Università degli Studi di Trento – Facoltà di Scienze Cognitive
CdL in Interfacce e Tecnologie della Comunicazione
CdL in Scienze e Tecniche di Psicologia Cognitiva
Terza Prova Intermedia di ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
Esame scritto di ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2008 – 2009 – Rovereto, 26 gennaio 2009

Terza Prova Intermedia (A)

1) i) $\lim_{x\to-\infty} \dfrac{3x^2+2^x}{\log|x|+x^2} = \underline{\underline{3}}$ $\left(\text{infatti, } \lim_{x\to-\infty} 2^x = 0 \text{ e } \lim_{x\to-\infty} \dfrac{\log|x|}{x^2} = 0\right.$

e quindi

$$\frac{3x^2+2^x}{\log|x|+x^2} = \frac{\cancel{x^2}\left(3+\underbrace{\frac{2^x}{x^2}}_{\to 0}\right)}{\cancel{x^2}\left(\underbrace{\frac{\log|x|}{x^2}}_{\to 0}+1\right)} \xrightarrow[x\to-\infty]{} 3 \Big)$$

$\lim_{x\to3^-} \dfrac{1-x^2}{x-3} = \underline{\underline{+\infty}}$ $\left(\text{infatti, } \dfrac{\overbrace{(1-x^2)}^{\to -8}}{\underbrace{(x-3)}_{\text{infinitesimo negativo per } x\to3^-}} \xrightarrow[x\to3^-]{} +\infty\right)$

$\lim_{x\to0} \dfrac{e^{x^2}-1}{2x} = \underline{\underline{0}}$ $\left(\text{infatti, } \dfrac{e^{x^2}-1}{2x} = \underbrace{\left(\dfrac{e^{x^2}-1}{x^2}\right)}_{x\to0 \ \to\ 1}\cdot \overbrace{\left(\dfrac{x}{2}\right)}^{\to 0} \xrightarrow[x\to0]{} 0\right).$

ii) $\dfrac{1}{3}-\dfrac{4}{5}+\dfrac{9}{7}-\dots-\dfrac{256}{33} = \underline{\underline{\displaystyle\sum_{n=1}^{16} (-1)^{n+1}\frac{n^2}{2n+1}}}$

2) i) $\lim_{x\to-3^+} f(x) = \underline{\underline{1}}$; $\lim_{x\to1^-} f(x) = \underline{\underline{-1}}$; $\lim_{x\to1^+} f(x) = \underline{\underline{1}}$; $\lim_{x\to+\infty} f(x) = \underline{\underline{2}}$.

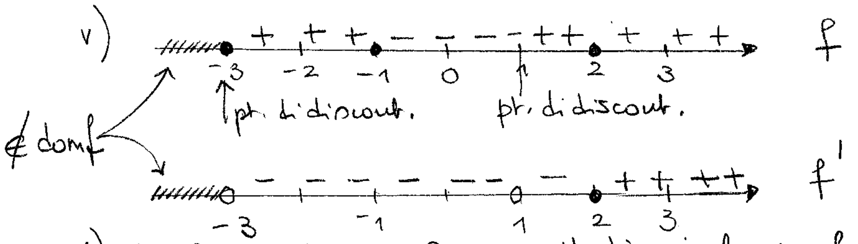
ii) $\underline{\underline{x=-3}}$ $\left(\text{infatti, } f(-3)=0 \neq \lim_{x\to-3^+} f(x)\right)$;

$\underline{\underline{x=1}}$ $\left(\text{infatti, } f(1) = -1 = \lim_{x\to1^-} f(x) \neq \lim_{x\to1^+} f(x)\right)$.

iii) nell' intervallo $]-3, 1]$ f è decrescente,
 " $]1, 2]$ " ;
nell' intervallo $[2, +\infty[$ f è crescente.

iv) $y = 2$ è asintoto orizzontale per f, per $x\to+\infty$.

v)



vi) $x=-3$, $x=1$, $x=2$ sono pt. di min. loc. per f. Nota: $f'(2)=0$, mentre $\nexists f'(-3)$, $\nexists f'(1)$.

3) i) $$\int_0^1 \frac{-1+e^{3x}}{e^x}\,dx = \int_0^1 \left(-e^{-x}+e^{2x}\right)dx = \int_0^1 -e^{-x}\,dx + \frac{1}{2}\int_0^1 2e^{2x}\,dx$$

$$= \left[e^{-x}\right]_0^1 + \frac{1}{2}\left[e^{2x}\right]_0^1 = \left(e^{-1}-1\right)+\frac{1}{2}\left(e^2-1\right)$$

