Università degli Studi di Trento — Facoltà di Scienze Cognitive
CdL in Interfacce e Tecnologie della Comunicazione
CdL in Scienze e Tecniche di Psicologia Cognitiva
Seconda Prova Intermedia di ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2008 – 2009 — Rovereto, 21/11/2008

Compito Ⓐ

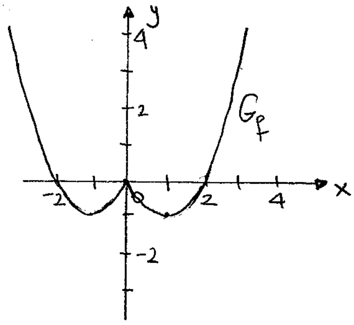
1) i) $f:\mathbb{R}\to\mathbb{R}$, $f(x)=x^2-2|x|$

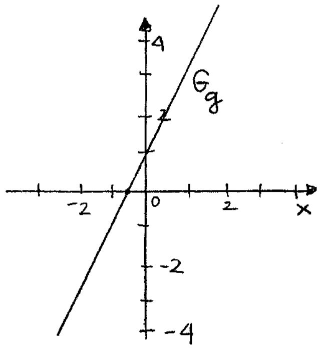
(f è una funzione pari, cioè $f(x)=f(-x)\ \forall x\in\mathbb{R}$ e quindi il grafico di f è simmetrico rispetto all'asse y; basta quindi studiare il grafico di f su $[0,+\infty[$; su $]-\infty,0]$ si ottiene per simmetria).

Abbiamo

$$f(x)=\begin{cases} x^2-2x & \text{se } x\geq 0 \\ x^2+2x & \text{se } x<0. \end{cases}$$

$g:\mathbb{R}\to\mathbb{R}$ $g(x)=2x+1$.





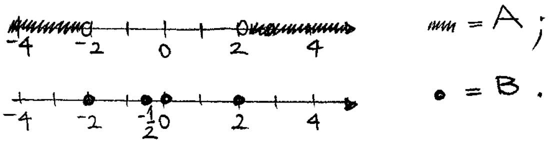
Notiamo che $x^2-2x=(x-1)^2-1$

$x^2+2x=(x+1)^2-1$

☐

ii) $A=\left\{x\in\mathbb{R}: \frac{1}{f(x)}>0\right\} = \underline{]-\infty,-2[\ \cup\ ]2,+\infty[}$;

$B=\{x\in\mathbb{R}: f(x)\cdot g(x)=0\} = \{x\in\mathbb{R}: f(x)=0 \text{ o } g(x)=0\} = \underline{\{-2,-\tfrac{1}{2},0,2\}}$.

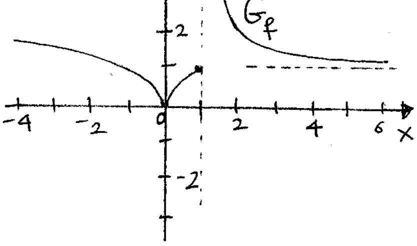


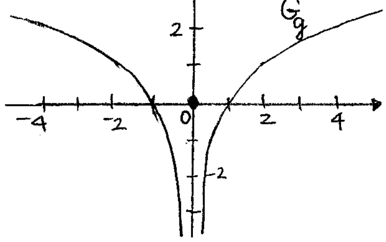
☐

iii) $A\cup B = \underline{]-\infty,-2]\cup\{-\tfrac{1}{2},0\}\cup[2,+\infty[}$; $A\cap B=\underline{\emptyset}$.

■

2) i) $f:\mathbb{R}\to\mathbb{R}$ $f(x)=\begin{cases} |\sqrt[3]{x}| & \text{se } x\leq 1 \\ \frac{1}{(x-1)^2}+1 & \text{se } x>1; \end{cases}$ $g:\mathbb{R}\to\mathbb{R}$ $g(x)=\begin{cases} \log_2|x| & \text{se } x\neq 0 \\ 0 & \text{se } x=0. \end{cases}$





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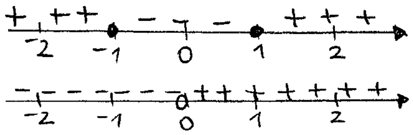
ii) se $k<0$, $\nexists$ono soluzioni dell'eq. $f(x)=k$;
se $k=0$, $\exists$ una soluzione " ;
se $k>0$, $\exists$ono due soluzioni " . ☐

iii) $(f+g)(0)= f(0)+g(0) = 0+0=\underline{\underline{0}}$; $g(-2)\cdot g(2) = \underline{1}$; $\left(\frac{f}{g}\right)(1)\ \nexists$
poichè $g(1)=0$! ☐

iv) Osserviamo che, per $x>1$, $f(x)>1$ e quindi $(g\circ f)(x) = g(f(x)) >0$
poichè $g(\square)>0$ se $\square>1$.
Abbiamo quindi che $\underline{\underline{g\circ f \text{ è positiva su }]1,+\infty[}}$. ■

3) i) $f(x)=|x|-1$: insieme di definizione $=\underline{\underline{\mathbb{R}}}$;
$g(x)=\frac{1}{e^x-1}$: insieme di definizione $=\{x\in\mathbb{R} : e^x-1\neq 0\}=$
$=\{x\in\mathbb{R} : e^x\neq 1\} = \underline{\underline{\mathbb{R}\setminus\{0\}}}$.



(basta oss. che $e^x>1$ per $x>0$, $e^0=1$, e $e^x<1$ per $x<0$).

ii) $(g\circ f)(x) \underset{\nearrow}{=} g(f(x)) = \underline{\underline{\frac{1}{e^{|x|-1}-1}}}$. ☐

$\{x\in\mathbb{R} : f(x)\neq 0\} = \{x\in\mathbb{R} : |x|-1\neq 0\}$
$= \underline{\underline{\mathbb{R}\setminus\{-1,1\}}}$

$(f\circ g)(x) \underset{\nearrow}{=} f(g(x)) = \underline{\underline{\left|\frac{1}{e^x-1}\right| - 1}}$.

