} + 18 + \cancel{6x} - \cancel{2x^2} - \cancel{12x}}{(3+x)^3}$$

$$= \frac{18}{(3+x)^3}$$



$$\begin{array}{cccccc} - - - - - & -3 & + + + + + & f''(x) \\ \frown & & \smile & f(x) \end{array}$$

••• $f(x) = \dfrac{x-1}{x^2+3}$

i) $\mathrm{dom}\, f = \mathbb{R} =]-\infty, +\infty[$

ii) $- - - - - - \; 1 \; + + + \quad f(x)$ $\qquad$ (nota: $x^2+3 > 0 \;\; \forall x \in \mathbb{R}$)

iii) $\lim\limits_{x\to-\infty} f(x) = 0$ $\qquad$ (infatti: $\dfrac{\cancel{x}\left(1-\frac{1}{x}\right)}{x^{\cancel{2}}\left(1+3/x^2\right)} \xrightarrow[\substack{x\to-\infty \\ (x\to+\infty)}]{} 0$)

$\lim\limits_{x\to+\infty} f(x) = 0$

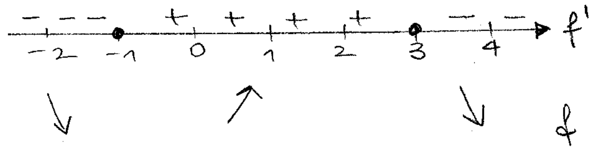
<u>$y=0$ è asintoto orizzontale</u> per f (per $x\to-\infty$, per $x\to+\infty$)

iv) f è continua su tutto $\mathbb{R}$ (rapporto di funz. continue e il denominatore non si annulla mai!)

□

v) $\text{dom } f' = \mathbb{R}$ $\qquad f'(x) = \dfrac{x^2+3 - (x-1)\cdot 2x}{(x^2+3)^2} = \dfrac{-x^2+2x+3}{(x^2+3)^2}$

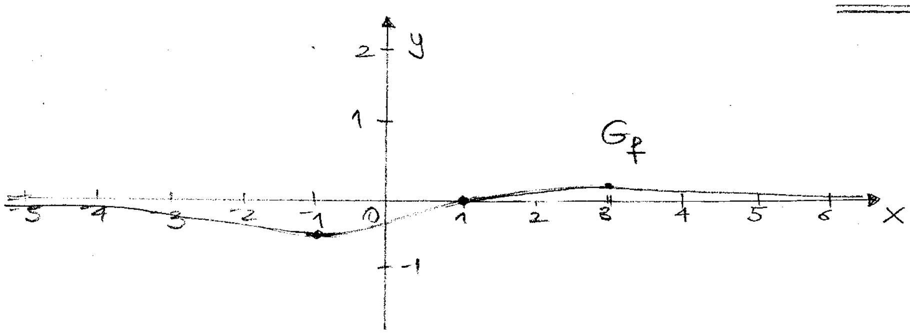
$f'(x) = 0 \iff -x^2+2x+3 = 0 \iff x_{1/2} = \dfrac{-2 \pm\sqrt{4+12}}{-2} = \dfrac{-2\pm 4}{-2} \begin{cases} -1 \\ 3 \end{cases}$



$\left.\begin{array}{l} x=-1 \\ x=3 \end{array}\right\}$ pt. critici di f

$x=-1$ pt. di min. loc. stretto $\quad \underline{\underline{f(-1) = -\frac{1}{2}}}$

$x=3$ pt. di max. loc. stretto $\quad \underline{\underline{f(3) = \frac{1}{6}}}$

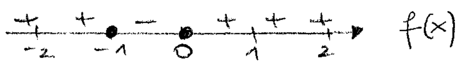


$\square$

•••• $\underline{\underline{f(x) = \dfrac{x^2+x}{x^2+1}}}$

i) $\text{dom } f = \mathbb{R} =]-\infty, +\infty[$

ii) $f(x) = \dfrac{x(x+1)}{x^2+1}$



(nota: $x^2+1>0 \ \forall x \in \mathbb{R}$)

iii) $\lim\limits_{x\to-\infty} f(x) = 1 \qquad \left(\dfrac{\cancel{x^2}(1+\frac{1}{x})}{\cancel{x^2}(1+\frac{1}{x^2})} \xrightarrow[\substack{x\to-\infty \\ (x\to+\infty)}]{} 1 \right)$