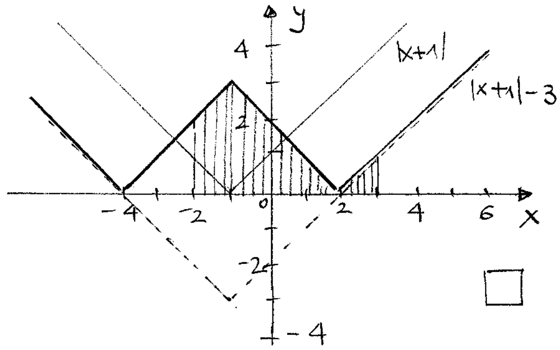
$$= \underline{\underline{\frac{e^2}{2}+\frac{1}{e}-\frac{3}{2}}}\,.$$

$$\int_1^2 x\left(1-x^{-2}\right)dx = \int_1^2 \left(x-\frac{1}{x}\right)dx = \left[\frac{x^2}{2}-\log|x|\right]_1^2 = \left(\frac{4}{2}-\log 2\right)-\left(\frac{1}{2}-\cancel{\log 1}\right)$$

$$= \underline{\underline{\frac{3}{2}-\log 2}}\,.$$

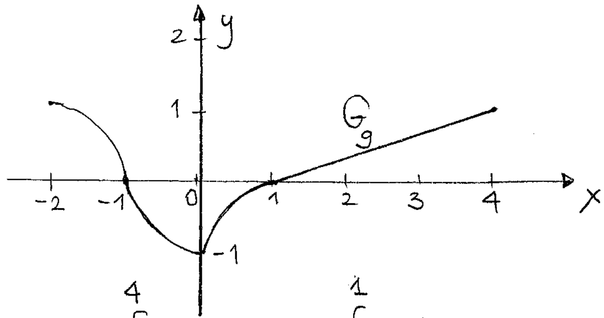
$$\int_{-2}^3 \big||x+1|-3\big|\,dx = \frac{3+2}{2}+\frac{3\cdot 3}{2}+\frac{1\cdot 1}{2}$$

$$= \underline{\underline{\frac{15}{2}}}\,.$$



□

ii)



$$\int_{-2}^4 g(x)\,dx = \int_0^4 g(x)\,dx = \int_0^1 -(x-1)^2\,dx + \int_1^4 \left(\frac{x}{3}-\frac{1}{3}\right)dx$$

nota: $\int_{-2}^0 g(x)\,dx = 0$

$$= -\left[\frac{(x-1)^3}{3}\right]_0^1 + \frac{1\cdot 3}{2} = -\frac{1}{3}+\frac{3}{2} = \underline{\underline{\frac{7}{6}}}\,.$$

$$\sum_{k=1}^4 \left|g\left(\frac{2}{k}\right)\right| = |g(2)| + |g(1)| + \left|g\left(\frac{2}{3}\right)\right| + \left|g\left(\frac{1}{2}\right)\right| =$$

$$= \frac{1}{3}+0+\frac{1}{9}+\frac{1}{4} = \frac{7}{12}+\frac{1}{9} = \underline{\underline{\frac{25}{36}}}$$

■

4) $f(x) = \dfrac{x^3}{x^2+1}$ i) • dom $f = \mathbb{R} = \left]-\infty,+\infty\right[$

nota: f è una funzione dispari !!

• segno di f: $- - - \; 0 \; + + + + +$ f (−2, 0, 2, 4) poiché $x^2+1>0 \;\forall x\in\mathbb{R}$

• $\lim_{x\to-\infty} f(x) = -\infty$ $\quad \lim_{x\to+\infty} f(x) = +\infty$.

Notiamo che f non ha asintoti orizzontali. Osserviamo però che f ha un asintoto obliquo (per $x\to-\infty$, per $x\to+\infty$):

$$\begin{cases} \lim\limits_{\substack{x\to-\infty\\(+\infty)}} \dfrac{f(x)}{x} = \lim\limits_{\substack{x\to-\infty\\(+\infty)}} \dfrac{x^3}{x^3+x} = 1 = a \\ \lim\limits_{\substack{x\to-\infty\\(+\infty)}} \left[f(x)-ax\right] = \lim\limits_{\substack{x\to-\infty\\(+\infty)}} \left(\dfrac{x^3}{x^2+1} - x\right) = \lim\limits_{\substack{x\to-\infty\\(+\infty)}} \left(\dfrac{-x}{x^2+1}\right) = 0 = b \end{cases}$$

e quindi $y=ax+b$, ossia $\underline{\underline{y=x \text{ è asintoto obliquo per } f}}$ (per $x\to-\infty$, per $x\to+\infty$).

- f è continua su tutto $\mathbb{R}$, perché prodotto e rapporto di funzioni continue e il denominatore non si annulla mai.