$\underline{\underline{x\in\mathbb{R}\setminus\{0\}}}$ ■

4) i) $|x^2-2x|>3 \iff x^2-2x<-3$ o $x^2-2x>3$;
$\iff x^2-2x+3<0$ o $x^2-2x-3>0$;
$\Rightarrow \underline{\underline{S=]-\infty,-1[\cup]3,+\infty[}}$;

$1+2|x-1|\leq 4 \iff |x-1|\leq 3/2 \iff -\frac{3}{2}\leq x-1\leq 3/2$
$\Rightarrow \underline{\underline{S=\left[-\frac{1}{2},\frac{5}{2}\right]}}$;

$$3x^2 - |x-2| \geqslant 0 \iff \begin{cases} x-2 \geqslant 0 \\ 3x^2-(x-2) \geqslant 0 \end{cases} \text{ o } \begin{cases} x-2<0 \\ 3x^2+(x-2) \geqslant 0 \end{cases}$$

$$\iff \begin{cases} x \geqslant 2 \\ 3x^2-x+2 \geqslant 0 \end{cases} \text{ o } \begin{cases} x<2 \\ 3x^2+x-2 \geqslant 0 \end{cases}$$

$$\iff \begin{cases} x \geqslant 2 \\ \mathbb{R} \end{cases} \text{ o } \begin{cases} x<2 \\ x \in \left]-\infty,-1\right] \cup \left[\tfrac{2}{3},+\infty\right[\end{cases}$$

$$\Rightarrow \underline{\underline{S = \left]-\infty,-1\right] \cup \left[\tfrac{2}{3},+\infty\right[}} \; .$$

□

ii) $$\frac{2^x \cdot 4^{x^2-1}}{2^{-2x}} - 1 = 0 \iff 2^x \cdot 2^{2(x^2-1)} \cdot 2^{2x} = 1$$

$$\iff 2^{x+2x^2-2+2x} = 2^0$$

$$\iff 2^{2x^2+3x-2} = 2^0$$

$$\iff 2x^2+3x-2=0 \Rightarrow \underline{\underline{S=\left\{-2,\tfrac{1}{2}\right\}}};$$

$$\log_2(1-|x|) = \log_2(3x^2) \iff \begin{cases} 1-|x|>0 \\ 3x^2>0 \\ 1-|x|=3x^2 \end{cases} \iff \begin{cases} |x|<1 \\ x \neq 0 \\ 1-|x|=3x^2 \end{cases}$$

$$\iff \begin{cases} -1<x<0 \\ 1+x=3x^2 \end{cases} \text{ o } \begin{array}{l} 0<x<1 \\ 1-x=3x^2 \end{array}$$

$$\iff \begin{cases} -1<x<0 \\ x_{1/2} = \dfrac{1 \pm \sqrt{13}}{6} \end{cases} \text{ o } \begin{cases} 0<x<1 \\ x_{1/2} = \dfrac{-1 \pm \sqrt{13}}{6} \end{cases} \Rightarrow \underline{\underline{S=\left\{\frac{1-\sqrt{13}}{6}, \frac{-1+\sqrt{13}}{6}\right\}}}.$$

□

iii) $$\log_{\frac{1}{2}}(x+3) - \log_{1/2} x \leq \log_{1/2}(x-1) \iff \begin{cases} x+3>0 \\ x>0 \\ x-1>0 \\ \dfrac{x+3}{x} \geqslant x-1 \end{cases}$$

$$\iff \begin{cases} x>1 \\ \dfrac{x+3}{x} - x+1 \geqslant 0 \end{cases} \iff \begin{cases} x>1 \\ \dfrac{-x^2+2x+3}{x} \geqslant 0 \end{cases}$$

$$\iff \begin{cases} x>1 \\ \left]-\infty,-1\right] \cup \left]0,3\right] \end{cases} \Rightarrow \underline{\underline{S=\left]1,3\right]}};$$

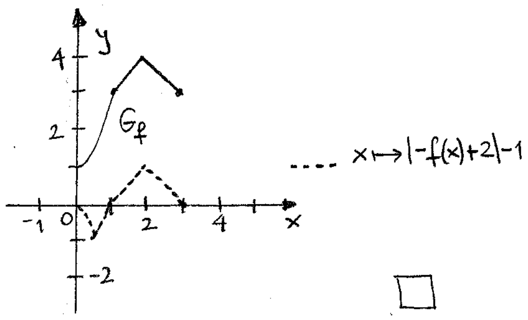
$$\left(\frac{1}{2}\right)^{x^2-1} \leq 2^{|x|+1} \iff 2^{-x^2+1} \leq 2^{|x|+1} \iff -x^2+\cancel{1} \leq |x|+\cancel{1}$$

$$\Rightarrow \underline{\underline{S=\mathbb{R}.}}$$

■

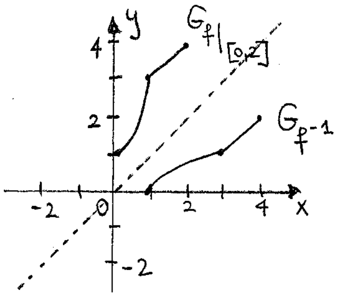
5) i) $f : [0,3] \to [1,4]$

$$f(x) = \begin{cases} 2x^2+1 & \text{se} \quad 0 \leq x < 1 \\ -|x-2|+4 & \text{se} \quad 1 \leq x \leq 3 \end{cases}$$



☐

ii) $I = [0,2]$

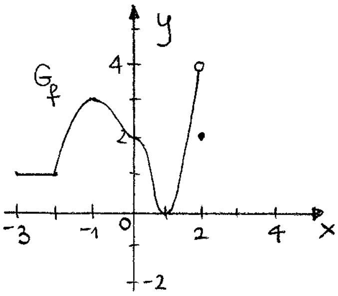


$f|_{[0,2]} : [0,2] \to [1,4]$

$f^{-1} : [1,4] \to [0,2]$.

■

6)



i) f è limitata : infatti $0 \leq f(x) \leq 4 \quad \forall x \in [-3,2]$.

☐

ii) $\min\limits_{[-3,2]} f = 0 \qquad x = 1$ pt. di minimo;

$\nexists \max\limits_{[-3,2]} f$.

☐

iii) $f([-3,2]) = [0,4[$.

☐

iv) in $[-3,-1]$, f è debolmente crescente;

in $[-1,1]$, f è decrescente;

in $[1,2[$, f è crescente.

■

Compito Ⓑ

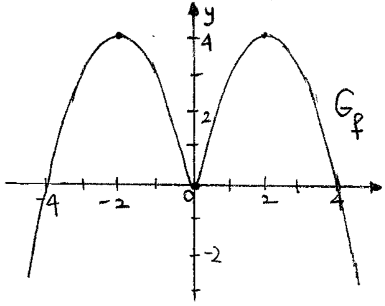
1) i) $f:\mathbb{R}\to\mathbb{R}$, $f(x)=-x^2+4|x|$ (vedi commento Es. 1 i) compito Ⓐ).

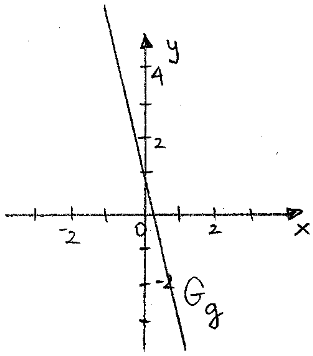
Abbiamo

$$f(x)=\begin{cases}-x^2+4x & \text{se } x\geq 0\\ -x^2-4x & \text{se } x<0.\end{cases}$$

Notiamo che $-x^2+4x=-(x-2)^2+4$
$-x^2-4x=-(x+2)^2+4$

$g:\mathbb{R}\to\mathbb{R}$ $g(x)=-4x+1$.

