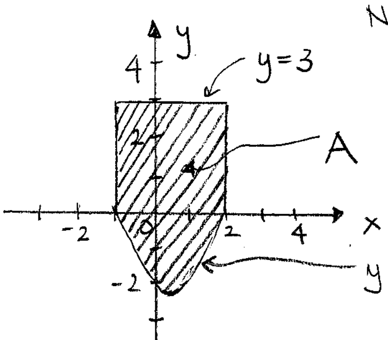
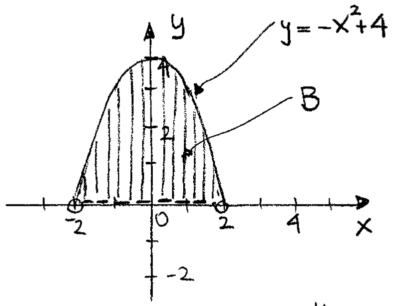
Università degli studi di Trento — Facoltà di Scienze Cognitive
CdL in Interfacce e Tecnologie della Comunicazione
CdL in Scienze e Tecniche di Psicologia Cognitiva
Verifica settimanale di ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2008 – 2009 — Rovereto, 13 – 17/10/08

1) i) $A = \{(x,y) \in \mathbb{R}^2 : -1 \leq x \leq 2 ,\ (x+1)(x-2) \leq y \leq 3\}$

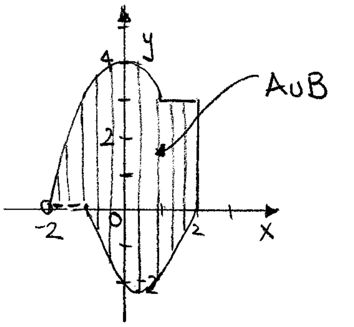
NOTA: $(x+1)(x-2) = x^2 - x - 2 = \left(x - \frac{1}{2}\right)^2 - \frac{9}{4}$

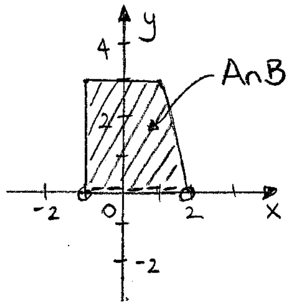


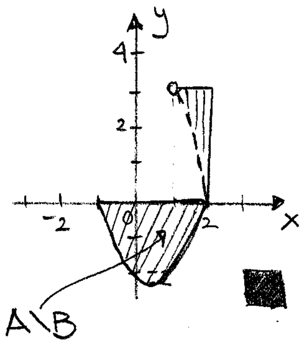
$B = \{(x,y) \in \mathbb{R}^2 : 0 < y \leq -x^2 + 4\}$



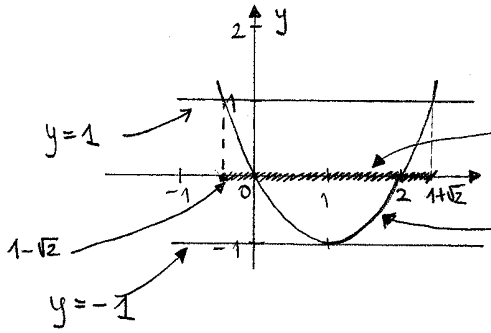
ii)







2) i) $-1 \leq x^2 - 2x \leq 1 \iff \begin{cases} 0 \leq x^2 - 2x + 1 \\ x^2 - 2x - 1 \leq 0 \end{cases} \iff \begin{cases} 0 \leq (x-1)^2 \\ 1-\sqrt{2} \leq x \leq 1+\sqrt{2} \end{cases}$

