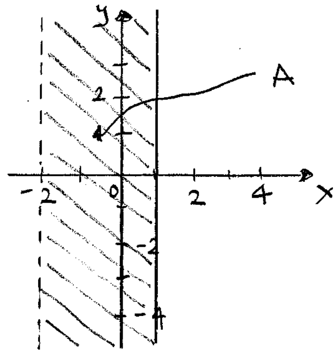
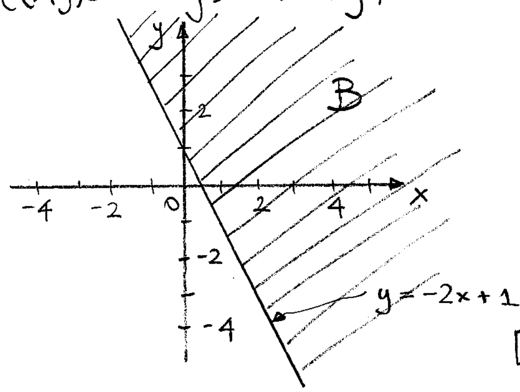
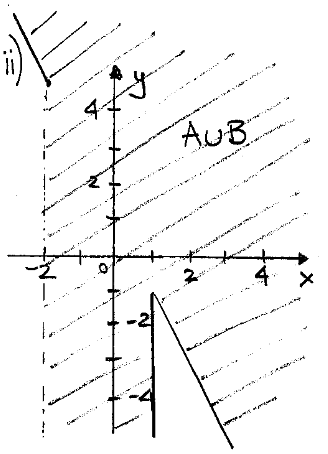
Università degli Studi di Trento – Facoltà di Scienze Cognitive
CdL in Interfacce e Tecnologie della Comunicazione
CdL in Scienze e Tecniche di Psicologia Cognitiva
Verifica settimanale di ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2008 – 2009 – Rovereto, 6–10/10/08

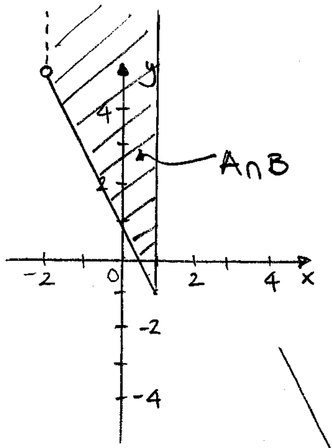
1) i) $A = \{(x,y) \in \mathbb{R}^2 : -2 < x \leq 1\}$ $\qquad$ $B = \{(x,y) \in \mathbb{R}^2 : y \geq -2x+1\}$.

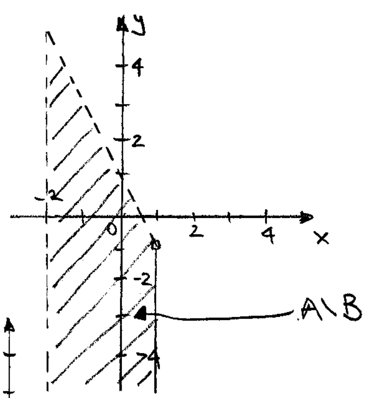




ii)



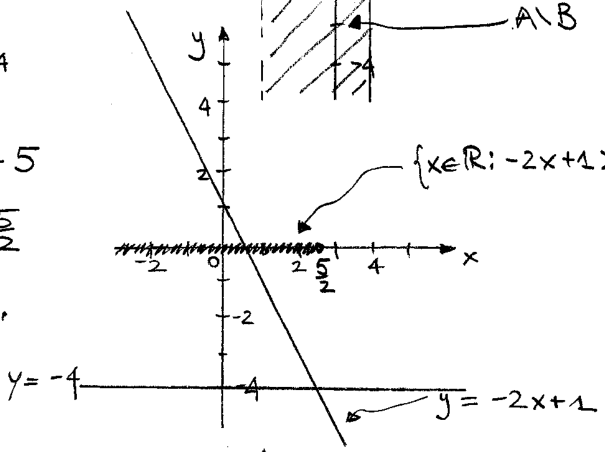




2) i) $-2x+1 \geq -4 \iff -2x \geq -5$

$\iff x \leq \frac{5}{2}$

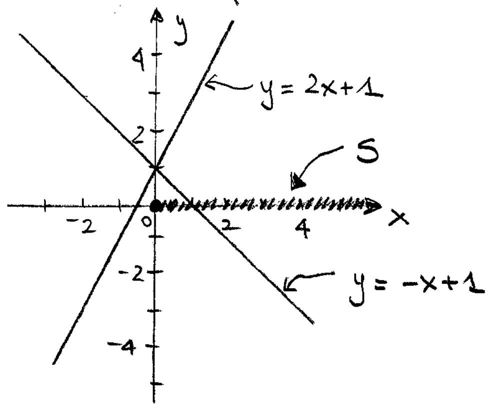
$S = \left]-\infty, \frac{5}{2}\right]$.