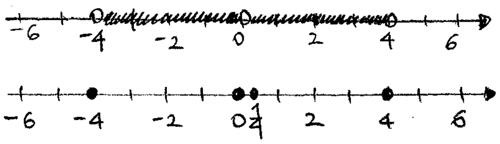




☐

ii) $A=\left\{x\in\mathbb{R}:\ \dfrac{1}{f(x)}>0\right\}=\underline{\underline{]-4,4[\ \setminus\{0\}}}$;

$B=\{x\in\mathbb{R}:\ f(x)\cdot g(x)=0\}=\{x\in\mathbb{R}: f(x)=0 \text{ o } g(x)=0\}=\underline{\underline{\left\{-4,0,\tfrac{1}{4},4\right\}}}$.



▨ = A ;

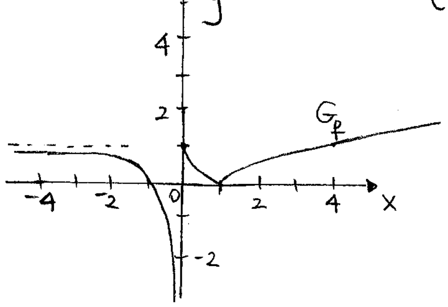
● = B .

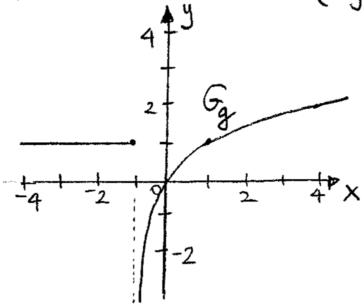
☐

iii) $A\cup B=\underline{\underline{[-4,4]}}$; $A\cap B=\underline{\underline{\left\{\tfrac{1}{4}\right\}}}$.

■

2) i) $f:\mathbb{R}\to\mathbb{R}$ $f(x)=\begin{cases}\dfrac{1}{x^3}+1 & \text{se } x<0\\ |\sqrt{x}-1| & \text{se } x\geq 0\end{cases}$; $g:\mathbb{R}\to\mathbb{R}$ $g(x)=\begin{cases}1 & \text{se } x\leq -1\\ \log_2(x+1) & \text{se } x>-1.\end{cases}$





☐

ii) se $k<0$, ∃ una soluzione dell'eq. $f(x)=k$;
se $k=0$, ∃ono 2 soluzioni " ;
se $0<k<1$, ∃ono 3 sol. " ;
se $k=1$, ∃ono 2 soluz. " ;
se $k>1$, ∃ una soluzione " .

☐

iii) $(f+g)(0) = f(0)+g(0) = 1+0 = \underline{\underline{1}};$ $\quad g(-1)\cdot g(1) = 1\cdot 1 = \underline{\underline{1}};$

$$\left(\frac{f}{g}\right)(-1) \underset{\substack{\uparrow \\ g(-1)\neq 0}}{=} \frac{f(-1)}{g(-1)} = \frac{0}{1} = \underline{\underline{0}}.$$

□

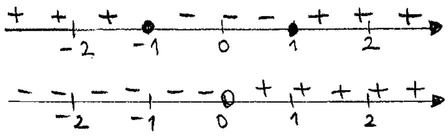
iv) Osserviamo che, per $x \leq -1$, $g(x) \equiv 1$, e quindi $(f\circ g)(x) = f(g(x)) = 0$ per ogni $x \leq -1$.

■

3) i) $f(x) = \sqrt[3]{|x|-1}$: insieme di definizione $= \underline{\underline{\mathbb{R}}};$

$g(x) = \dfrac{1}{3^x-1}$: insieme di definizione $= \{x\in\mathbb{R} : 3^x - 1 \neq 0\} =$

$= \{x\in\mathbb{R} : 3^x \neq 1\} = \underline{\underline{\mathbb{R}\setminus\{0\}}}.$



(basta osservare che $3^x > 1$ per $x>0$, $3^0 = 1$, e $3^x < 1$ per $x<0$).

ii) $(g\circ f)(x) \underset{\uparrow}{=} g(f(x)) = \underline{\underline{\dfrac{1}{3^{\sqrt[3]{|x|-1}}-1}}}.$

□

$\{x\in\mathbb{R} : f(x)\neq 0\} = \{x\in\mathbb{R} : |x|-1 \neq 0\}$ (nota : $\sqrt[3]{x} = 0 \iff x = 0$)

$= \underline{\underline{\mathbb{R}\setminus\{-1,1\}}}$

$$(f\circ g)(x) \underset{\substack{\uparrow \\ \underline{\underline{x\in\mathbb{R}\setminus\{0\}}}}}{=} f(g(x)) = \underline{\underline{\sqrt[3]{\left|\frac{1}{3^x-1}\right| - 1}}}.$$

■

4) i) $|x^2-3x| \leq 4 \iff -4 \leq x^2-3x \leq 4 \iff \begin{cases} x^2-3x+4 \geq 0 \\ x^2-3x-4 \leq 0 \end{cases}$

$\iff \begin{cases} \mathbb{R} \\ x\in[-1,4] \end{cases} \Rightarrow \underline{\underline{S = [-1,4]}};$

□

$2+|x-5| > 4 \iff |x-5| > 2 \iff (x-5 < -2 \text{ o } x-5 > 2)$

$\Rightarrow \underline{\underline{S =]-\infty,3[\, \cup\,]7,+\infty[}};$

□

$3x^2 - |x+2| \leq 0 \iff \begin{cases} x+2 \geq 0 \\ 3x^2-(x+2) \leq 0 \end{cases}$ o $\begin{cases} x+2 < 0 \\ 3x^2+(x+2) \leq 0 \end{cases}$

$\iff \begin{cases} x \geq -2 \\ 3x^2-x-2 \leq 0 \end{cases}$ o $\begin{cases} x < -2 \\ 3x^2+x+2 \leq 0 \end{cases}$

$$\begin{cases} x \geq -2 \\ x \in \left[-\frac{2}{3}, 1\right] \end{cases} \quad \text{o} \quad \begin{cases} x < -2 \\ \phi \end{cases} \quad \Rightarrow \quad \underline{\underline{S = \left[-\tfrac{2}{3}, 1\right].}}$$

☐

ii) $$\frac{\left(\frac{1}{2}\right)^{x^2-1} \cdot 4^{x+1}}{2^{3-x}} - 4 = 0 \iff 2^{-x^2+1} \cdot 2^{2x+2} \cdot 2^{-3+x} = 2^2$$

$$\iff 2^{-x^2+3x} = 2^2 \iff -x^2+3x = 2$$

$$\iff -x^2+3x-2=0 \Rightarrow \underline{\underline{S=\{1,2\}}}\ ;$$

$$\log_{\frac{1}{2}}(2-|x|) = \log_{\frac{1}{2}}(4x^2) \iff \begin{cases} 2-|x|>0 \\ 4x^2>0 \\ 2-|x| = 4x^2 \end{cases} \iff \begin{cases} |x|<2 \\ x \neq 0 \\ 2-|x| = 4x^2 \end{cases}$$

$$\iff \begin{cases} -2<x<0 \\ 2+x = 4x^2 \end{cases} \quad \text{o} \quad \begin{cases} 0<x<2 \\ 2-x = 4x^2 \end{cases}$$

$$\iff \begin{cases} -2<x<0 \\ x_{1/2} = \frac{1 \pm \sqrt{33}}{8} \end{cases} \quad \text{o} \quad \begin{cases} 0<x<2 \\ x_{1/2} = \frac{-1 \pm \sqrt{33}}{8} \end{cases}$$

$$\Rightarrow \underline{\underline{S = \left\{\frac{1-\sqrt{33}}{8}, \frac{-1+\sqrt{33}}{8}\right\}}}.$$

