Università degli Studi di Trento – Facoltà di Scienze Cognitive
CdL in Interfacce e Tecnologie della Comunicazione
CdL in Scienze e Tecniche di Psicologia Cognitiva
Esame scritto di ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2008 – 2009 – Rovereto, 8 giugno 2009

Compito (A)

1) i) $\mathcal{C}$ ha eq. $x^2-4x+y^2+2y+4=0$. L'eq. canonica di $\mathcal{C}$ è

$$(x-2)^2+(y+1)^2=1$$

e il $C=(2,-1)$ è il centro di $\mathcal{C}$. L'eq. della retta r di pendenza $m=\frac{1}{2}$ e passante per C è $y=-1+\frac{1}{2}(x-2)$, ossia

$$y=\frac{1}{2}x-2. \qquad \square$$

ii) $P=(0,-2)$. Dobbiamo calcolare la distanza tra P e C:

$$d(P,C)=\sqrt{(0-2)^2+(-2+1)^2}=\sqrt{4+1}=\sqrt{5}. \qquad \square$$

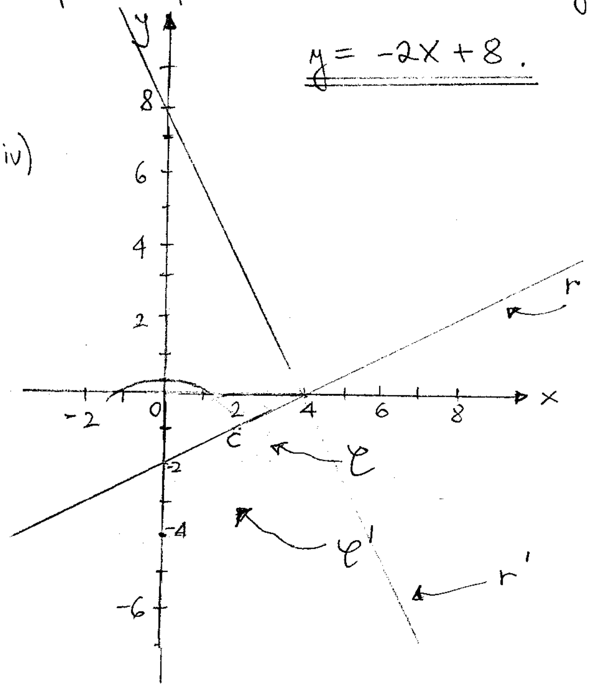
L'eq. canonica di $\tilde{\mathcal{C}}$ è quindi $(x-0)^2+(y-(-2))^2=(\sqrt{5})^2$, ossia

$$x^2+(y+2)^2=5. \qquad \square$$

iii) $S=(4,0)$. L'eq. della retta r' perpendicolare alla retta r e passante per il punto S è data da $y=0-2(x-4)$, ossia

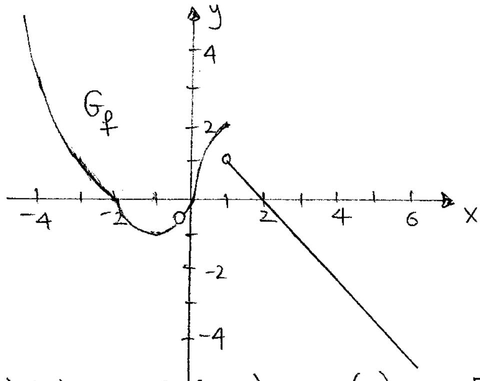
$$y=-2x+8. \qquad \square$$

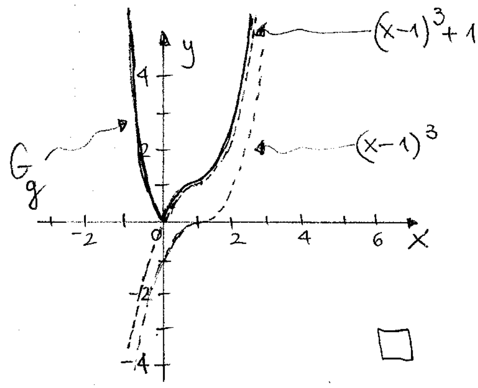
iv)