$2x+1 \geq -x+1 \iff 3x \geq 0$

$\iff x \geq 0$

$S = [0, +\infty[$.

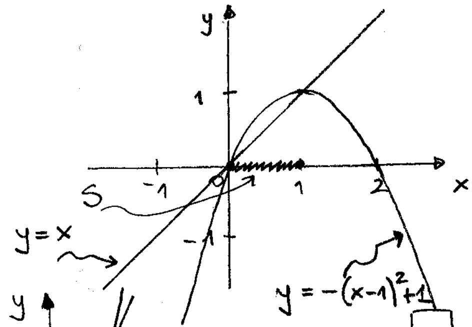


$-(x-1)^2+1 \geqslant x \iff -x^2+2x-\cancel{1}+\cancel{1} \geqslant x$

$\iff -x^2+x \geqslant 0$

$\iff x(x-1) \leqslant 0$

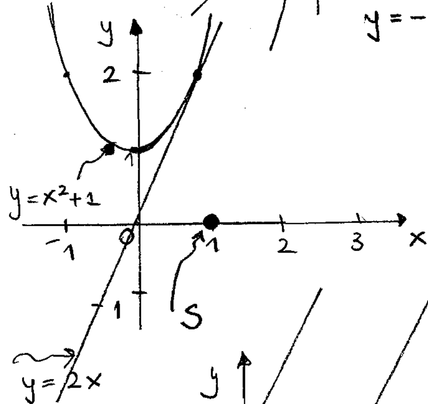
$\Rightarrow \underline{\underline{S=[0,1]}}$



ii) $x^2+1=2x \iff x^2-2x+1=0$

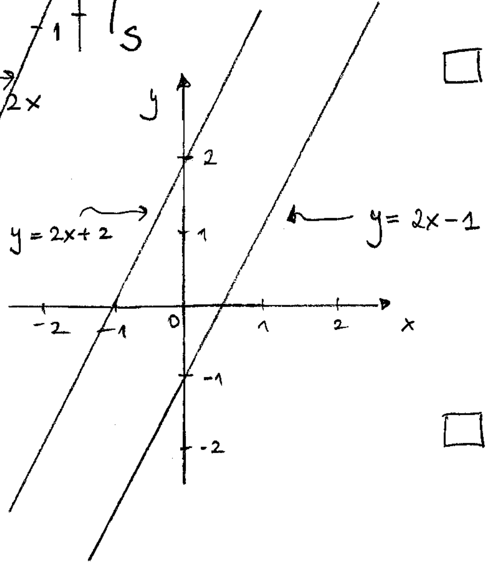
$\iff (x-1)^2=0$

$\Rightarrow \underline{\underline{S=\{1\}}}$



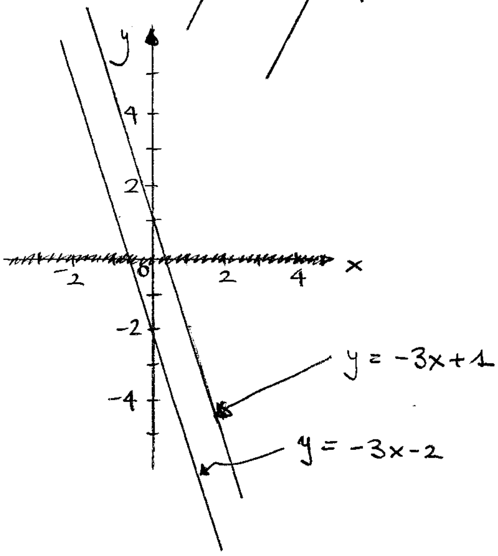
$2x-1 \geqslant 2x+2 \iff -1 \geqslant 2$

