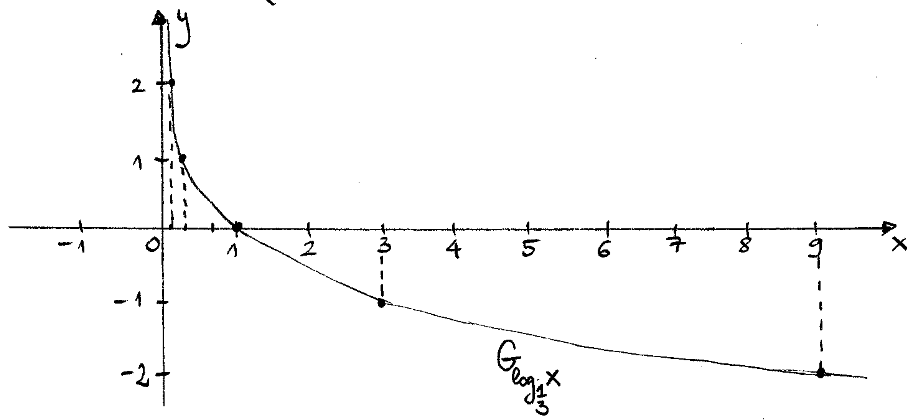
Università degli Studi di Trento — Facoltà di Scienze Cognitive
CdL in Interfacce e Tecnologie della Comunicazione
CdL in Scienze e Tecniche di Psicologia Cognitiva
Verifica settimanale di ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2008-2009 — Rovereto, 17 – 21/11/08

1) i) $\log_{\frac{1}{3}} \frac{1}{9} = \underline{\underline{2}}$ poiché $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$; $\log_{\frac{1}{3}} \frac{1}{3} = \underline{\underline{1}}$ poiché $\left(\frac{1}{3}\right)^1 = \frac{1}{3}$;

$\log_{\frac{1}{3}} 1 = \underline{\underline{0}}$ poiché $\left(\frac{1}{3}\right)^0 = 1$; $\log_{\frac{1}{3}} 3 = \underline{\underline{-1}}$ poiché $\left(\frac{1}{3}\right)^{-1} = 3$;

$\log_{\frac{1}{3}} 9 = \underline{\underline{-2}}$ poiché $\left(\frac{1}{3}\right)^{-2} = 9$



$\square$

ii) $\log_{\frac{1}{3}} 81 = \underline{\underline{-4}}$, poiché $\left(\frac{1}{3}\right)^x = 81 \iff 3^{-x} = 3^4 \iff x = -4$;

$\log_4 \frac{1}{16} = \underline{\underline{-2}}$, poiché $4^x = \frac{1}{16} \iff 4^x = 4^{-2} \iff x = -2$;

$\log_4 64 = \underline{\underline{3}}$, poiché $4^x = 64 \iff 4^x = 4^3 \iff x = 3$;

$\log_3 3^{-6} = \underline{\underline{-6}}$, poiché $\log_3 3^{-6} = -6 \log_3 3 = -6 \cdot 1 = -6$;

$e^{\log 2} = \underline{\underline{2}}$ poiché $\log x$ è la funzione inversa della funzione e^x. $\blacksquare$

2) $3^{-x+2} = 27 \iff 3^{-x+2} = 3^3 \iff -x+2 = 3 \iff x = -1$.

$\underline{\underline{S = \{-1\}}}$;

$2^x = -\frac{1}{2}$ $\underline{\underline{S = \phi}}$ poiché $2^x > 0 \quad \forall x \in \mathbb{R}$;

$\log_2 x = -2$ $\underline{\underline{S = \{2^{-2}\}}} = \left\{\frac{1}{4}\right\}$;

$\log_5 (x^2+1) = 1 \iff x^2+1 = 5 \iff x^2 = 4 \iff \underline{\underline{S = \{-2, 2\}}}$;

$2^{|x|+1} < 4^x \iff 2^{|x|+1} < 2^{2x} \underset{2^x \nearrow}{\iff} |x|+1 < 2x \iff$

$\begin{cases} x \geq 0 \\ x+1 < 2x \end{cases}$ o $\begin{cases} x < 0 \\ -x+1 < 2x \end{cases}$

$$\begin{cases} x \geq 0 \\ x > 1 \end{cases} \quad o \quad \begin{cases} x < 0 \\ 3x > 1 \end{cases} \iff \begin{cases} x > 1 \end{cases} \quad o \quad \underbrace{\begin{cases} x < 0 \\ x > \frac{1}{3} \end{cases}}_{\emptyset}$$

$$\Rightarrow \underline{\underline{S =]1, +\infty[}};$$

$$e^{x^2-2x} > 1 \iff x^2-2x > 0 \Rightarrow \underline{\underline{S =]-\infty, 0[\cup]2, +\infty[}};$$