2) $$f(x)=\begin{cases}\left(\frac{1}{2}\right)^{x+2}-1 & \text{se } x<-2\\ (x+1)^2-1 & \text{se } -2\le x<0\\ 2\sqrt{x} & \text{se } 0\le x\le 1\\ -x+2 & \text{se } x>1\end{cases} \qquad g(x)=\left|(x-1)^3+1\right|$$

i)





ii) $(f\circ g)(1) \underset{\substack{\uparrow\\ \text{se esiste}\\ g(1) \text{ e } f(g(1))}}{=} f(g(1)) = f(1) = 2\sqrt{1} = \underline{\underline{2}}\,;$

$(g\circ f)(1) \underset{\substack{\uparrow\\ \text{se esiste } f(1)\\ \text{e } g(f(1))}}{=} g(f(1)) = g(2\sqrt{1}) = g(2) = \underline{\underline{2}}\,;$

$(f-g)(1) = f(1)-g(1) = 2\sqrt{1} - 1 = 2-1 = \underline{\underline{1}}\,;$

$(fg)(-2) = f(-2)\,g(-2) = \Big[\underbrace{(-2+1)^2-1}_{=0}\Big]\,\Big|\underbrace{(-3)^3+1}_{=26}\Big| = \underline{\underline{0}}\,;$

$\sqrt{f(-1)} = \nexists$ poiché $f(-1)=-1$ e $\sqrt{-1}\ \nexists$.

iii) Abbiamo

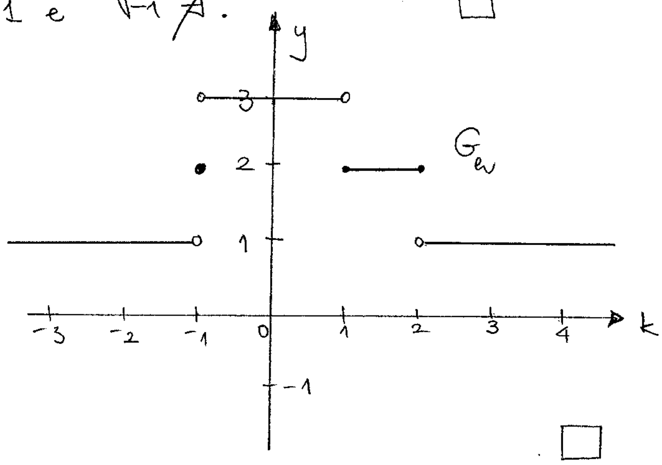
$h(k) = 1$ se $k<-1$

$h(k) = 2$ se $k=-1$

$h(k) = 3$ se $-1<k<1$

$h(k) = 2$ se $1\le k\le 2$

$h(k) = 1$ se $k>2$



iv) a) V basta prendere un $x\in\mathbb{R}$: $f(x)<0$; per esempio, va sempre bene $x\in\,]-2,0[$ oppure un $x\in\,]2,+\infty[$.

b) F basta prendere, per esempio, $x=-3$ e si ha $x\cdot f(x) = -3\cdot f(-3)<0$; oppure $x=4$ e si ha $x\cdot f(x) = 4\cdot f(4)<0$.

c) V basta osservare che $g(x)\ge 0\ \forall x\in\mathbb{R}$.