$\Rightarrow \underline{\underline{S=\emptyset}}$



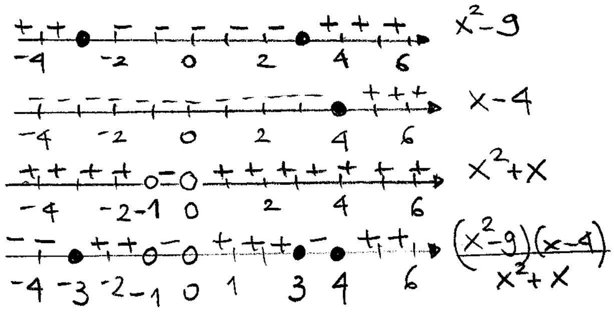
$-3x-2 < -3x+1 \iff -2 < 1$

$\Rightarrow \underline{\underline{S=\mathbb{R}}}$

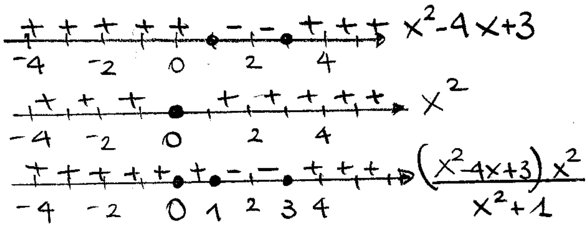


3) $A=\left\{x\in\mathbb{R} : \dfrac{(x^2-9)(x-4)}{x^2+x} \geqslant 0\right\}$

$\underline{\underline{=[-3,-1[\cup]0,3]\cup[4,+\infty[}}$

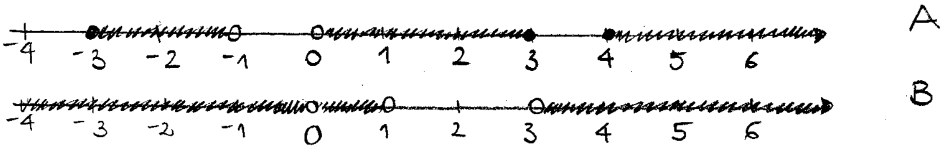


$B = \left\{ x \in \mathbb{R} : \frac{(x^2-4x+3)\,x^2}{x^2+1} > 0 \right\}$



<u>nota</u>: $x^2+1>0$
$\forall x \in \mathbb{R}$!!

$$B = \,]-\infty, 0[\, \cup \,]0,1[\, \cup \,]3,+\infty[$$

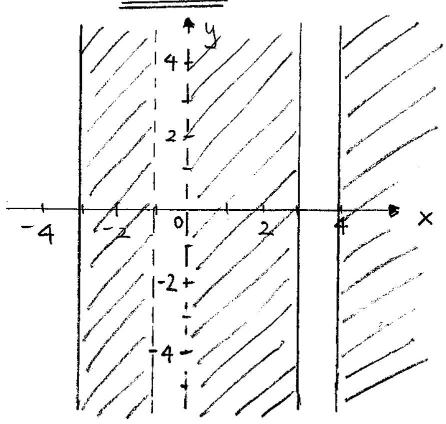


A non è un intervallo di $\mathbb{R}$; B non è un intervallo di $\mathbb{R}$.

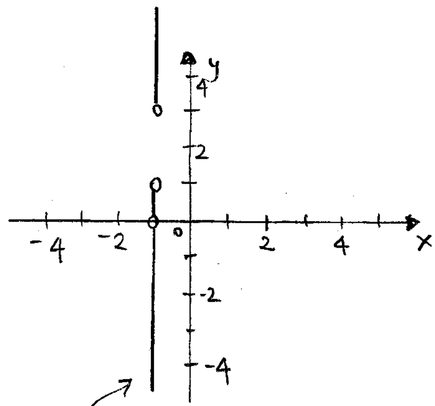
ii) $A \cup B = \mathbb{R} \setminus \{0\}$; $\quad A \cap B = [-3,-1[\, \cup \,]0,1[\, \cup \,[4,+\infty[$;

$A \setminus B = [1,3]$

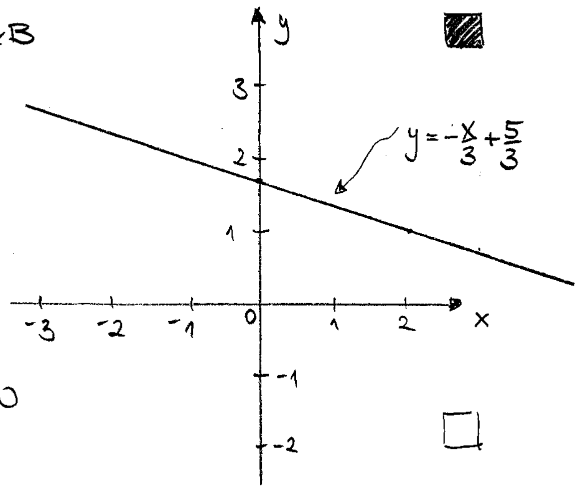
iii)



$A \times \mathbb{R}$



$\{-1\} \times B$



4) i) $y = 1 - \frac{1}{3}(x-2) \implies y = -\frac{1}{3}x + \frac{5}{3}$

ii) $y = 3x + q \quad q \in \mathbb{R}$.

