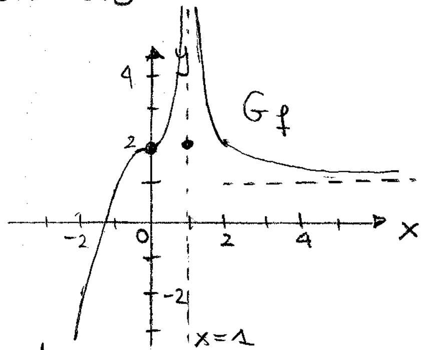
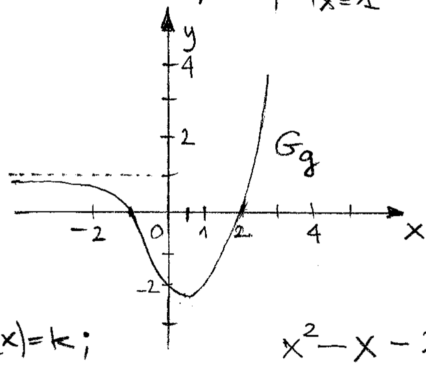
Università degli Studi di Trento - Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione
ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2009/10 — Rovereto, 19 — 23 ottobre 2009

1) i) $f : \mathbb{R} \to \mathbb{R}$

$$f(x)=\begin{cases} -x^2+2 & \text{se } x<0 \\ \dfrac{1}{(x-1)^2}+1 & \text{se } x\in[0,+\infty[\setminus\{1\} \\ 2 & \text{se } x=1 \end{cases}$$



$$g(x)=\begin{cases} 1-\dfrac{1}{x^4} & \text{se } x\le -1 \\ \left(x-\dfrac{1}{2}\right)^2-\dfrac{9}{4} & \text{se } x>-1 \end{cases}$$



ii) Se $k<-\frac{9}{4}$ $\nexists$ sono soluzioni di $g(x)=k$;

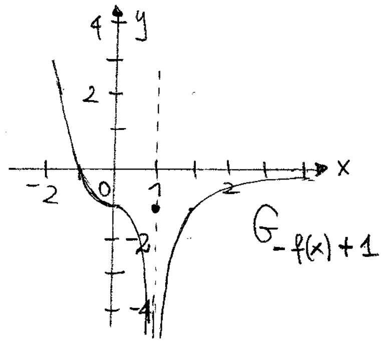
$k=-\frac{9}{4}$ $\exists$ una soluzione di $g(x)=k$;

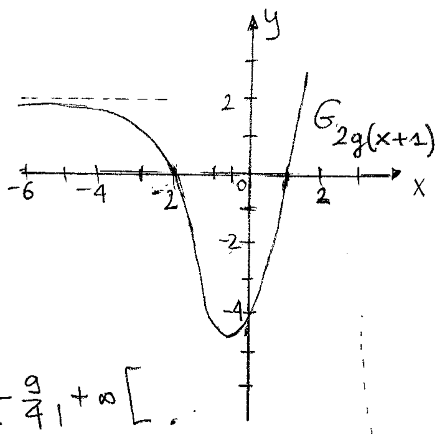
$-\frac{9}{4}<k<1$ $\exists$ sono due soluzioni " ;

$k\ge 1$ $\exists$ una soluzione di " .

$x^2-x-2=(x-2)(x+1)$

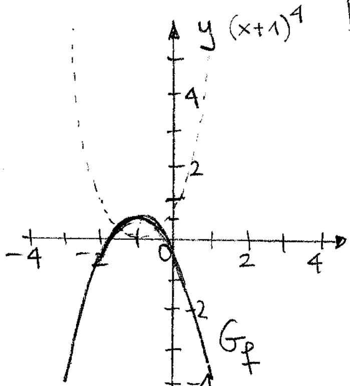
iii)



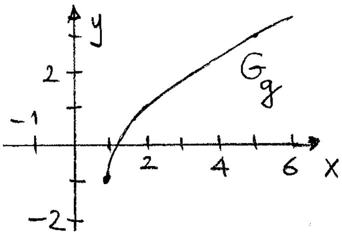


iv) $f(\mathbb{R})=\mathbb{R}$, $g(\mathbb{R})=\left[-\frac{9}{4},+\infty\right[$.