3) i) $|x^2-2|+x>0 \iff \begin{cases} x^2-2\geq 0 \\ x^2-2+x>0 \end{cases}$ o $\begin{cases} x^2-2<0 \\ -x^2+2+x>0 \end{cases}$

$\iff \begin{cases} x\leq -\sqrt{2} \text{ o } x\geq \sqrt{2} \\ x^2+x-2>0 \end{cases}$ o $\begin{cases} -\sqrt{2}\leq x\leq \sqrt{2} \\ x^2-x-2<0 \end{cases}$

$\iff \begin{cases} x\leq -\sqrt{2} \text{ o } x\geq \sqrt{2} \\ x<-2 \text{ o } x>1 \end{cases}$ o $\begin{cases} -\sqrt{2}\leq x\leq \sqrt{2} \\ -1<x<2 \end{cases}$

$\Rightarrow S =]-\infty,-2[\cup [\sqrt{2},+\infty[\cup]-1,\sqrt{2}]$

$= \underline{\underline{]-\infty,-2[\cup]-1,+\infty[}}\,.$ $\square$

$\log_{\frac{1}{2}}|x+2|<0 \iff \begin{cases} |x+2|>0 \\ |x+2|>1 \end{cases} \iff x+2<-1$ o $x+2>1$

$\Rightarrow S = \underline{\underline{]-\infty,-3[\cup]-1,+\infty[}}\,.$ $\square$

$\frac{7x}{x^2+3}\geq 2 \iff \frac{7x}{x^2+3}-2\geq 0 \iff \frac{-2x^2+7x-6}{x^2+3}\geq 0$

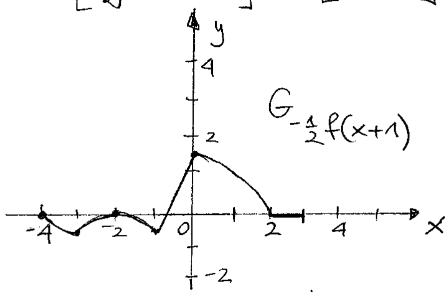
$\iff 2x^2-7x+6\leq 0 \Rightarrow S = \underline{\underline{\left[\frac{3}{2},2\right]}}\,.$ $\square$

ii) $\int_0^1 \sqrt[n]{x}\,dx = \int_0^1 x^{\frac{1}{n}}\,dx = \left.\frac{x^{\frac{1}{n}+1}}{\frac{1}{n}+1}\right]_0^1 = \frac{1}{\frac{1}{n}+1} = \frac{n}{n+1}\,.$

$\sum_{n=1}^{5}\int_0^1 \sqrt[n]{x}\,dx = \sum_{n=1}^{5}\frac{n}{n+1} = \frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6} = \frac{5}{4}+\frac{\cancel{9}^{\,3}}{\cancel{6}_{\,2}}+\frac{4}{5} =$

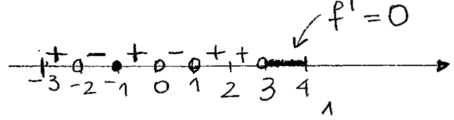
$= \frac{11}{4}+\frac{4}{5} = \frac{71}{20} > \frac{7}{2}\,.$ ■

4) i) $\text{dom}\left[-\frac{1}{2}f(x+1)\right] = [-4,3]$



$\square$

ii)



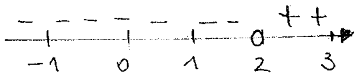
$\square$

iii) a) $F(1)-F(-2) = \int_{-3}^{1} f(t)\,dt - \int_{-3}^{-2} f(t)\,dt = \cancel{\int_{-3}^{-2} f(t)\,dt} + \int_{-2}^{1} f(t)\,dt - \cancel{\int_{-3}^{-2} f(t)\,dt}$

$= \int_{-2}^{1} f(t)\,dt$ V ■

5) $f(x) = \dfrac{1}{2^x - 4}$ i) • $\operatorname{dom} f = \mathbb{R} \setminus \{2\} =]-\infty, 2[\cup]2, +\infty[$

• segno di f



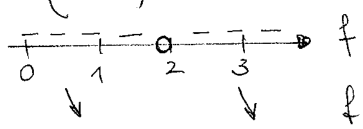
• $\lim_{x \to -\infty} f(x) = -\frac{1}{4}$ $\quad y = -\frac{1}{4}$ è asintoto orizzontale, per $x \to -\infty$;

$\lim_{x \to 2^-} f(x) = -\infty$ $\quad x = 2$ è asintoto verticale (per $x \to 2$);

$\lim_{x \to 2^+} f(x) = +\infty$ $\quad$ " $\quad$ "

$\lim_{x \to +\infty} f(x) = 0$ $\quad y = 0$ è asintoto orizzontale, per $x \to +\infty$.