☐

iii) $$3^{|x|+1} \cdot \left(\frac{1}{9}\right)^{-x} \geq 1 \iff 3^{|x|+1} \cdot 3^{2x} \geq 3^0 \iff |x|+1+2x \geq 0$$

$$\iff \begin{cases} x \geq 0 \\ 3x \geq -1 \end{cases} \quad \text{o} \quad \begin{cases} x<0 \\ x \geq -1 \end{cases} \quad \Rightarrow \underline{\underline{S = [-1, +\infty[}}.$$

$$\log_2(x+3) - \log_2 x \leq \log_2(x-1) \iff \begin{cases} x+3>0 \\ x>0 \\ x-1>0 \\ \frac{x+3}{x} \leq x-1 \end{cases}$$

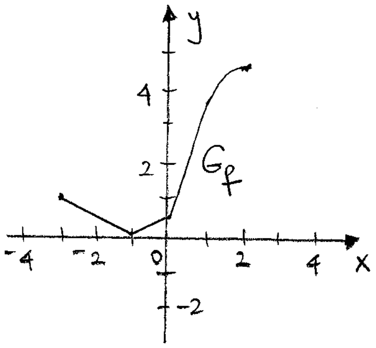
$$\iff \begin{cases} x>1 \\ \frac{x+3}{x} - x+1 \leq 0 \end{cases} \iff \begin{cases} x>1 \\ \frac{-x^2+2x+3}{x} \leq 0 \end{cases}$$

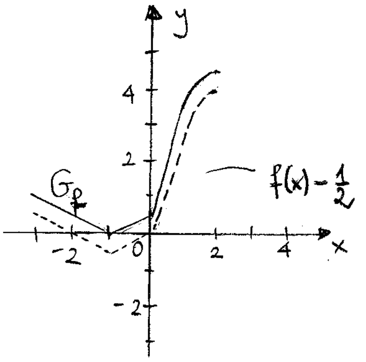
$$\iff \begin{cases} x>1 \\ x \in [-1,0[\ \cup [3,+\infty[\end{cases} \quad \Rightarrow \quad \underline{\underline{S = [3, +\infty[}}.$$

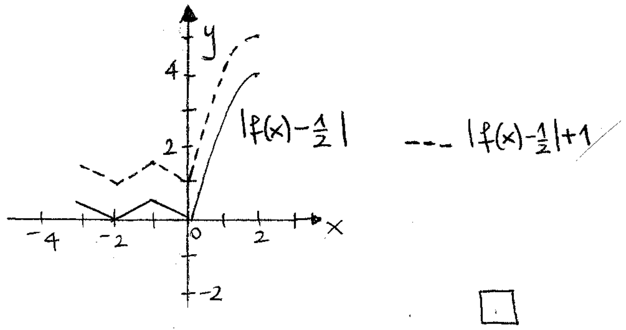
■

5) i) $f : [-3,2] \rightarrow \left[0, \frac{9}{2}\right]$

$$f(x) = \begin{cases} \frac{1}{2}|x+1| & \text{se} \quad -3 \leq x < 0 \\ -(x-2)^2 + \frac{9}{2} & \text{se} \quad 0 \leq x \leq 2 \end{cases}$$







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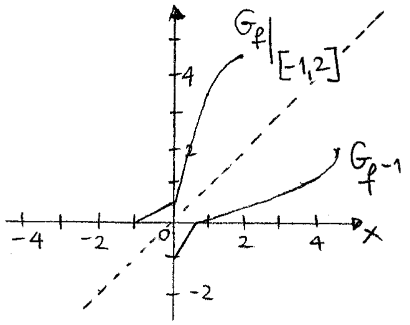
ii) $I = [-1,2]$

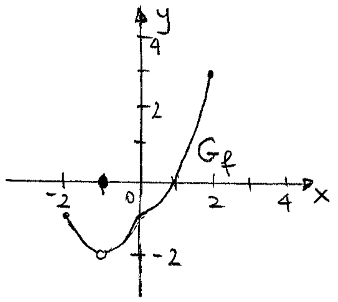


$f|_{[-1,2]} : [-1,2] \to [0,\frac{9}{2}]$

$f^{-1} : [0,\frac{9}{2}] \to [-1,2]$.

■

6)



i) f è limitata : infatti $-2 \le f(x) \le 3$ $\forall x \in [-2,2]$.

☐

ii) $\nexists \min\limits_{[-2,2]} f$;

$\max\limits_{[-2,2]} f = 3$ $x=2$ pt. di massimo.

☐

iii) $f([-2,2]) =]-2,3]$.

☐

iv) In $[-2,-1[$ f è decrescente ;

in $]-1,2]$ f è crescente.

■

Compito Ⓒ

1) i) $f:\mathbb{R}\to\mathbb{R}$, $f(x)=-x^2+2|x|$ (vedi commento Es. 1 i) compito Ⓐ)

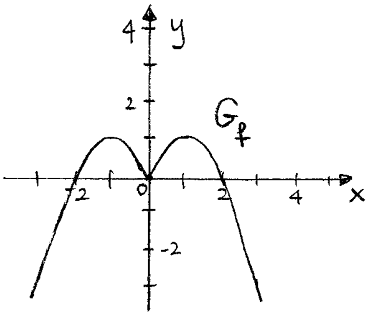
Abbiamo

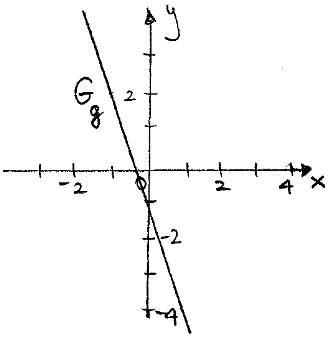
$$f(x)=\begin{cases}-x^2+2x & \text{se } x\geqslant 0\\ -x^2-2x & \text{se } x<0 .\end{cases}$$

Notiamo che $-x^2+2x = -(x-1)^2+1$

$-x^2-2x = -(x+1)^2+1$

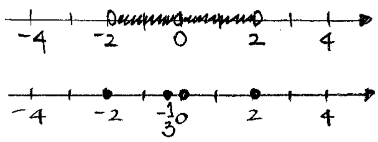
$g:\mathbb{R}\to\mathbb{R}$ $g(x)=-3x-1$





ii) $A=\left\{x\in\mathbb{R} : \frac{1}{f(x)}>0\right\} = \underline{\underline{\left]-2,2\right[\setminus \{0\}}}$;

$B=\left\{x\in\mathbb{R} : f(x)\cdot g(x)=0\right\} = \left\{x\in\mathbb{R} : f(x)=0 \text{ o } g(x)=0\right\} = \underline{\underline{\left\{-2,-\frac{1}{3},0,2\right\}}}$.