$$\log(x+2) < 0 \iff \begin{cases} x+2 > 0 \\ x+2 < 1 \end{cases} \iff \begin{cases} x > -2 \\ x < -1 \end{cases} \Rightarrow \underline{\underline{S =]-2, -1[}};$$

$$\log_{\frac{1}{2}} |x| \geq 0 \iff \begin{cases} |x| \neq 0 \\ |x| \leq 1 \end{cases} \iff \begin{cases} x \neq 0 \\ -1 \leq x \leq 1 \end{cases} \Rightarrow \underline{\underline{S = [-1, 1] \setminus \{0\}}};$$

$$\log_2 |x-1| > 0 \iff \begin{cases} |x-1| \neq 0 \\ |x-1| > 1 \end{cases} \iff \begin{cases} x \neq 1 \\ x-1 < -1 \quad o \quad x-1 > 1 \end{cases}$$

$$\iff \begin{cases} x \neq 1 \\ x < 0 \quad o \quad x > 2 \end{cases} \Rightarrow \underline{\underline{S =]-\infty, 0[\cup]2, +\infty[}}.$$

■

3) i) $(\sqrt{x})^2 = x$ $\quad \underline{\underline{\forall x \geq 0}}$ (altrimenti non è definito $\sqrt{x}$);

$\sqrt[3]{x^3} = x$ $\quad \underline{\underline{\forall x \in \mathbb{R}}}$;

$\sqrt{x^2} = |x|$ $\quad \underline{\underline{\forall x \in \mathbb{R}}}$;

$3^{\log_3(x+1)} = x+1$ $\quad \forall x : \ x+1 > 0$ ossia $\underline{\underline{\forall x > -1}}$;

$\log_2 (2^{x+1}) = x+1$ $\quad \underline{\underline{\forall x \in \mathbb{R}}}.$

□

ii) a) $$2^{|x|} \cdot 2^{-x} \leq 4^{x^2} \iff 2^{|x|-x} \leq 2^{2x^2} \underset{2^x \nearrow}{\iff} |x| - x \leq 2x^2$$

$$\iff \begin{cases} x \geq 0 \\ x - x \leq 2x^2 \end{cases} \quad o \quad \begin{cases} x < 0 \\ -x - x \leq 2x^2 \end{cases}$$

$$\iff \begin{cases} x \geq 0 \\ 2x^2 \geq 0 \end{cases} \quad o \quad \begin{cases} x < 0 \\ 0 \leq 2x^2 + 2x \end{cases}$$

$$\iff \begin{cases} x \geq 0 \end{cases} \quad o \quad \begin{cases} x < 0 \\ x \leq -1 \quad o \quad x \geq 0 \end{cases} \iff \underline{\underline{S =]-\infty, -1] \cup [0, +\infty[}};$$

$$\left(\frac{1}{3}\right)^{x^2-2} \cdot 3^x \geq 1 \iff 3^{-x^2+2} \cdot 3^x \geq 1 \iff 3^{-x^2+x+2} \geq 1$$

$$\iff -x^2+x+2 \geq 0 \iff x^2-x-2 \leq 0 \Rightarrow \underline{\underline{S = [-1, 2]}};$$

$$4^x = 2 \cdot 2^{x^2-4} \iff 2^{2x} = 2^{x^2-3} \iff 2x = x^2-3$$

$$\iff x^2-2x-3=0 \qquad \underline{\underline{S=\{-1,3\};}}$$

b) $$\log(1+3x^2) - \log x^2 \geq \log 4 \iff \begin{cases} 1+3x^2>0 \\ x^2>0 \\ \log\left(\frac{1+3x^2}{x^2}\right) \geq \log 4 \end{cases}$$

$$\iff \begin{cases} \mathbb{R} \\ \mathbb{R}\setminus\{0\} \\ \frac{1+3x^2}{x^2} \geq 4 \end{cases} \iff \begin{cases} \mathbb{R}\setminus\{0\} \\ \frac{1-x^2}{x^2} \geq 0 \end{cases} \iff \begin{cases} \mathbb{R}\setminus\{0\} \\ -1 \leq x \leq 1 \ , \ x \neq 0 \end{cases}$$