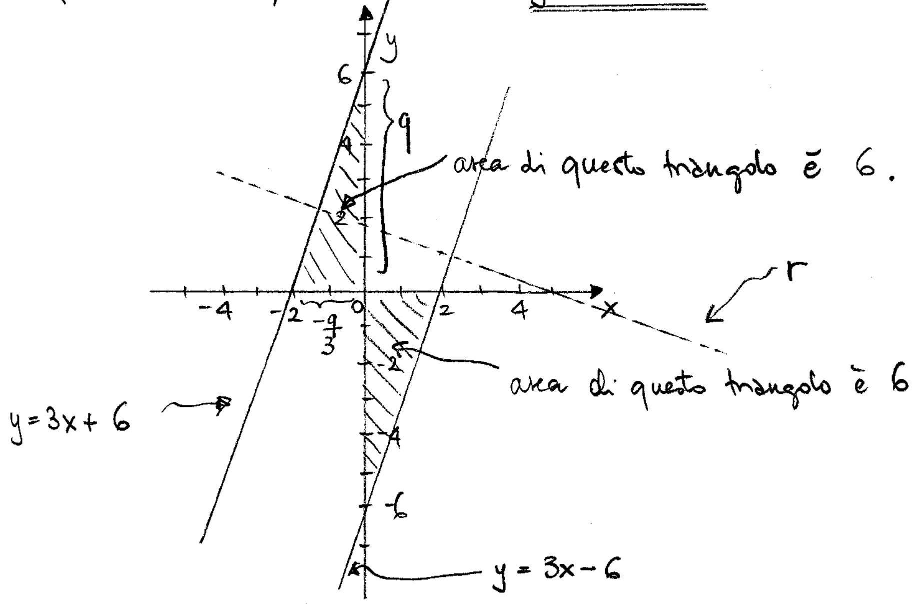
iii) la retta $y = 3x+q$ interseca l'asse $y=0$ nel pt. $x = -\frac{q}{3}$. Si ha allora

$$\frac{\text{base} \cdot \text{altezza}}{2} = \frac{q}{3} \cdot \frac{q}{2} = 6 \iff q^2 = 36 \implies q = \pm 6.$$

Quindi l'eq. di una retta soddisfacente quanto richiesto è

$\underline{\underline{y = 3x + 6}}$;

essa non è unica; Un'altra è $\underline{\underline{y = 3x - 6}}$.

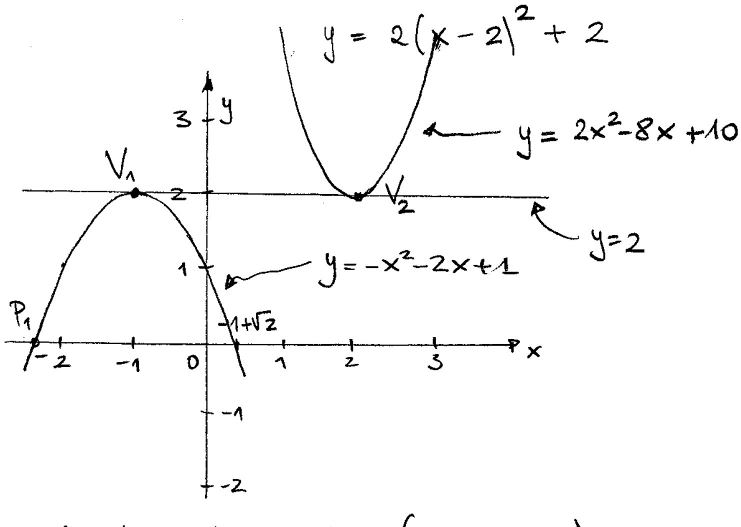


5) i) $y = -x^2 - 2x + 1 \iff y = -(x^2 + 2x) + 1$

$= -(x+1)^2 + 2 \qquad V_1 = (-1, 2)$

$y = 2x^2 - 8x + 10 \iff y = 2(x^2 - 4x) + 10$

$y = 2(x-2)^2 + 2 \qquad V_2 = (2, 2)$



ii) $y = 2$ è l'eq. della retta (orizzontale) passante per V_1 e V_2.

iii) La prima parabola interseca la retta di eq. $y=0$ nei punti

$P_1 = (-1-\sqrt{2}, 0)$ e $P_2 = (-1+\sqrt{2}, 0)$: infatti $\begin{cases} -(x+1)^2 + 2 = 0 \\ y = 0 \end{cases} \iff$

$\begin{cases} (x+1)^2 = 2 \\ y = 0 \end{cases} \iff \begin{cases} x+1 = \pm\sqrt{2} \\ y = 0 \end{cases}$ e quindi si hanno immediatamente P_1 e P_2.

La seconda parabola non interseca la retta di eq. $y=0$!!