▨ = A ;

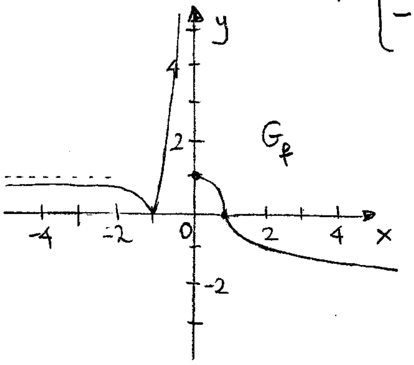
● = B .

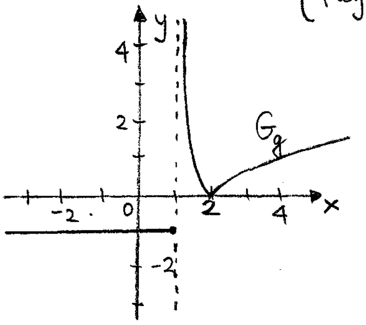
☐

iii) $A\cup B = \underline{\underline{[-2,2]}}$; $A\cap B = \underline{\underline{\left\{-\frac{1}{3}\right\}}}$.

■

2) i) $f:\mathbb{R}\to\mathbb{R}$ $f(x)=\begin{cases}\left|\frac{1}{x^4}-1\right| & \text{se } x<0\\ -\sqrt[3]{x-1} & \text{se } x\geqslant 0\end{cases}$ $g:\mathbb{R}\to\mathbb{R}$ $g(x)=\begin{cases}-1 & \text{se } x\leqslant 1\\ |\log_2(x-1)| & \text{se } x>1\end{cases}$





☐

ii) se $k<0$ o $k>1$, $\exists$ una soluzione dell'eq. $f(x)=k$;

se $k=0$ o $k=1$, $\exists$ono 2 soluzioni " ;

se $0<k<1$, $\exists$ono 3 soluzioni " .

☐

iii) $(f+g)(0) = f(0)+g(0) = -\sqrt[3]{-1} + (-1) = 1-1 = \underline{\underline{0}}$; $\quad g(-1)\cdot g(1) = (-1)(-1) = \underline{\underline{1}}$;

$$\left(\frac{f}{g}\right)(-1) \underset{\substack{\uparrow \\ g(-1)=-1\neq 0}}{=} \frac{f(-1)}{g(-1)} = \frac{0}{-1} = \underline{\underline{0}}.$$

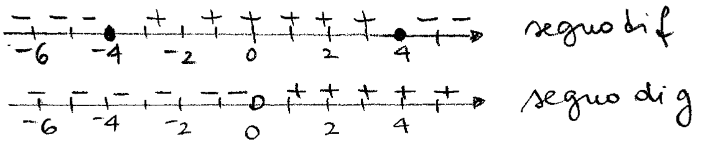
☐

iv) Osserviamo che, per $x \leq 1$, $g(x) \equiv -1$, e quindi $(f\circ g)(x) = f(g(x)) = 0$ per ogni $x \leq 1$. ■

3) i) $f(x) = 4-|x|$: insieme di definizione $= \mathbb{R}$;

$g(x) = \dfrac{1}{2^x-1}$: insieme di definizione $= \{x\in\mathbb{R} : 2^x-1 \neq 0\} =$

$= \{x\in\mathbb{R} : 2^x \neq 1\} = \underline{\underline{\mathbb{R}\setminus\{0\}}}.$



(basta osservare che $2^x > 1$ per $x>0$, $2^0 = 1$, e $2^x < 1$ per $x<0$).

☐

ii) $$(g\circ f)(x) \underset{\uparrow}{=} g(f(x)) = \underline{\underline{\frac{1}{2^{4-|x|}-1}}}.$$

$\{x\in\mathbb{R} : f(x)\neq 0\} = \{x\in\mathbb{R} : 4-|x|\neq 0\}$

$= \underline{\underline{\mathbb{R}\setminus\{-4,4\}}}$

$$(f\circ g)(x) \underset{\substack{\uparrow \\ \underline{\underline{x\in\mathbb{R}\setminus\{0\}}}}}{=} f(g(x)) = \underline{\underline{4 - \left|\frac{1}{2^x-1}\right|}}.$$

■

4) i) $|x^2+3x| \leq 4 \iff -4 \leq x^2+3x \leq 4 \iff \begin{cases} x^2+3x+4 \geq 0 \\ x^2+3x-4 \leq 0 \end{cases}$

$\iff \begin{cases} \mathbb{R} \\ x\in[-4,1] \end{cases} \Rightarrow \underline{\underline{S = [-4,1]}};$ ☐

$3-|x+4| < 2 \iff -|x+4| < -1 \iff |x+4| > 1$

$\iff (x+4<-1 \quad \text{o} \quad x+4>1)$

$\Rightarrow \underline{\underline{S =]-\infty,-5[\cup]-3,+\infty[}};$ ☐

$4x^2-|x+5| \leq 0 \iff \begin{cases} x+5 \geq 0 \\ 4x^2-(x+5) \leq 0 \end{cases} \quad \text{o} \quad \begin{cases} x+5 < 0 \\ 4x^2+(x+5) \leq 0 \end{cases}$

$$\begin{cases} x \geq -5 \\ 4x^2 - x - 5 \leq 0 \end{cases} \quad \text{o} \quad \begin{cases} x < -5 \\ 4x^2 + x + 5 \leq 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x \geq -5 \\ -1 \leq x \leq 5/4 \end{cases} \quad \text{o} \quad \begin{cases} x < -5 \\ \phi \end{cases} \quad \Rightarrow \underline{\underline{S = \left[-1, \tfrac{5}{4}\right]}}. \quad \square$$

ii) $$\frac{\left(\frac{1}{9}\right)^{x^2-1} \cdot 3^{x-4}}{3^{-2x}} - \frac{1}{3} = 0 \quad \Leftrightarrow \quad 3^{-2x^2+2} \cdot 3^{x-4} \cdot 3^{2x} = 3^{-1}$$

$$\Leftrightarrow \quad 3^{-2x^2+3x-2} = 3^{-1}$$

$$\Leftrightarrow \quad -2x^2 + 3x - 2 = -1 \quad \Leftrightarrow 2x^2 - 3x + 1 = 0$$

$$\Rightarrow \quad \underline{\underline{S = \left\{\tfrac{1}{2}, 1\right\}}}; \quad \square$$

