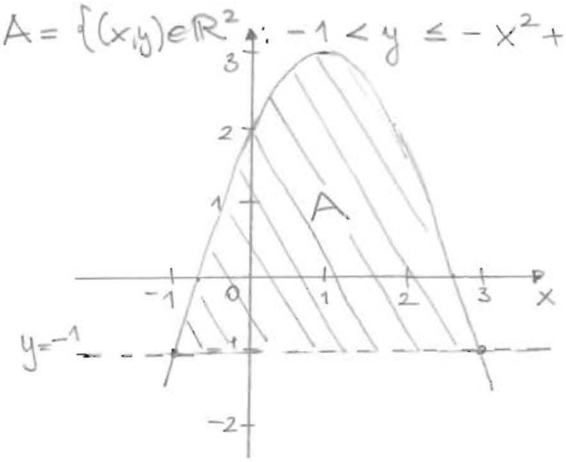
Università degli Studi di Trento — Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione
ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2009/10 — Rovereto, 5 – 9 ottobre 2009

1) i) $A = \{(x,y) \in \mathbb{R}^2 : -1 < y \le -x^2+2x+2\}$

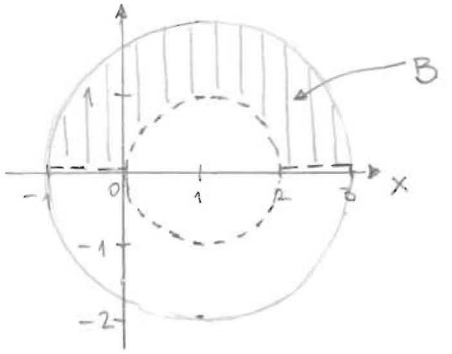


$$\begin{cases} y = -x^2+2x+2 \\ \quad = -(x^2-2x)+2 \\ \quad = -(x-1)^2+3 \end{cases}$$

$$\begin{cases} -x^2+2x+2=0 \qquad x_{1/2} = \dfrac{-2\pm\sqrt{4+8}}{-2} \\ \qquad x_{1/2} = 1\pm\sqrt{3} \end{cases}$$

$$\begin{cases} -x^2+2x+2=-1 \\ -x^2+2x+3=0 \\ \qquad x_{1/2} = \dfrac{-2\pm\sqrt{4+12}}{-2} \\ \qquad x_{1/2} = -\dfrac{2\pm 4}{-2} = \begin{matrix} 3 \\ -1 \end{matrix} \end{cases}$$

$B = \{(x,y) \in \mathbb{R}^2 : y > 0,\ 1 < (x-1)^2+y^2 \le 4\}$



$\square$

ii) • tutte le rette orizzontali che non intersecano mai l'insieme A hanno eq. $\underline{y = k \text{ con } k > 3 \text{ e } k \le -1}$.

• le eq. delle rette verticali che non intersecano mai l'insieme B sono $\underline{\underline{x = k \text{ con } k \in \,]-\infty, -1[\, \cup\,]3, +\infty[}}$.

2) i) $-1 \le -x^2+2x \le 1 \iff \begin{cases} -x^2+2x+1 \ge 0 \\ -x^2+2x-1 \le 0 \end{cases}$