- $\operatorname{dom} f' = \mathbb{R}$ e $f'(x) = \dfrac{3x^2(x^2+1) - x^3\cdot 2x}{(x^2+1)^2} = \dfrac{3x^4+3x^2-2x^4}{(x^2+1)^2}$

$$= \frac{x^4+3x^2}{(x^2+1)^2} = \frac{x^2(x^2+3)}{(x^2+1)^2}$$

$f'(x)=0 \iff x=0$ e $f(0)=0$

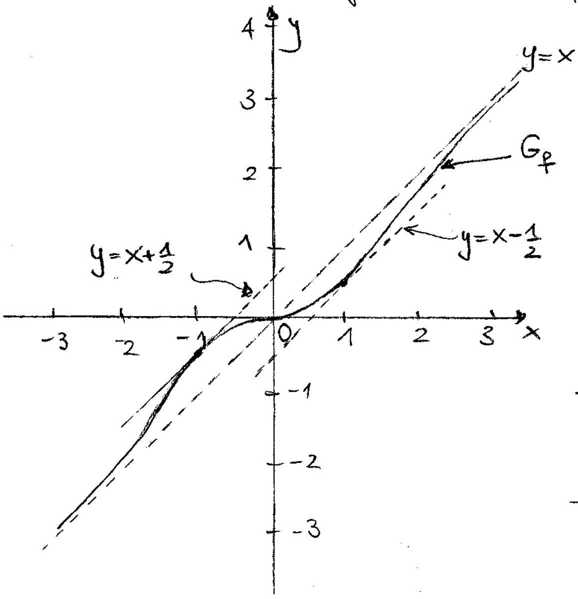
x	-2	-1	0	1	2
$f'(x)$	+ +	+ +	•	+ +	+ +
$f(x)$		↗		↗	

$x=0$ non è né un pt. di min. loc. né un pt. di massimo loc. per f.

- $\operatorname{dom} f'' = \mathbb{R}$ $\quad f''(x) = \dfrac{(4x^3+6x)(x^2+1)^{\cancel{2}} - (x^4+3x^2)\,2\cancel{(x^2+1)}\cdot 2x}{(x^2+1)^{\cancel{4}3}}$

$$= \frac{\cancel{4x^5}+4x^3+6x^3+6x-\cancel{4x^5}-12x^3}{(x^2+1)^3}$$

$$= \frac{-2x^3+6x}{(x^2+1)^3} = \frac{2x(-x^2+3)}{(x^2+1)^3}$$



x	-3	-2	$-\sqrt{3}$	-1	0	1	$\sqrt{3}$	2	3
$2x$	−	−	−	−	•	+	+	+	+
$-x^2+3$	−	−	•	+	+	+	•	−	−
$f''(x)$	+	+	•	−	•	+	•	−	−
$f(x)$	∪			∩		∪		∩	

□

ii) La pendenza della retta tangente al grafico di f in un pt. $(x, f(x))$ è dato da $f'(x)$. Quindi $f'(x) = 1 \iff \dfrac{x^4+3x^2}{(x^2+1)^2} = 1$

$\iff x^4+3x^2 = (x^2+1)^2 \iff \cancel{x^4}+3x^2 = \cancel{x^4}+2x^2+1 \iff x^2 = 1$,

ossia $x = \pm 1$.

Le rette tangenti ricercate avranno dunque:

nel pt. $(-1, f(-1)) = (-1, -\frac{1}{2})$ l'eq. $y = -\frac{1}{2} + (x+1)$

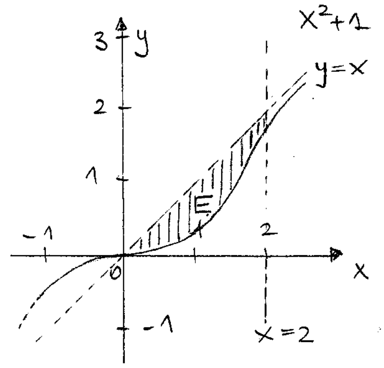
$\underline{\underline{y = x + \frac{1}{2}}}$

nel pt. $(1, f(1)) = (1, \frac{1}{2})$ l'eq. $y = \frac{1}{2} + (x-1)$

$\underline{\underline{y = x - \frac{1}{2}}}$. $\square$

iii) Notiamo che $x - \dfrac{x}{x^2+1} = \dfrac{x^3+\cancel{x}-\cancel{x}}{x^2+1} = \dfrac{x^3}{x^2+1} = f(x) \quad \forall x \in \mathbb{R}$. $\square$

iv)



$$\text{area } E = \int_0^2 (x - f(x))\,dx = \int_0^2 \left(\cancel{x} - \cancel{x} + \frac{x}{x^2+1}\right)dx$$

$$= \frac{1}{2}\int_0^2 \frac{2x}{x^2+1}\,dx = \frac{1}{2}\Big[\log(x^2+1)\Big]_0^2$$

$$= \frac{1}{2}\Big[\log 5 - \cancel{\log 1}\Big] = \underline{\underline{\frac{\log 5}{2}}}$$ $\blacksquare$

5) a) (F) : basta osservare che $f(x) = x^4+x^3-1$ si annulla almeno in un $c \in \,]0,1[$. Infatti f è continua in $\mathbb{R}$, $f(0) = -1$, $f(1) = 1$ e quindi per il teorema di esistenza degli zeri $\exists c \in \,]0,1[: f(c) = 0$. $\square$

b) (V) : Infatti, $\displaystyle\lim_{x\to+\infty} \frac{x^m}{f'(x)} = \lim_{x\to+\infty} \frac{x^m}{4x^3+3x^2} = \frac{1}{4}$ se $m = 3$. $\square$

c) (V) : basta osservare che $f(x) = x^4+x^3-1 \geq 1+\cancel{1}-\cancel{1} \quad \forall x \geq 1$

e quindi $\displaystyle\int_1^2 f(x)\,dx \geq \int_1^2 1\,dx = 1 > 0$ (oppure integrare!!) $\blacksquare$