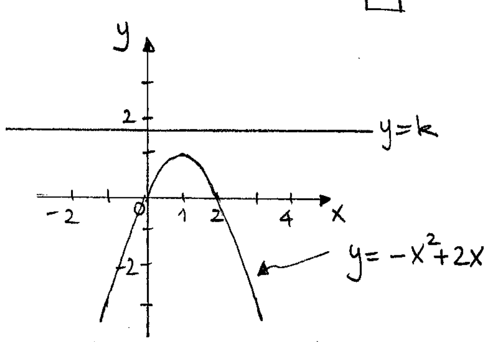


$\underline{S = [1-\sqrt{2},\ 1+\sqrt{2}]}$

$y = x^2 - 2x \iff y = (x-1)^2 - 1$

$-x^2 + 2x = k$ al variare di $k \in \mathbb{R}$

$-(x-1)^2 + 1 = k$ "



Dalla rappresentazione grafica si deduce che se $k > 1$ non esiste una

Soluzione dell'eq. $-x^2+2x=k$ (cioè la retta $y=k$ non interseca la parabola $y=-x^2+2x$);

se $k=1$ esiste unica e corrisponde ad $x=1$;

se $k<1$ esistono due soluzioni dell'eq. $-x^2+2x=k$.

Per determinarle esplicitamente si risolve

$$-x^2+2x-k=0.$$

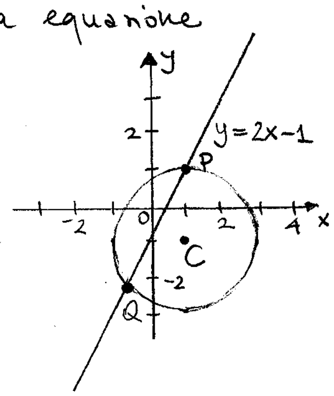
Si ottiene $x_{1/2}=\dfrac{-2\pm\sqrt{4-4k}}{-2}=\dfrac{-\cancel{2}\pm\cancel{2}\sqrt{1-k}}{-\cancel{2}}=+1\pm\sqrt{1-k}$.

Si deduce allora se $k>1$ $\nexists$ soluzione,
se $k=1$ $\exists!$ soluzione $x=1$,
se $k<1$ si hanno $\underline{\underline{x_{1/2}=1\pm\sqrt{1-k}}}$.

$\square$

ii) La circonferenza di centro $(1,-1)$ e raggio 2 ha equazione

$$(x-1)^2+(y+1)^2=4$$



Abbiamo la retta di equazione $y=2x-1$.

Le coordinate di P e Q si ottengono risolvendo il sistema di equazioni

$$\begin{cases}(x-1)^2+(y+1)^2=4\\ y=2x-1.\end{cases}$$

Si ha $\begin{cases}(x-1)^2+(2x-\cancel{1}+\cancel{1})^2=4\\ y=2x-1\end{cases} \Leftrightarrow \begin{cases}x^2-2x+1+4x^2=4\\ y=2x-1\end{cases}$

$$\Leftrightarrow \begin{cases}5x^2-2x-3=0\\ y=2x-1\end{cases} \Leftrightarrow \begin{cases}x_{1/2}=\dfrac{2\pm\sqrt{4+60}}{10}\\ y=2x-1\end{cases} \Leftrightarrow \begin{cases}x_{1/2}= -\frac{3}{5},\ 1\\ y=2x-1\end{cases}$$

$$\Rightarrow Q=\left(-\frac{3}{5},-\frac{11}{5}\right) \qquad P=(1,1).$$

Infine

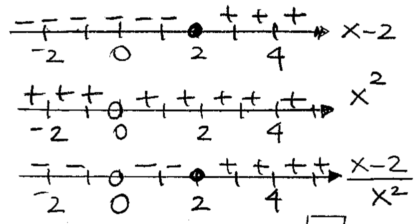
$$d(P,Q)=\sqrt{\left(1+\frac{3}{5}\right)^2+\left(1+\frac{11}{5}\right)^2}=\sqrt{\frac{64}{25}+\frac{256}{25}}=\sqrt{\frac{320}{25}}$$

$$=\underline{\underline{\frac{8\sqrt{5}}{5}}}.$$

$\blacksquare$

3) • $\frac{(x+2)(x-1)}{x^2} \geqslant 1 \iff \frac{\cancel{x^2}+x-2-\cancel{x^2}}{x^2} \geqslant 0$