$$\iff \begin{cases} x^2-2x-1 \le 0 \\ x^2-2x+1 \ge 0 \end{cases}$$

$x^2-2x-1=0$

$x_{1/2} = \dfrac{2\pm\sqrt{4+4}}{2} = 1\pm\sqrt{2}$

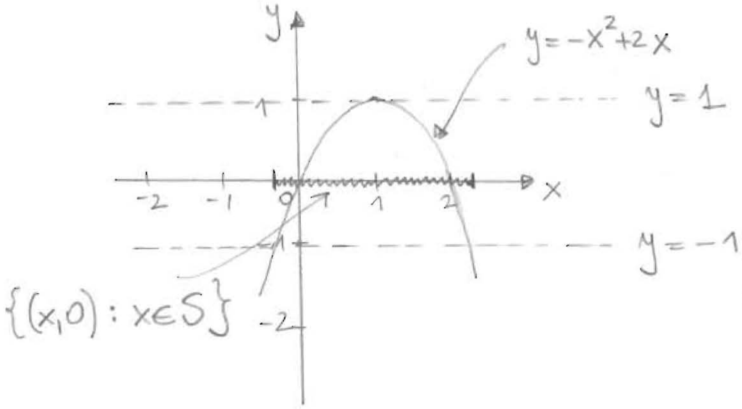
$x^2-2x+1=0$

$x_{1/2} = \dfrac{2\pm\sqrt{4-4}}{2} = 1$

$$\begin{cases} x \in [1-\sqrt{2}, 1+\sqrt{2}] \\ x \in \mathbb{R} \end{cases}$$

$\Rightarrow S = [1-\sqrt{2}, 1+\sqrt{2}]$

$$y = -x^2+2x$$
$$= -(x^2-2x)$$
$$y = -(x-1)^2+1$$

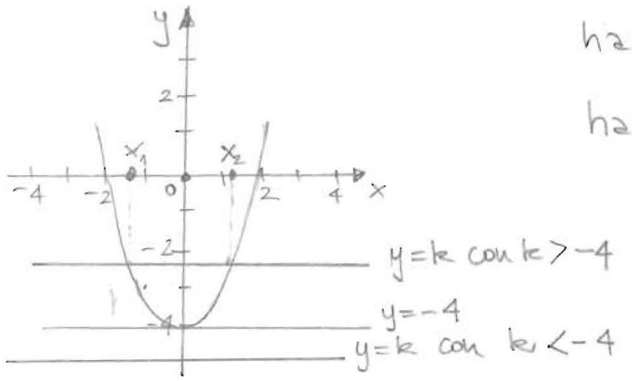


$\square$

$x^2-4 = k \iff x^2 = 4+k$ ha nessuna soluz. se $k<-4$

ha l'unica sol. $x=0$ se $k=-4$

ha due soluz. $x_{1/2} = \pm\sqrt{4+k}$ se $k>-4$



$\square$

ii) Eq. dell'ellisse di centro $C=(1,-1)$ e semiassi $a=1$, $b=2$

ha eq. $(x-1)^2+\dfrac{(y+1)^2}{4} = 1$.

La retta di eq. $x-y-1=0 \iff y=x-1$. Allora P, Q sono pt. di intersezione dell'ellisse con la retta $y=x-1$ se e solo se (x,y) soddisf.

$$\begin{cases}(x-1)^2+\dfrac{(y+1)^2}{4}=1\\ y=x-1\end{cases} \iff \begin{cases}(x-1)^2+\dfrac{x^2}{4}=1\\ y=x-1\end{cases}$$

$$\iff \begin{cases}4(x-1)^2+x^2-4=0\\ y=x-1\end{cases} \iff \begin{cases}5x^2-8x=0\\ y=x-1\end{cases}$$

$$\iff P=(0,-1) \quad Q=\left(\frac{8}{5},\frac{3}{5}\right)$$

$$d(P,Q)=\sqrt{\left(\frac{8}{5}-0\right)^2+\left(\frac{3}{5}+1\right)^2}=\sqrt{\frac{64}{25}+\frac{64}{25}}=\underline{\frac{8}{5}\sqrt{2}}$$

$\blacksquare$

3) $\dfrac{x(x^2-4)}{x^2+1} > x \iff \dfrac{x(x^2-4)-x(x^2+1)}{x^2+1} > 0 \iff \dfrac{-4x-x}{x^2+1} > 0$

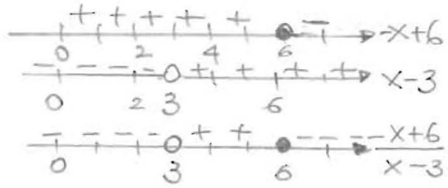
$\iff \dfrac{5x}{x^2+1} < 0$.