6) $C_{9,3} = \dfrac{9!}{3!\,6!} = \dfrac{\overset{3}{\cancel{9}} \cdot \overset{4}{\cancel{8}} \cdot 7 \cdot \cancel{6!}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{6!}} = 12 \cdot 7 = \underline{\underline{84}}$. $\blacksquare$

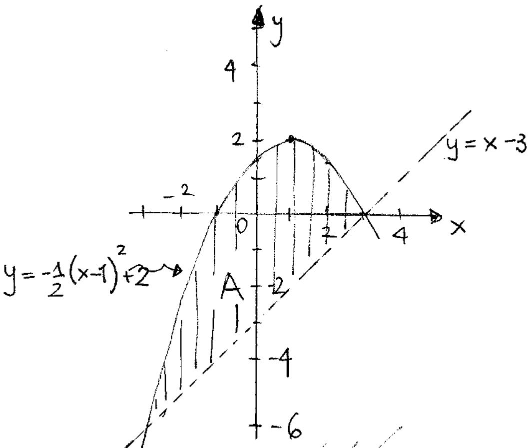
Esame scritto (A)

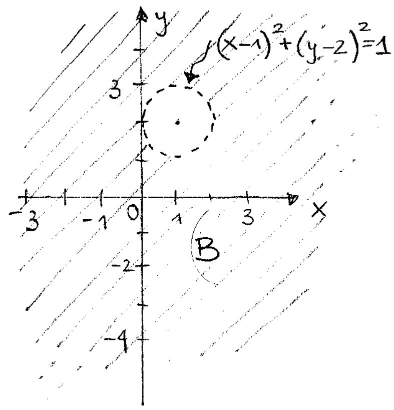
1) i) $A = \{(x,y) \in \mathbb{R}^2 : \; x-3 < y \leq -\frac{1}{2}(x-1)^2+2\}$,

$B = \{(x,y) \in \mathbb{R}^2 : \; x^2-2x+y^2-4y > -4\}$

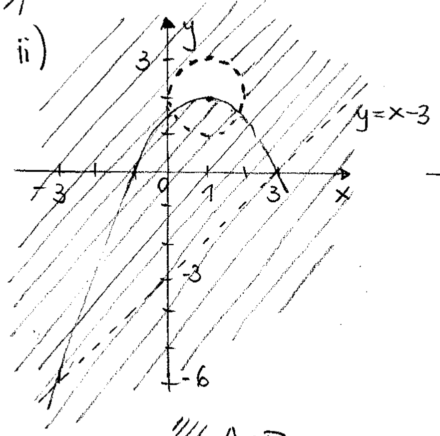
$$x^2-2x+y^2-4y+4>0$$
$$(x-1)^2-1+(y-2)^2>0$$
$$(x-1)^2+(y-2)^2>1$$

$$x-3 = -\tfrac{1}{2}(x-1)^2+2$$
$$\cancel{2x}-6-4 = -x^2+\cancel{2x}-1$$
$$x^2 = 9 \qquad x=\pm 3$$

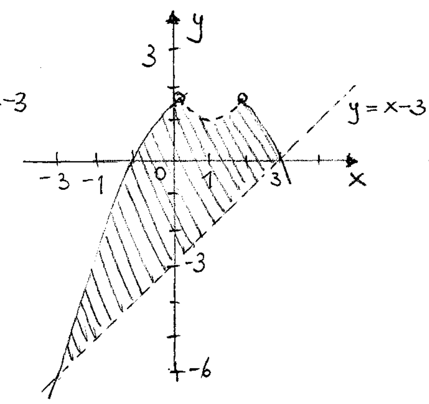




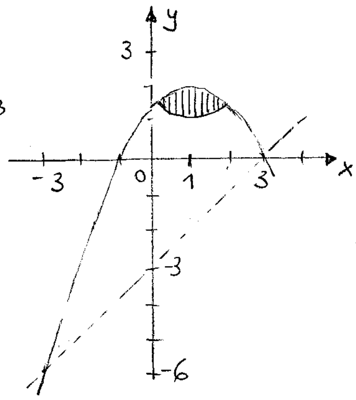
ii)