$$\Rightarrow \underline{\underline{S=[-1,1]\setminus\{0\}}}$$

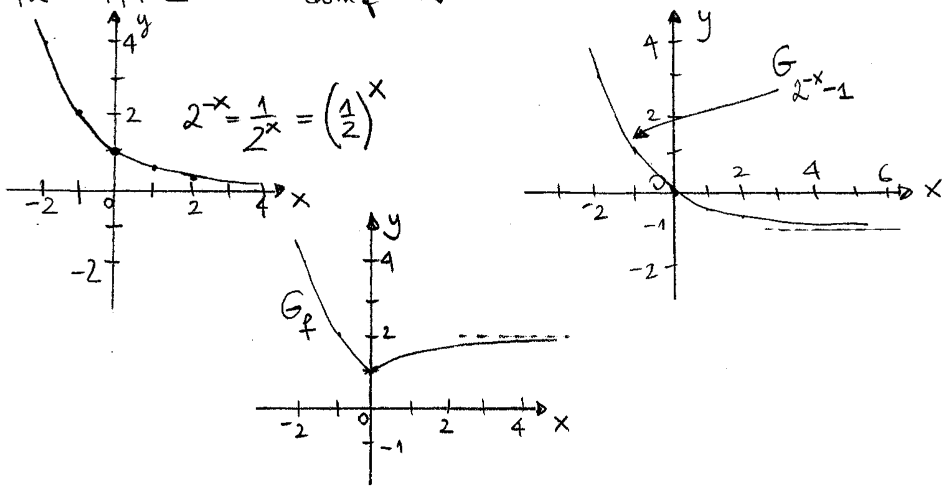
$$\log_{\frac{1}{3}}(|x|+1) + \log_{\frac{1}{3}}\frac{1}{3} < 0 \iff \begin{cases} |x|+1>0 \\ \log_{\frac{1}{3}}\left(\frac{1}{3}(|x|+1)\right) < 0 \end{cases} \iff \begin{cases} x \in \mathbb{R} \\ \frac{1}{3}(|x|+1) > 1 \end{cases}$$

$$\iff \begin{cases} x \in \mathbb{R} \\ |x|+1>3 \end{cases} \iff \begin{cases} x \in \mathbb{R} \\ |x|>2 \end{cases} \Rightarrow \underline{\underline{S=]-\infty,-2[\ \cup\ ]2,+\infty[;}}$$

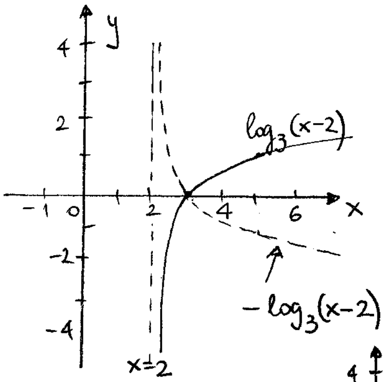
$$\log(2x) = \log(x^2-3) \iff \begin{cases} x>0 \\ x^2-3>0 \\ 2x = x^2-3 \end{cases} \iff \begin{cases} x>0 \\ x<-\sqrt{3} \text{ o } x>\sqrt{3} \\ x^2-2x-3=0 \end{cases}$$

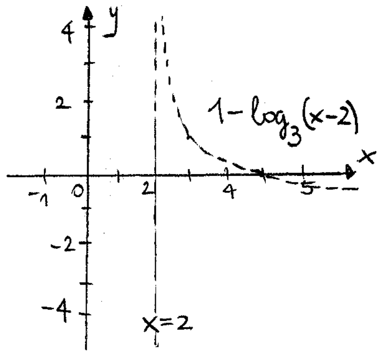
$$\iff \begin{cases} x>0 \\ x<-\sqrt{3} \text{ o } x>\sqrt{3} \\ x=-1 \text{ o } x=3 \end{cases} \Rightarrow \underline{\underline{S=\{3\}}}$$

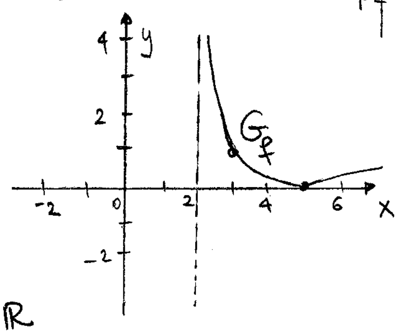
4) $f(x) = |2^{-x}-1|+1 \qquad \operatorname{dom} f = \mathbb{R}$



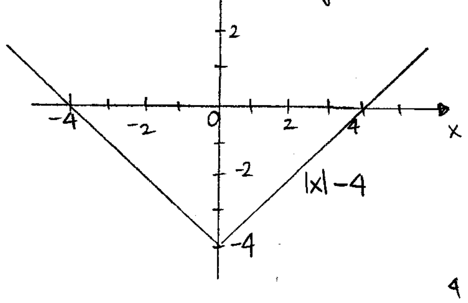
$f(x) = |1 - \log_3(x-2)|$ $\qquad$ $\text{dom} f =]2, +\infty[$