$$\log_{\frac{1}{2}}(3 - |x|) = \log_{\frac{1}{2}}(2x^2) \quad \Leftrightarrow \begin{cases} 3 - |x| > 0 \\ 2x^2 > 0 \\ 3 - |x| = 2x^2 \end{cases} \Leftrightarrow \begin{cases} |x| < 3 \\ x \neq 0 \\ 3 - |x| = 2x^2 \end{cases}$$

$$\Leftrightarrow \begin{cases} -3 < x < 0 \\ 3 + x = 2x^2 \end{cases} \quad \text{o} \quad \begin{cases} 0 < x < 3 \\ 3 - x = 2x^2 \end{cases}$$

$$\Leftrightarrow \begin{cases} -3 < x < 0 \\ x_{1/2} \in \left\{-1, \tfrac{3}{2}\right\} \end{cases} \quad \text{o} \quad \begin{matrix} 0 < x < 3 \\ x \in \left\{-\tfrac{3}{2}, 1\right\} \end{matrix} \quad \Rightarrow \quad \underline{\underline{S = \{-1, 1\}}}. \quad \square$$

iii) $$4^{1-|x|} \cdot \left(\tfrac{1}{2}\right)^x \leq 1 \quad \Leftrightarrow \quad 2^{2-2|x|} \cdot 2^{-x} \leq 2^0$$

$$\Leftrightarrow \quad 2^{2-2|x|-x} \leq 2^0 \quad \Leftrightarrow \quad 2 - 2|x| - x \leq 0$$

$$\Leftrightarrow \begin{cases} x \geq 0 \\ 2 - 3x \leq 0 \end{cases} \quad \text{o} \quad \begin{cases} x < 0 \\ 2 + 2x - x \leq 0 \end{cases}$$

$$\Rightarrow \quad \underline{\underline{S = \left]-\infty, -2\right] \cup \left[\tfrac{2}{3}, +\infty\right[}} \;;$$

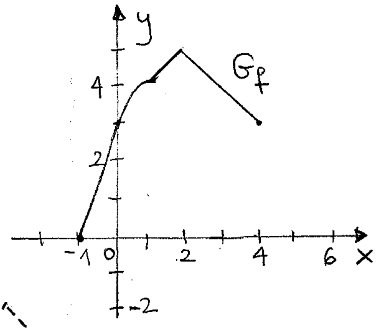
$$\log_3(x+1) - \log_3(2x) \geq \log_3(2x-1) \quad \Leftrightarrow \begin{cases} x + 1 > 0 \\ x > 0 \\ 2x - 1 > 0 \\ \dfrac{x+1}{2x} \geq 2x - 1 \end{cases}$$

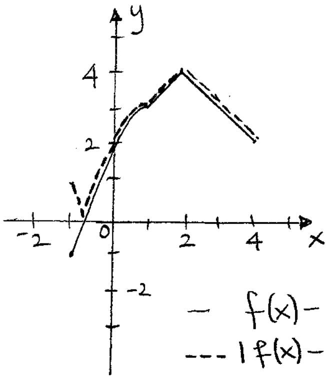
$$\Leftrightarrow \begin{cases} x > \frac{1}{2} \\ \dfrac{x+1}{2x} - 2x + 1 \geq 0 \end{cases} \Leftrightarrow \begin{cases} x > \frac{1}{2} \\ \dfrac{-4x^2 + 3x + 1}{2x} \geq 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x > \frac{1}{2} \\ x \in \left]-\infty, -\tfrac{1}{4}\right] \cup \left]0, 1\right] \end{cases} \quad \Rightarrow \quad \underline{\underline{S = \left]\tfrac{1}{2}, 1\right]}}. \quad \blacksquare$$

5) i) $f : [-1,4] \to [0,5]$

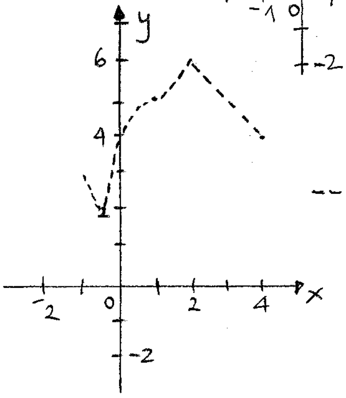
$$f(x) = \begin{cases} -(x-1)^2+4 & \text{se } -1 \le x < 1 \\ -|x-2|+5 & \text{se } 1 \le x \le 4 \end{cases}$$





— $f(x)-1$

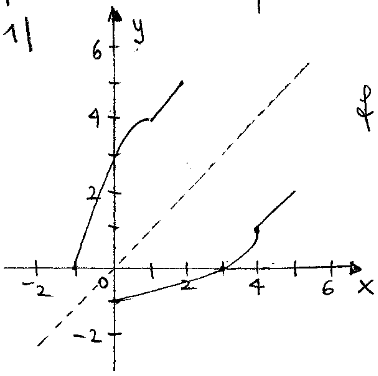
--- $|f(x)-1|$



--- $|f(x)-1|+2$.

☐

ii) $I = [-1, 2]$

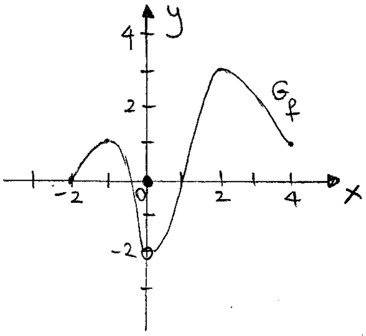


$f|_{[-1,2]} : [-1,2] \to [0,5]$

$f^{-1} : [0,5] \to [-1,2]$

■

6)



i) f è limitata : infatti $-2 \le f(x) \le 3 \quad \forall x \in [-2,4]$.

☐

ii) $\nexists \min\limits_{[-2,4]} f$;

$\max\limits_{[-2,4]} f = 3 \qquad x = 2$ pt. di massimo.

☐

iii) $f([-2,4]) = \,]-2, 3]$.

☐

iv) in $[-2,-1]$, f è crescente ;
in $[-1, 0[$, f è decrescente;
in $]0, 2]$, f è crescente ;
in $[2, 4]$, f è decrescente.

■

Compito Ⓓ

1) i) $f:\mathbb{R}\to\mathbb{R}$, $f(x)=x^2-4|x|$ (vedi commento Es. 1 i) compito Ⓐ).