$\Rightarrow \underline{\underline{S=[2,+\infty[}}$



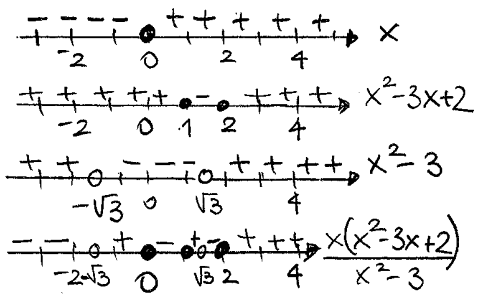
□

• $\frac{(x^2-3x+3)x}{x^2-3} \leq \frac{x}{x^2-3} \iff$

$\iff \frac{x^3-3x^2+3x-x}{x^2-3} \leq 0$

$\iff \frac{x^3-3x^2+2x}{x^2-3} \leq 0$

$\iff \frac{x(x^2-3x+2)}{x^2-3} \leq 0$



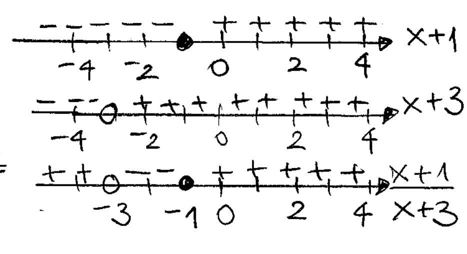
$\Rightarrow \underline{\underline{S=]-\infty,-\sqrt{3}[\cup[0,1]\cup]\sqrt{3},2]}}$

□

• $\frac{x(x+1)}{x+3} < x+1 \iff \frac{x^2+x-(x+1)(x+3)}{x+3} < 0$

$\iff \frac{\cancel{x^2}+x-\cancel{x^2}-4x-3}{x+3} < 0$

$\iff \frac{3(x+1)}{x+3} > 0$

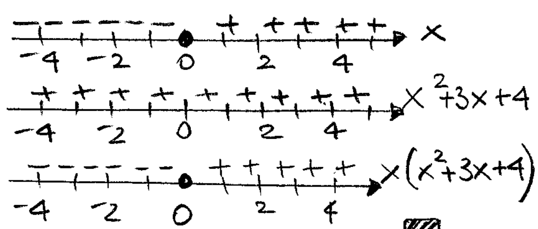


$\Rightarrow \underline{\underline{S=]-\infty,-3[\cup]-1,+\infty[}}$

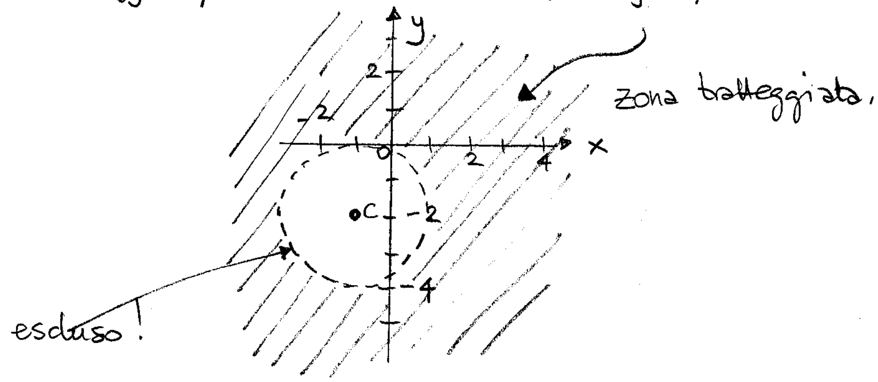
□

• $x^3+3x^2+4x>0 \iff x(x^2+3x+4)>0$

$\Rightarrow \underline{\underline{S=]0,+\infty[}}$.