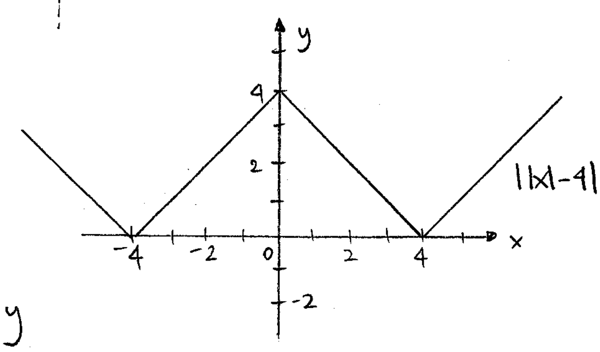


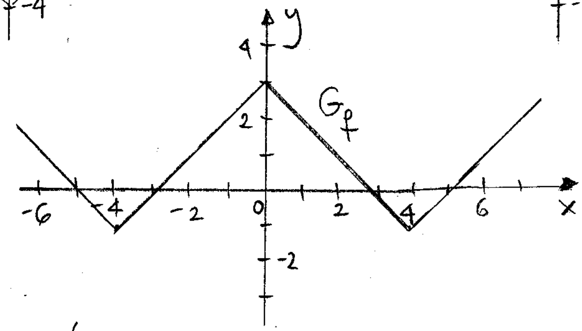




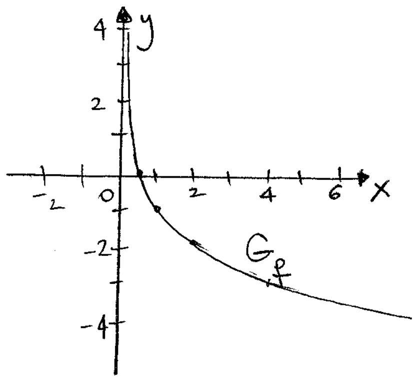
$f(x) = ||x| - 4| - 1$ $\qquad$ $\text{dom} f = \mathbb{R}$





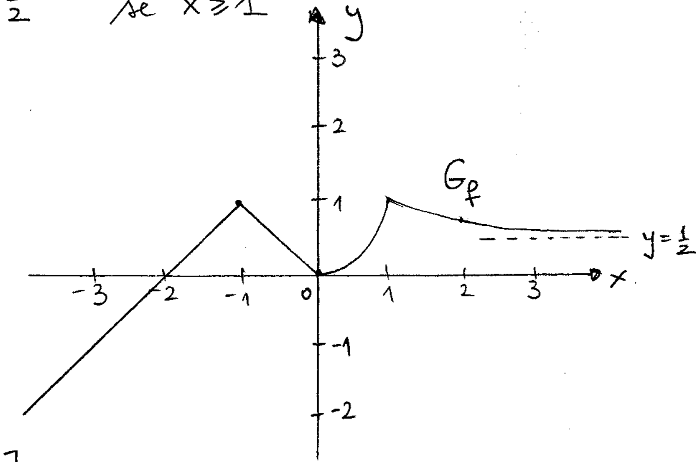


$f(x) = \log_{\frac{1}{2}} x + \log_{\frac{1}{2}} 2 = \left(\log_{\frac{1}{2}} x\right) - 1$ $\qquad$ $\text{dom} f =]0, +\infty[$



5) i) $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} -|x+1|+1 & \text{se } x<0 \\ x^2 & \text{se } 0 \le x < 1 \\ \left(\frac{1}{2}\right)^x + \frac{1}{2} & \text{se } x \ge 1 \end{cases}$$



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ii) $f(\mathbb{R}) =]-\infty, 1]$.

Nota: $[-2,2]$ è un intervallo chiuso e limitato.

f è una funzione continua su $[-2,2]$: notiamo che f è continua su $]-\infty,0[$, su $[0,1[$ e su $[1,+\infty[$.

Inoltre osserviamo che $f(x)$ è arbitrariamente vicino a $f(0)=0$ per "x che si avvicina a 0 da destra" (e per "x che si avvicina a 0 da sinistra").

Inoltre $f(x)$ è arbitrariamente vicino a $f(1)=1$ per "x che si avvicina a 1 da sinistra" (e per "x che si avvicina a 1 da destra").

Quindi risulta che f è continua in tutti i pt. di $[-2,2]$.

Allora f soddisfa le ipotesi del teorema di Weierstrass su $[-2,2]$.

Abbiamo $\min\limits_{[-2,2]} f = f(-2) = \underline{\underline{0}}$ — $\underline{\underline{x=-2}}$ pt. di minimo di f su $[-2,2]$; $\underline{\underline{x=0}}$ pt. di minimo di f su "

$\max\limits_{[-2,2]} f = f(-1) = f(1) = \underline{\underline{1}}$ — $\underline{\underline{x=-1,\ x=1}}$ pt. di massimo di f su $[-2,2]$.

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