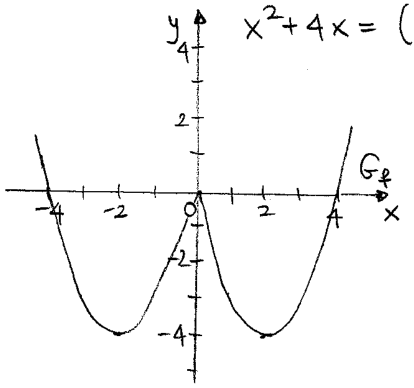
Abbiamo

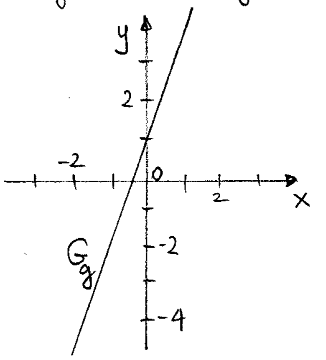
$$f(x)=\begin{cases} x^2-4x & \text{se } x\geq 0\\ x^2+4x & \text{se } x<0.\end{cases}$$

Notiamo che $x^2-4x=(x-2)^2-4$

$x^2+4x=(x+2)^2-4$

$g:\mathbb{R}\to\mathbb{R}$ $g(x)=3x+1$.

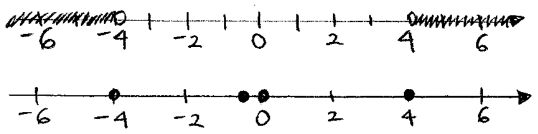




☐

ii) $A=\left\{x\in\mathbb{R}:\ \frac{1}{f(x)}>0\right\}=\ ]-\infty,-4[\ \cup\ ]4,+\infty[$;

$B=\{x\in\mathbb{R}: f(x)\cdot g(x)=0\}=\{x\in\mathbb{R}: f(x)=0 \text{ o } g(x)=0\}=\left\{-4,-\frac{1}{3},0,4\right\}$.



▨ = A ;

● = B .

☐

iii) $A\cup B=\ ]-\infty,-4]\cup\left\{-\frac{1}{3},0\right\}\cup[4,+\infty[$; $A\cap B=\emptyset$.

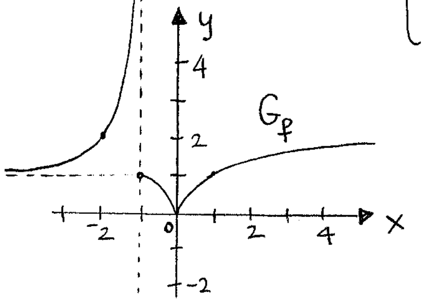
■

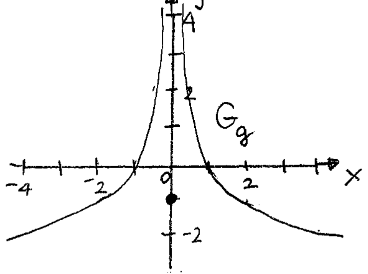
2) i) $f:\mathbb{R}\to\mathbb{R}$

$$f(x)=\begin{cases}\frac{1}{(x+1)^2}+1 & \text{se } x<-1\\ |\sqrt[3]{x}| & \text{se } x\geq -1;\end{cases}$$

$g:\mathbb{R}\to\mathbb{R}$

$$g(x)=\begin{cases}\log_{1/2}|x| & \text{se } x\neq 0\\ -1 & \text{se } x=0.\end{cases}$$





☐

ii) se $k<0$ $\nexists$ono soluzioni dell'eq. $f(x)=k$;

se $k=0$ $\exists$ una soluzione " ;

se $k>0$ $\exists$ono due soluzioni " .

☐

iii) $(f+g)(0) = f(0)+g(0) = 0-1 = \underline{\underline{-1}}$; $\quad g(-2)\cdot g(2) = (-1)\cdot(-1) = \underline{\underline{1}}$;

$\left(\frac{f}{g}\right)(1) \; \nexists$ poichè $g(1) = 0$! □

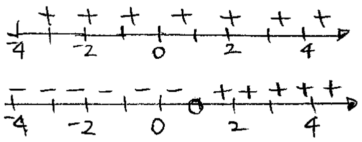
iv) Osserviamo che, per $x>1$, $f(x)>1$ e quindi $(g\circ f)(x) = g(f(x)) < 0$ poichè $g(\square) < 0$ se $\square > 1$.

Abbiamo quindi che $\underline{\underline{g\circ f \text{ è negativa su }]1,+\infty[}}$. ■

3) i) $f(x) = |x|+1$: insieme di definizione $= \underline{\underline{\mathbb{R}}}$;

$g(x) = \frac{1}{x^3-1}$: insieme di definizione $= \{x\in\mathbb{R} : x^3-1 \neq 0\} =$

$= \{x\in\mathbb{R} : x^3 \neq 1\} = \underline{\underline{\mathbb{R}\setminus\{1\}}}$.



segno di f

segno di g (basta oss. che $x^3 > 1$ per $x>1$, $x^3 = 1$ se $x = 1$ e $x^3 < 1$ per $x<1$).

□

ii) $(g\circ f)(x) \underset{\uparrow}{=} g(f(x)) = \underline{\underline{\frac{1}{(|x|+1)^3 - 1}}}$.

$\{x\in\mathbb{R} : f(x) \neq 1\} = \{x\in\mathbb{R} : |x|+1 \neq 1\}$

$= \underline{\underline{\mathbb{R}\setminus\{0\}}}$

$(f\circ g)(x) \underset{\uparrow}{=} f(g(x)) = \underline{\underline{\left|\frac{1}{x^3-1}\right| + 1}}$.

$\underline{\underline{x\in\mathbb{R}\setminus\{1\}}}$ ■

4) i) $|x^2+2x| > 3 \iff x^2+2x < -3$ o $x^2+2x > 3$;

$\iff x^2+2x+3 < 0$ o $x^2+2x-3 > 0$;

$\Rightarrow \underline{\underline{S =]-\infty,-3[\cup]1,+\infty[}}$;

$2+3|x+2| \leq 5 \iff |x+2| \leq 1 \iff -1 \leq x+2 \leq 1$

$\iff -3 \leq x \leq -1 \Rightarrow \underline{\underline{S = [-3,-1]}}$;

$5x^2 - |2x+3| \geq 0 \iff \begin{cases} 2x+3 \geq 0 \\ 5x^2-(2x+3) \geq 0 \end{cases}$ o $\begin{cases} 2x+3 < 0 \\ 5x^2+(2x+3) \geq 0 \end{cases}$

$$\Longleftrightarrow \begin{cases} x \geq -3/2 \\ 5x^2-2x-3 \geq 0 \end{cases} \quad \text{o} \quad \begin{cases} x < -3/2 \\ 5x^2+2x+3 \geq 0 \end{cases}$$

$$\Longleftrightarrow \begin{cases} x \geq -3/2 \\ x \in \left]-\infty, -\frac{3}{5}\right] \cup [1,+\infty[\end{cases} \quad \text{o} \quad \begin{cases} x<-3/2 \\ \mathbb{R} \end{cases}$$

$$\Longrightarrow \underline{\underline{S = \left]-\infty, -\frac{3}{5}\right] \cup [1,+\infty[.}}$$ □

ii) $$\frac{9^x \cdot 3^{x^2-4}}{3^{-2x}} - 3 = 0 \Longleftrightarrow 3^{2x} \cdot 3^{x^2-4} \cdot 3^{2x} = 3^1$$