Poiché $x^2+1>0 \ \forall x$ si ha immed. $\underline{\underline{S =]-\infty, 0[}}$. $\square$

$\dfrac{x^2-2x}{x-3} \le x+2 \iff \dfrac{x^2-2x-(x+2)(x-3)}{x-3} \le 0$

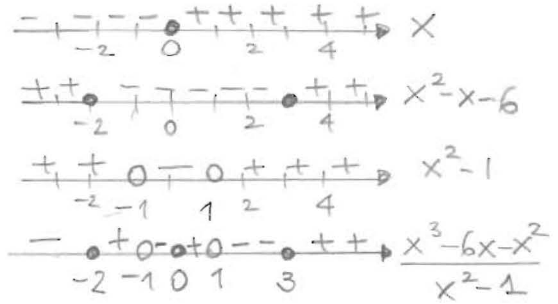
$\iff \dfrac{-2x+x+6}{x-3} \le 0 \iff \dfrac{-x+6}{x-3} \le 0$

$\Rightarrow \underline{\underline{S =]-\infty, 3[\cup [6, +\infty[}}$.



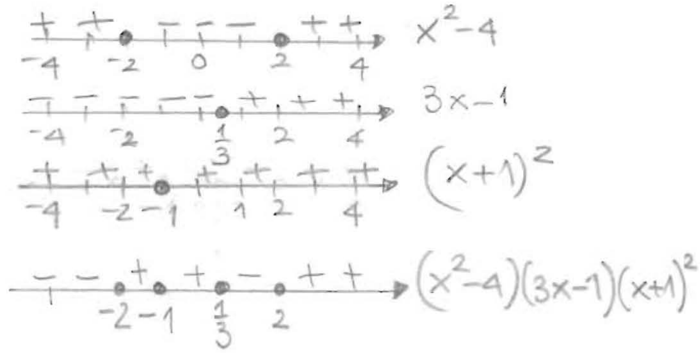
$\dfrac{x^3-6x-x^2}{x^2-1} < 0 \iff \dfrac{x(x^2-x-6)}{x^2-1} < 0$

$\underline{\underline{S =]-\infty, -2[\cup]-1, 0[\cup]1, 3[}}$



$(x^2-4)(3x-1)(x+1)^2 \le 0$

$\underline{\underline{S =]-\infty, -2] \cup \{-1\} \cup [\tfrac{1}{3}, 2]}}$

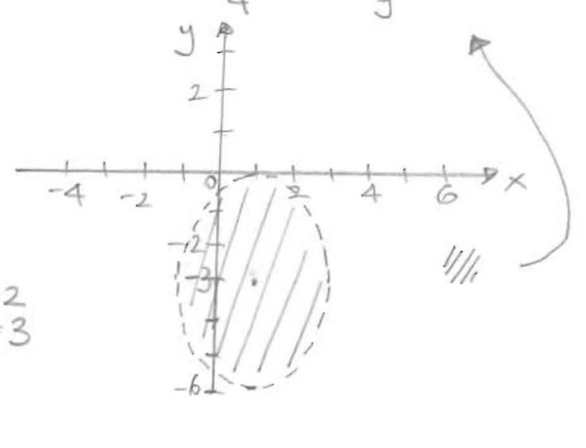


4) i) $9(x-1)^2 + 4(y+3)^2 < 36 \iff \dfrac{(x-1)^2}{4} + \dfrac{(y+3)^2}{9} < 1$