$A \cup B$



$A \cap B$



$A \setminus B$

$\square$

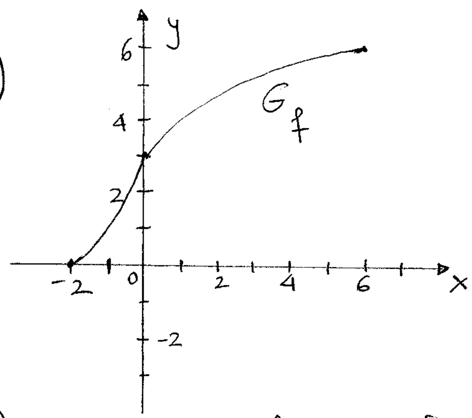
2) $f : [-2,6] \to \mathbb{R}$

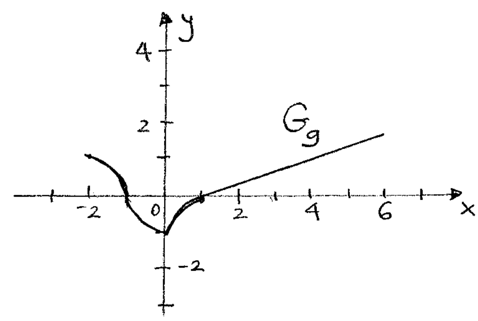
$$f(x) = \begin{cases} 4\cdot 2^x - 1 & \text{se } -2 \leq x \leq 0 \\ 3\log_3(x+3) & \text{se } 0 < x \leq 6; \end{cases}$$

$g : [-2,6] \to \mathbb{R}$

$$g(x) = \begin{cases} -\sqrt[3]{x+1} & \text{se } -2 \leq x \leq 0 \\ -(x-1)^2 & \text{se } 0 < x < 1 \\ \frac{x}{3} - \frac{1}{3} & \text{se } 1 \leq x \leq 6. \end{cases}$$

i)





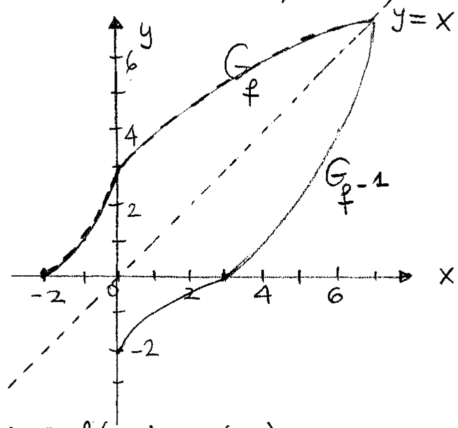
$\square$

ii) Osserviamo che f è una funzione iniettiva: infatti $4\cdot 2^x - 1$ è crescente su $[-2,0]$ e $3\log_3(x+3)$ è crescente su $]0,6]$. Infine

$$4\cdot 2^x - 1 \leqslant 3 \quad \forall x \in [-2,0] \quad e \quad 3 < 3\log_3(x+3) \quad \forall x \in]0,6].$$

Basta scegliere $B = f([-2,6])$ e si ha che f è suriettiva; quindi essendo allora iniettiva e suriettiva, f è biettiva.

Abbiamo $B = f([-2,6]) = [0,6]$.



$f^{-1} : [0,6] \to [-2,6]$

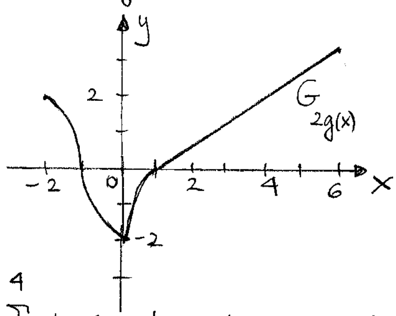
iii) $(f+g)(-2) \doteq f(-2) + g(-2) = 0 + 1 = \underline{\underline{1}}$;

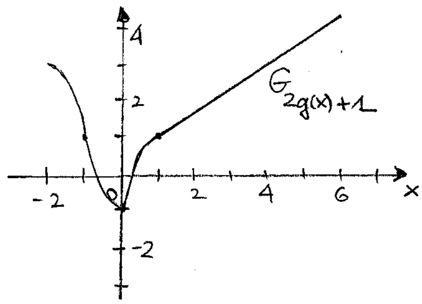
$(fg)(0) \doteq f(0)\cdot g(0) = 3\cdot(-1) = \underline{\underline{-3}}$;

$\left(\frac{f}{g}\right)(1) \ \nexists$ poichè $g(1) = 0$

iv) $f(0) = 3$, $g(3) = \frac{2}{3}$. Quindi $(g\circ f)(0) \underset{se\ \exists}{\doteq} g(f(0)) = g(3) = \frac{2}{3}$.

v) $x \longmapsto 2g(x) + 1$





vi) $\sum_{k=1}^{4} \left|g\left(\frac{2}{k}\right)\right| = |g(2)| + |g(1)| + \left|g\left(\frac{2}{3}\right)\right| + \left|g\left(\frac{1}{2}\right)\right|$

$$= \frac{1}{3} + 0 + \frac{1}{9} + \frac{1}{4} = \underline{\underline{\frac{25}{36}}}$$

3) i) • $3x^2 - |x^2-1| + 3x \geqslant 0 \iff \begin{cases} x^2 - 1 \geqslant 0 \\ 3x^2 - (x^2-1) + 3x \geqslant 0 \end{cases}$ o $\begin{cases} x^2 - 1 < 0 \\ 3x^2 - (-x^2+1) + 3x \geqslant 0 \end{cases}$

$$\iff \begin{cases} x^2 \geqslant 1 \\ 2x^2 + 3x + 1 \geqslant 0 \end{cases} \quad o \quad \begin{cases} x^2 < 1 \\ 4x^2 + 3x - 1 \geqslant 0. \end{cases}$$