$$\Longleftrightarrow 3^{2x+x^2-4+2x} = 3^1$$

$$\Longleftrightarrow x^2+4x-4 = 1$$

$$\Longleftrightarrow x^2+4x-5=0 \Longrightarrow \underline{\underline{S=\{-5,1\}}};$$

$$\log_2(3-|x|) = \log_2(2x^2) \Longleftrightarrow \begin{cases} 3-|x|>0 \\ 2x^2>0 \\ 3-|x| = 2x^2 \end{cases} \Longleftrightarrow \begin{cases} |x|<3 \\ x\neq 0 \\ 3-|x|=2x^2 \end{cases}$$

$$\Longleftrightarrow \begin{cases} -3<x<0 \\ 3+x=2x^2 \end{cases} \quad \text{o} \quad \begin{cases} 0<x<3 \\ 3-x=2x^2 \end{cases}$$

$$\Longleftrightarrow \begin{cases} -3<x<0 \\ x_{1/2} = \frac{1\pm 5}{4} \end{cases} \quad \text{o} \quad \begin{matrix} 0<x<3 \\ x_{1/2} = \frac{-1\pm 5}{4} \end{matrix} \Longrightarrow \underline{\underline{S=\{-1,1\}}}.$$ □

iii) $$\log_{\frac{1}{3}}(x+4) - \log_{\frac{1}{3}} x \leq \log_{\frac{1}{3}}(x-2) \Longleftrightarrow \begin{cases} x+4>0 \\ x>0 \\ x-2>0 \\ \frac{x+4}{x} \geq x-2 \end{cases}$$

$$\Longleftrightarrow \begin{cases} x>2 \\ \frac{x+4}{x} - x+2 \geq 0 \end{cases} \Longleftrightarrow \begin{matrix} x>2 \\ \frac{-x^2+3x+4}{x} \geq 0 \end{matrix}$$

$$\Longleftrightarrow \begin{cases} x>2 \\ ]-\infty,-1] \cup]0,4] \end{cases} \Longrightarrow \underline{\underline{S =]2,4]}};$$ □

$$\left(\frac{1}{2}\right)^{1-x^2} \geq 2^{|x|+1} \Longleftrightarrow 2^{x^2-1} \geq 2^{|x|+1} \Longleftrightarrow x^2-1 \geq |x|+1$$

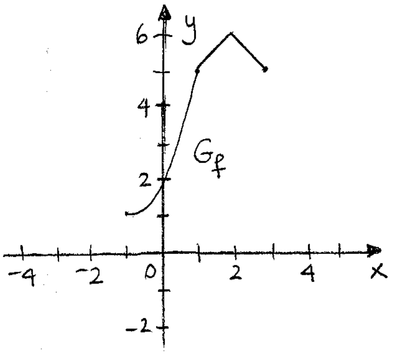
$$\Longleftrightarrow \begin{cases} x\geq 0 \\ x^2-x-2 \geq 0 \end{cases} \quad \text{o} \quad \begin{cases} x<0 \\ x^2+x-2\geq 0 \end{cases}$$

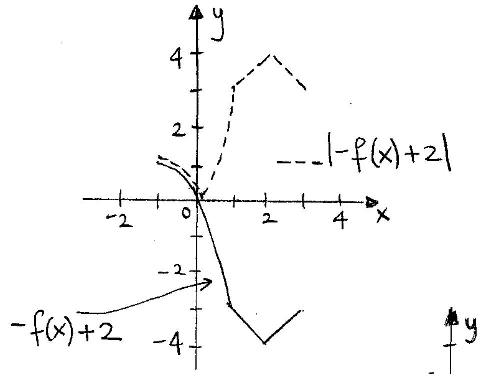
$$\Longleftrightarrow \begin{cases} x\geq 0 \\ x\in]-\infty,-1] \cup [2,+\infty[\end{cases} \quad \text{o} \quad \begin{cases} x<0 \\ x\in]-\infty,-2] \cup [1,+\infty[\end{cases}$$

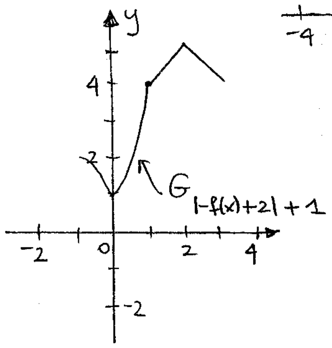
$$\Longrightarrow \underline{\underline{S=]-\infty,-2] \cup [2,+\infty[.}}$$ ■

5) i) $f:[-1,3]\to[1,6]$

$$f(x)=\begin{cases}(x+1)^2+1 & \text{se } -1\le x\le 1\\ -|x-2|+6 & \text{se } 1<x\le 3\end{cases}$$

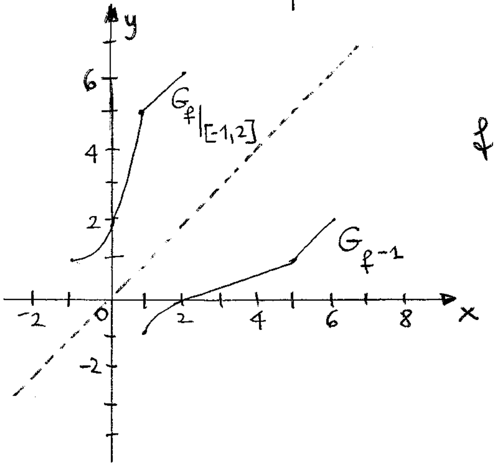






□

ii) $I=[-1,2]$

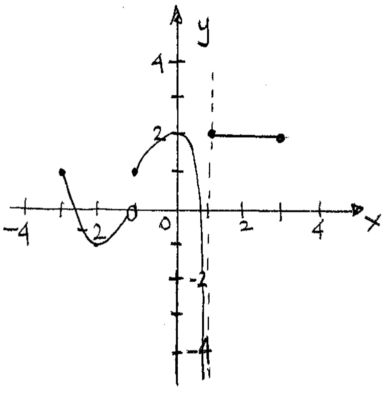


$f|_{[-1,2]}:[-1,2]\to[1,6]$

$f^{-1}:[1,6]\to[-1,2].$

■

6)



i) f è limitata superiormente: $f(x)\le 2\ \forall x\in[-3,3]$

non è limitata inferiormente: infatti, $\forall m<0$ fissato, $\exists\ x\in[-3,3]$ t.c. $f(x)<m$.

Quindi f non è limitata. □

ii) $\max\limits_{[-3,3]} f = 2$, $x\in\{0\}\cup[1,3]$ sono pti di max.

$\nexists \min\limits_{[-3,3]} f$. □

iii) $f([-3,3])=]-\infty,2]$. □

iv) In $[-3,-2]$, f è decrescente; in $[-2,0]$, f è crescente; in $[0,1[$ f è decrescente; in $[1,3]$ f è costante (f è debolmente crescente come debolmente decrescente).

■