$\lim\limits_{x\to+\infty} f(x) = 1 \qquad$ $\underline{y=1 \text{ asintoto orizzontale per } f}$ (per $x\to-\infty$, per $x\to+\infty$).

iv) f è continua in $\mathbb{R}$, poiché rapporto di due funzioni continue e il denominatore non si annulla mai.

v) $\text{dom } f' = \mathbb{R}$; $\quad f'(x) = \dfrac{(2x+1)(x^2+1) - (x^2+x)\cdot 2x}{(x^2+1)^2} =$

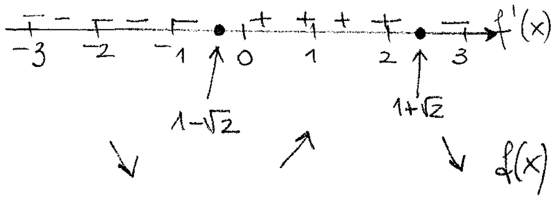
$= \dfrac{\cancel{2x^3}+2x+x^2+1-\cancel{2x^3}-2x^2}{(x^2+1)^2} =$

$= \dfrac{-x^2+2x+1}{(x^2+1)^2}$

$$f'(x)=0 \iff -x^2+2x+1=0 \iff x_{1/2}=\frac{-2\pm\sqrt{4+4}}{-2}$$

$$=\frac{-2\pm 2\sqrt{2}}{-2}=$$

$$=1\pm\sqrt{2} \quad \text{pt. critici di } f$$



$x=1-\sqrt{2}$ pt. di min. loc. stretto

$$f(1-\sqrt{2})=\frac{(1-\sqrt{2})^2+(1-\sqrt{2})}{(1-\sqrt{2})^2+1}$$

$$=\frac{1-2\sqrt{2}+2+1-\sqrt{2}}{1-2\sqrt{2}+2+1}=$$

$$=\frac{4-3\sqrt{2}}{4-2\sqrt{2}}\sim -0.2$$

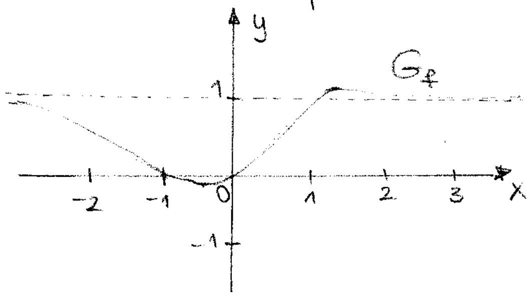
$x=1+\sqrt{2}$ pt. di max. loc. stretto

$$f(1+\sqrt{2})=\frac{(1+\sqrt{2})^2+(1+\sqrt{2})}{(1+\sqrt{2})^2+1}$$

$$=\frac{1+2\sqrt{2}+2+1+\sqrt{2}}{1+2\sqrt{2}+2+1}$$

$$=\frac{4+3\sqrt{2}}{4+2\sqrt{2}}=1+\frac{\sqrt{2}}{4+2\sqrt{2}}$$

$$\sim 1.2$$

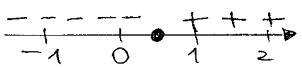


□

••••• $f(x)=\left(x-\frac{1}{2}\right)e^{-x^2+1}$

i) $\text{dom} f = \mathbb{R} =]-\infty,+\infty[$

ii)



(nota: $e^{-x^2+1}>0 \ \forall x\in\mathbb{R}$)

iii) $\lim_{x\to-\infty} f(x) = 0$ $\quad$ (infatti, $\dfrac{x-\frac{1}{2}}{e^{x^2-1}} \xrightarrow[x\to-\infty \ (x\to+\infty)]{} 0$)

$\lim_{x\to+\infty} f(x) = 0$; $y=0$ asintoto orizzontale per f (per $x\to-\infty$, per $x\to+\infty$).

iv) f è continua in $\mathbb{R}$ poiché composizione di funzioni continue e prodotto di funzioni continue.