Oss. che $2x^2 + 3x + 1 = 0 \iff x_{1/2} = \frac{-3 \pm \sqrt{9-8}}{4} = \frac{-3 \pm 1}{4} \begin{cases} -1 \\ -\frac{1}{2} \end{cases}$

$$4x^2+3x-1=0 \iff x_{1/2}=\frac{-3\pm\sqrt{9+16}}{8}=\frac{-3\pm5}{8}=\begin{cases}-1\\ \frac{1}{4}\end{cases}.$$

Ne segue

$$\begin{cases}x^2\geq 1\\ 2x^2+3x+1\geq 0\end{cases} \iff \begin{cases}x\leq -1 \text{ o } x\geq 1\\ x\leq -1 \text{ o } x\geq -\frac{1}{2}\end{cases} \implies S_1=]-\infty,-1]\cup[1,+\infty[;$$

o

$$\begin{cases}x^2<1\\ 4x^2+3x-1\geq 0\end{cases} \iff \begin{cases}-1<x<1\\ x\leq -1 \text{ o } x\geq \frac{1}{4}\end{cases} \implies S_2=[\tfrac{1}{4},1[.$$

Otteniamo infine $S=S_1\cup S_2=\underline{\underline{]-\infty,-1]\cup[\frac{1}{4},+\infty[}}$. $\square$

• $$\log_2(2-|x|)-\log_2 x^2<0 \iff \begin{cases}2-|x|>0\\ x^2>0\\ 2-|x|<x^2\end{cases} \quad (\log_2 x \nearrow)$$

$$\iff \begin{cases}|x|<2\\ x\neq 0\\ 2-|x|<x^2\end{cases} \iff \begin{cases}-2<x<0\\ 2-(-x)<x^2\end{cases} \text{ o } \begin{cases}0<x<2\\ 2-x<x^2\end{cases}$$

$$\iff \begin{cases}-2<x<0\\ x^2-x-2>0\end{cases} \text{ o } \begin{cases}0<x<2\\ x^2+x-2>0\end{cases}.$$

Osserviamo che $x^2-x-2=0 \iff x_{1/2}=\frac{1\pm\sqrt{1+8}}{2}=\frac{1\pm3}{2}\begin{cases}-1\\ 2\end{cases}$

$x^2+x-2=0 \iff x_{1/2}=\frac{-1\pm\sqrt{1+8}}{2}=\frac{-1\pm3}{2}=\begin{cases}-2\\ 1\end{cases}.$

Ne segue

$$\begin{cases}-2<x<0\\ x<-1 \text{ o } x>2\end{cases} \text{ o } \begin{cases}0<x<2\\ x<-2 \text{ o } x>1\end{cases}.$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$S_1=]-2,-1[\qquad\qquad S_2=]1,2[$$

Otteniamo infine $S=S_1\cup S_2=\underline{\underline{]-2,-1[\cup]1,2[}}$. $\square$

ii) $\displaystyle\lim_{x\to-\infty}\frac{x^2+2^x}{\log|x|+3x^2}=\underline{\underline{\frac{1}{3}}}$ (vedi anche Es. 1) i) TPI, A).

$$\int_1^3 x(1-x^{-2})\,dx=\int_1^3\left(x-\frac{1}{x}\right)dx=\left[\frac{x^2}{2}-\log|x|\right]_1^3=\left(\frac{9}{2}-\log 3\right)-\left(\frac{1}{2}-\cancel{\log 1}\right)$$

$$=\underline{\underline{4-\log 3}}.$$

$\blacksquare$

4) i) $\lim_{x\to-\infty} f(x) = \underline{\underline{1}}$; $\lim_{x\to-1^-} f(x) = \underline{\underline{1}}$; $\lim_{x\to-1^+} f(x) = \underline{\underline{0}}$; $\lim_{x\to+\infty} f(x) = \underline{\underline{+\infty}}$.

ii) $x = -1$: infatti $\lim_{x\to-1^-} f(x) = f(-1) \neq \lim_{x\to-1^+} f(x)$.

iii) su $]-\infty, -3]$, $[-2, 0]$ f è decrescente;
su $[-3, -2]$, $[0, 2[$ f è crescente.

iv) $y = 1$ è asintoto orizzontale per f, per $x\to-\infty$;
$x = 2$ è asintoto verticale per f (per $x\to 2^-$).

v) – – – – • + • – – o – – • + + + o ////////// $f'(x)$
-4 $\;-3$ $\;-2$ $\;-1$ $\;0$ $\;1$ $\;2$ $\longrightarrow \notin$ dom f

vi) $\left.\begin{array}{l} x=-3 \\ x=0 \end{array}\right\}$ pt. di min. locale per f e in questi pt. f' si annulla.