• $\operatorname{dom} f' = \mathbb{R} \setminus \{2\}$ $\quad f'(x) = \dfrac{-2^x \log 2}{(2^x - 4)^2}$



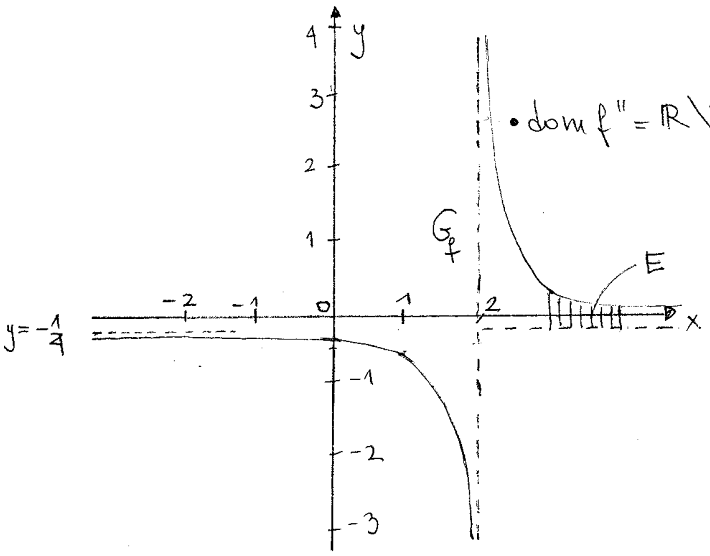
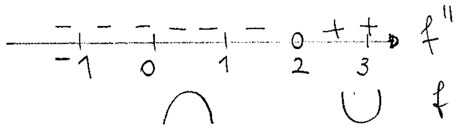


grafico qualitativo di f.

• $\operatorname{dom} f'' = \mathbb{R} \setminus \{2\}$ $\quad f''(x) = \dfrac{-2^x (\log 2)^2 (2^x - 4)^{\cancel{2}} + 2^x (\log 2)^2 2 \cancel{(2^x - 4)} 2^x}{(2^x - 4)^{\cancel{4}\,3}}$

$$= \frac{-2^{2x} (\log 2)^2 + 4 \cdot 2^x (\log 2)^2 + 2 \cdot 2^{2x} (\log 2)^2}{(2^x - 4)^3}$$

$$= \frac{[4 \cdot 2^x + 2^{2x}] (\log 2)^2}{(2^x - 4)^3}$$



□

ii) $y = -\frac{1}{3} + f'(0)(x - 0) \Rightarrow y = -\frac{1}{3} - \frac{\log 2}{9} x \Rightarrow \underline{\underline{y = -\frac{\log 2}{9} x - \frac{1}{3}}}.$

□

iii) $\operatorname{area}(E) = \displaystyle\int_3^4 \left(\frac{1}{2^x - 4} + \frac{1}{4} \right) dx = \int_3^4 \frac{\cancel{4} + 2^x - \cancel{4}}{4(2^x - 4)}\, dx =$

$$= \frac{1}{4 \log 2} \int_3^4 \frac{2^x \log 2}{2^x - 4}\, dx = \frac{1}{4 \log 2} \left[\log |2^x - 4| \right]_3^4 =$$

$$= \frac{1}{4 \log 2} \left[\log \frac{\cancel{12}^{\,3}}{\cancel{4}} \right. = \underline{\underline{\frac{\log 3}{4 \log 2}}}.$$

■

6) $C_{15,3} = \dfrac{15!}{3!\, 12!} = \dfrac{\overset{5}{\cancel{15}} \cdot \overset{7}{\cancel{14}} \cdot 13 \cdot \cancel{12!}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{12!}} = 35 \cdot 13 = \underline{\underline{455}}$

■