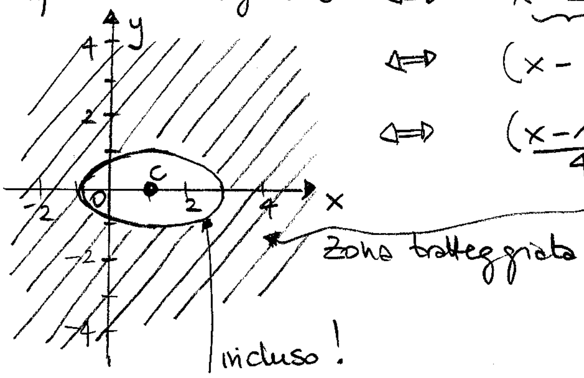
4) i) $4(x+1)^2+4(y+2)^2>16 \iff (x+1)^2+(y+2)^2>4$



ii) $x^2-2x+4y^2\geq 3 \iff \underbrace{x^2-2x+1}-1+4y^2\geq 3$

$\iff (x-1)^2+4y^2\geq 4$

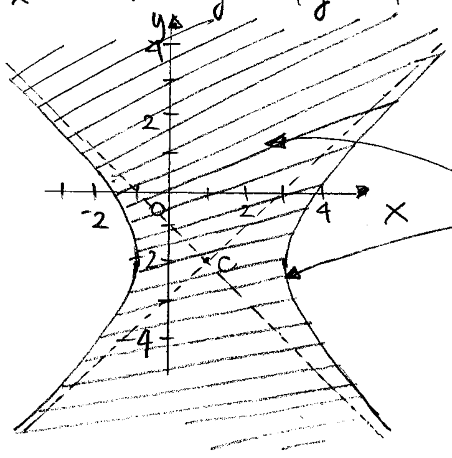
$\iff \frac{(x-1)^2}{4}+y^2\geq 1$



iii) $x^2-2x-y^2-4y-7\leq 0 \iff \underbrace{x^2-2x+1}-1-\underbrace{y^2-4y-4}+4-7\leq 0$

$\iff (x-1)^2-(y+2)^2\leq 4$

$\iff \frac{(x-1)^2}{4}-\frac{(y+2)^2}{4}\leq 1$

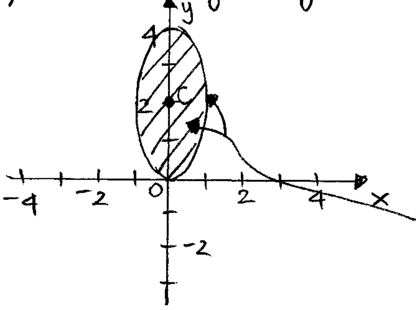


iv) $-4x^2-y^2+4y\geq 0 \iff -4x^2-(y^2-4y)\geq 0$

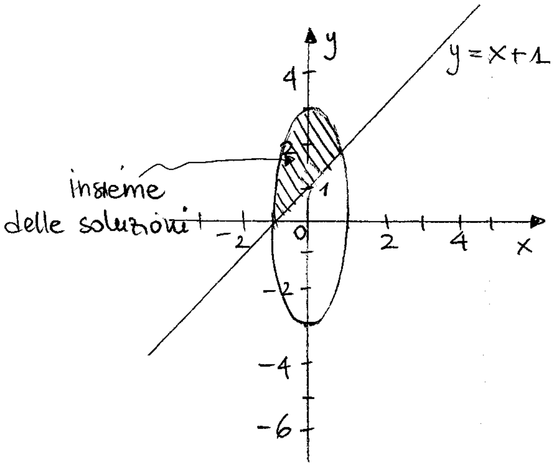
$\iff -4x^2-(y-2)^2+4\geq 0$

$\iff -4x^2-(y-2)^2\geq -4$

$\iff x^2+\frac{(y-2)^2}{4}\leq 1$



5) ⓐ $\begin{cases} y \geq x+1 \\ x^2 + \dfrac{y^2}{9} \leq 1 \end{cases}$



ⓑ $\begin{cases} y > 2 \\ x^2 - y^2 \leq 4 \end{cases}$

$\Leftrightarrow \begin{cases} y > 2 \\ \dfrac{x^2}{4} - \dfrac{y^2}{4} \leq 1 \end{cases}$