$x = -2$ pt. di max. locale per f e si ha $f'(2) = 0$

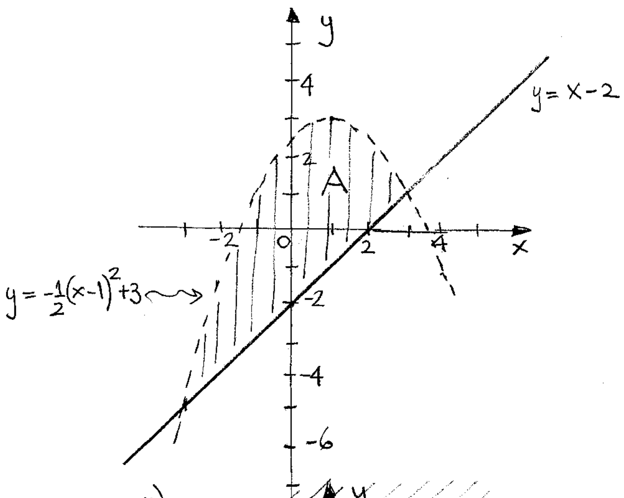
5) Vedi TPI, 26/01/09, Compito A Es. 4.

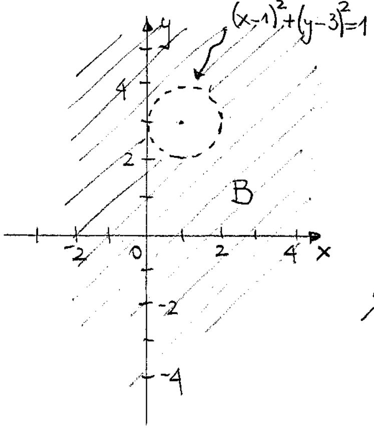
6) $C_{7,3} = \dfrac{7!}{3!\,4!} = \dfrac{7\cdot\cancel{6}\cdot5\cdot\cancel{4!}}{\cancel{3\cdot2}\cdot\cancel{4!}} = \underline{\underline{35}}$

Esame scritto (B)

1) i) $A = \{(x,y) \in \mathbb{R}^2 : \ x-2 \le y < -\frac{1}{2}(x-1)^2 + 3\}$,

$B = \{(x,y) \in \mathbb{R}^2 : \ x^2 - 2x + y^2 - 6y > -9\}$





$x^2 - 2x + y^2 - 6y + 9 > 0$

$(x-1)^2 - 1 + (y-3)^2 > 0$

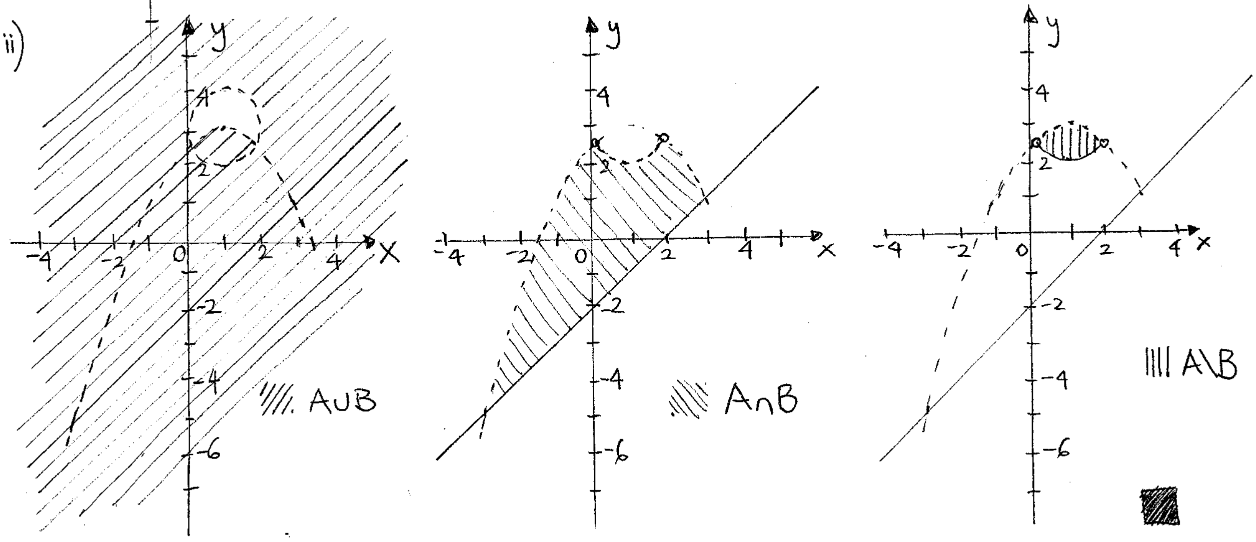
$(x-1)^2 + (y-3)^2 > 1$

$x - 2 = -\frac{1}{2}(x-1)^2 + 3$

$\cancel{2x} - 4 - 6 = -x^2 + \cancel{2x} - 1$

$x^2 = 9 \quad x = \pm 3$

ii)



2) i) $\lim\limits_{x \to -\infty} f(x) = \underline{\underline{1}}$; $\lim\limits_{x \to -1^-} f(x) = \underline{\underline{+\infty}}$; $\lim\limits_{x \to -1^+} f(x) = \underline{\underline{0}}$; $\lim\limits_{x \to 2^-} f(x) = \underline{\underline{1}}$.

ii) $x = -1$: infatti $\lim\limits_{x \to -1^-} f(x) = +\infty$;

$x = 2$: infatti $\lim\limits_{x \to 2^-} f(x) = 1 \neq f(2) = 0$.