v) $\text{dom} f' = \mathbb{R}$ $\quad f'(x)=e^{-x^2+1}+\left(x-\frac{1}{2}\right)e^{-x^2+1}\cdot(-2x)=$

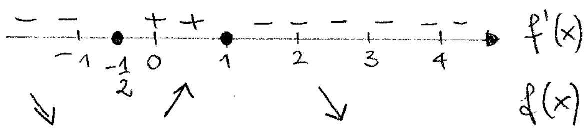
$$=e^{-x^2+1}\left(-2x^2+x+1\right)$$

$$f'(x)=0 \iff -2x^2+x+1=0 \iff x_{1/2}=\frac{-1\pm\sqrt{1+8}}{-4}=\begin{cases}1\\-\frac{1}{2}\end{cases}$$

$x = -\frac{1}{2}$, $x = 1$ sono pt. critici di f

$\underline{\underline{f\left(-\frac{1}{2}\right)}} = -1\, e^{-\frac{1}{4}+1} = \underline{\underline{-e^{3/4}}}$ (~ -2.1)

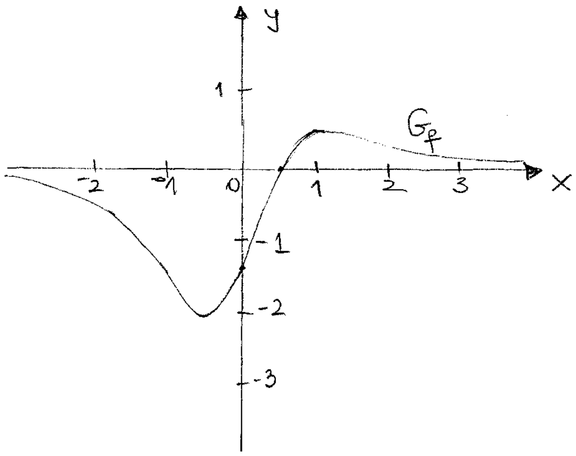
$\underline{\underline{f(1)}} = \frac{1}{2}\, e^{-1+1} = \underline{\underline{\frac{1}{2}}}$



$(e^{-x^2+1} > 0 \quad \forall x \in \mathbb{R})$

$x = -\frac{1}{2}$ pt. di min. loc. stretto

$x = 1$ pt. di max. loc. stretto

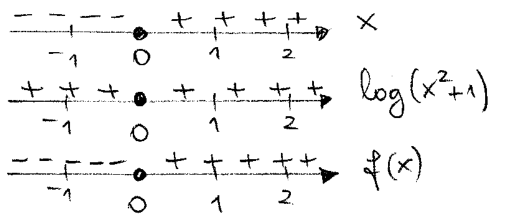


□

•••••• $\underline{f(x) = x \log(x^2+1)}$

i) $\operatorname{dom} f = \mathbb{R} =]-\infty, +\infty[$

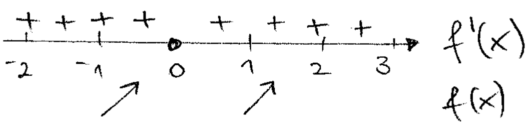
ii)



iii) $\lim_{x \to -\infty} f(x) = -\infty$ $\qquad \lim_{x \to +\infty} f(x) = +\infty$

iv) f continua in $\mathbb{R}$, poiché composizione e prodotto di funzioni continue.

v) $\operatorname{dom} f' = \mathbb{R}$ $\qquad f'(x) = \log(x^2+1) + \dfrac{2x^2}{x^2+1}$



vi) dom $f'' = \mathbb{R}$

$$f''(x) = \frac{2x}{x^2+1} + \frac{4x(x^2+1) - 2x^2 \cdot 2x}{(x^2+1)^2}$$

$$= \frac{2x(x^2+1) + \cancel{4x^3} + 4x - \cancel{4x^3}}{(x^2+1)^2} =$$

$$= \frac{2x(x^2+3)}{(x^2+1)^2}$$

$f''(x)$: $- - - - -$ (0) $+ + +$; $-1 \quad 0 \quad 1$

$f(x)$: $\cap$ $\uparrow$ $\cup$

pt. di flesso