$\dfrac{(x-1)^2}{4} + \dfrac{(y+3)^2}{9} = 1$

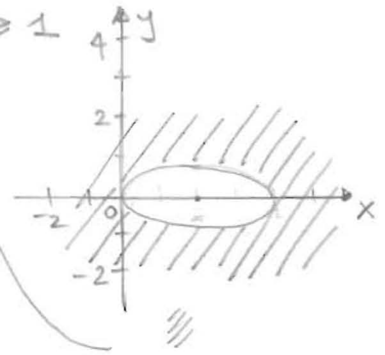
eq. ellisse di centro

$C = (1, -3)$ e semiasse $a = 2$
$b = 3$



ii) $x^2-4x+9y^2\geq 0 \iff (x-2)^2+9y^2\geq 4$

$\iff \frac{(x-2)^2}{4}+\frac{y^2}{4/9}\geq 1$



$$\frac{(x-2)^2}{4}+\frac{y^2}{4/9}=1$$

eq. ellisse di centro $C=(2,0)$

e semiassi $a=2$, $b=2/3$

iii) $-x^2-y^2+2x-6y\leq 6 \iff x^2+y^2-2x+6y\geq -6$

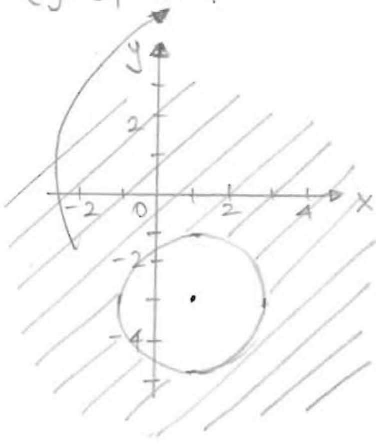
$(x-1)^2-1+(y+3)^2-9+6\geq 0$

$(x-1)^2+(y+3)^2\geq 4$

$$(x-1)^2+(y+3)^2=4$$

eq. circonferenza di centro $C=(1,-3)$

e raggio $r=2$

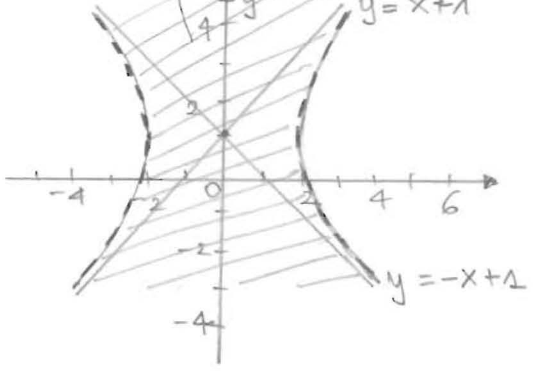


iv) $x^2-(y-1)^2-4<0 \iff \frac{x^2}{4}-\frac{(y-1)^2}{4}<1$

$$\frac{x^2}{4}-\frac{(y-1)^2}{4}=1$$

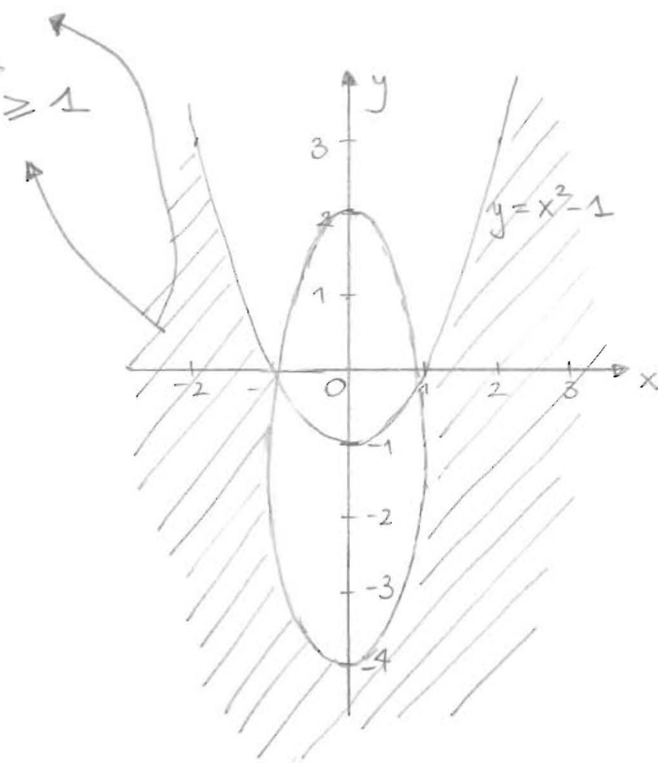
eq. iperbole di centro $C=(0,1)$ e semiassi $a=b=2$

asintoti $y=\pm x+1$



5) a) $y \leq x^2 - 1$

$x^2 + \dfrac{(y+1)^2}{9} \geq 1$



b) $\begin{cases} -1 < x \leq 1 \\ -4x^2 + y^2 \leq 4 \end{cases} \iff \begin{cases} -1 < x \leq 1 \\ -x^2 + \dfrac{y^2}{4} \leq 1 \end{cases}$