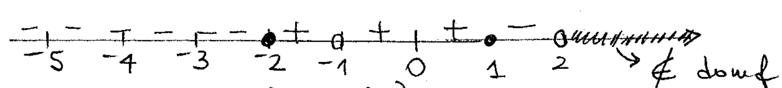
iii) in $]-\infty, -2]$, $[1,2]$ f è decrescente;

in $[-2, -1[$, $]-1, 1]$ f è crescente.

iv) $y = 1$ è asintoto orizzontale per f, per $x \to -\infty$;

$x = -1$ è asintoto verticale per f.

v)



$f'(x)$

vi) $x = -2$ pt. di min. loc. per f, $x = 1$ pt. di max. loc. per f $\}$ e in questi pt. f' si annulla.

$x = 2$ pt. di min. loc. per f e si ha $\nexists f'(2)$.

3) i) ii) vedi Compito (A), es. 2 i) ii);

iii) $(f+g)(-1) = f(-1)+g(-1) = \left(4\cdot 2^{-1}-1\right)+0 = \underline{\underline{1}}$;

$(fg)(6) = f(6)\cdot g(6) = \overset{2}{\cancel{6}}\cdot \frac{5}{\cancel{3}} = \underline{\underline{10}}$;

$\left(\frac{f}{g}\right)(-1)$ $\nexists$ poiché $g(-1)=0$.

iv), v) vedi Compito (A), es. 2 iv), v);

vi) $\sum_{k=1}^{4} \left|g\left(\frac{3}{k}\right)\right| = |g(3)| + \left|g\left(\frac{3}{2}\right)\right| + |g(1)| + \left|g\left(\frac{3}{4}\right)\right| = \frac{2}{3}+\frac{1}{6}+0+\frac{1}{16} = \underline{\underline{\frac{43}{48}}}$

4) i) • $3x^2-|x^2-1|+3x \le 0 \iff \begin{cases} x^2-1\ge 0 \\ 3x^2-(x^2-1)+3x\le 0 \end{cases}$ o $\begin{cases} x^2-1<0 \\ 3x^2-(-x^2+1)+3x\le 0 \end{cases}$

$\iff \begin{cases} x^2\ge 1 \\ 2x^2+3x+1\le 0 \end{cases}$ o $\begin{cases} x^2<1 \\ 4x^2+3x-1\le 0 \end{cases}$.

Si ha (vedi Compito (A), Es. 3i))

$\iff \begin{cases} x\le -1 \text{ o } x\ge 1 \\ -1\le x\le -\frac{1}{2} \end{cases} \implies S_1=\{-1\}$

o $\begin{cases} -1<x<1 \\ -1\le x\le \frac{1}{4} \end{cases} \implies S_2 = \left]-1,\frac{1}{4}\right]$

$\implies S = S_1\cup S_2 = \underline{\underline{\left[-1,\frac{1}{4}\right]}}$

• $\log_2(2-|x|)-\log_2 x^2 \ge 0 \iff \begin{cases} 2-|x|>0 \\ x^2>0 \\ 2-|x|\ge x^2 \end{cases}$ $(\log_2 x \nearrow)$

$\iff \begin{cases} |x|<2 \\ x\neq 0 \\ x^2+|x|-2\le 0 \end{cases} \iff \begin{cases} -2<x<0 \\ x^2-x-2\le 0 \end{cases}$ o $\begin{cases} 0<x<2 \\ x^2+x-2\le 0 \end{cases}$

Si ha (vedi Compito (A), Es. 3i))

$\begin{cases} -2<x<0 \\ -1\le x\le 2 \end{cases}$ o $\begin{cases} 0<x<2 \\ -2\le x\le 1 \end{cases} \iff S = \underline{\underline{[-1,0[\ \cup\ ]0,1]}}$

ii) $\lim_{x\to-\infty} \frac{x^2+2^x}{\log|x|+4x^2} = \underline{\underline{\frac{1}{4}}}$ (vedi anche Es. 1) i) TPI (A)).

$\int_2^3 x(1-x^{-2})\,dx = \int_2^3 \left(x-\frac{1}{x}\right)dx = \left[\frac{x^2}{2}-\log|x|\right]_2^3 = \left(\frac{9}{2}-\log 3\right)-\left(2-\log 2\right) = \underline{\underline{\frac{5}{2}+\log\frac{2}{3}}}$

5) Vedi Compito (A), Es. 5.

6) $C_{8,3} = \frac{8!}{5!\,3!} = \frac{8\cdot 7\cdot \cancel{6}\cdot \cancel{5!}}{\cancel{5!}\ \cancel{3\cdot 2}} = \underline{\underline{56}}$