2) i) $f(x)=-(x+1)^4+\frac{1}{2}$ dom $f=\mathbb{R}$



$g(x)=2\sqrt{x-1}-1$ dom $g=[1,+\infty[$



ii) f è limitata superiormente : $f(\mathbb{R}) =]-\infty, \frac{1}{2}]$
non è limitata inferiormente ;
g è limitata inferiormente : $g([1,+\infty[) = [-1,+\infty[$
non è limitata superiormente.

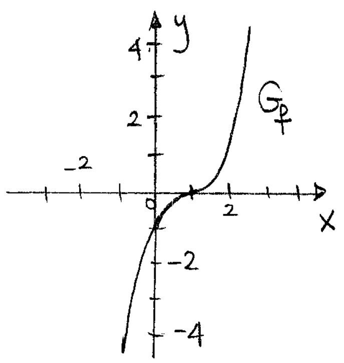
$\min_{x\in[1,2]} f(x) = f(2) = -81+\frac{1}{2}$ $\quad x=2$ pt. di minimo;

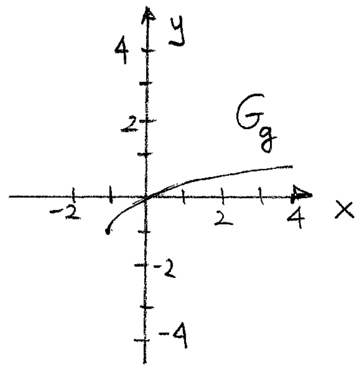
$\max_{x\in[1,2]} f(x) = f(1) = -16+\frac{1}{2}$ $\quad x=1$ pt. di massimo.

$\min_{x\in[1,2]} g(x) = g(1) = -1$ $\quad x=1$ pt. di minimo;

$\max_{x\in[1,2]} g(x) = g(2) = 1$ $\quad x=2$ pt. di massimo.

3) i) $f(x) = (x-1)^3$ $\quad \mathrm{dom}\, f = \mathbb{R}$
$g(x) = \sqrt[4]{x+1} - 1$ $\quad \mathrm{dom}\, g = [-1,+\infty[$





$f(\mathbb{R}) = \mathbb{R}$ $\quad g([-1,+\infty[) = [-1,+\infty[$.

ii) $(f-g)(-1) = f(-1) - g(-1) = -8 - (-1) = -7$.
$(fg)(0) = f(0)g(0) = (-1)\cdot 0 = 0$.
$\left(\frac{g}{f}\right)(-1) = \frac{g(-1)}{f(-1)} = \frac{-1}{-8} = \frac{1}{8}$.
$\left(\frac{g}{f}\right)(-2) \nexists$ poiché $\nexists\, g(-2)$.

iii) $f(\mathbb{R}) = \mathbb{R} = \text{dom} f$ e quindi posso considerare $\underset{x \in \mathbb{R}}{(f \circ f)(x) =} f((x-1)^3)$

$$= \left((x-1)^3 - 1\right)^3.$$

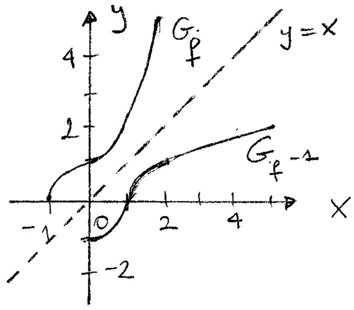
Poiché $f(\mathbb{R}) = \mathbb{R}$ e $\text{dom} g = [-1, +\infty[$, osserviamo che possiamo considerare solo $g \circ f$ per $x \in E = [0, +\infty[$ poiché $f(E) = [-1, +\infty[= \text{dom} g$. Quindi

$$\forall x \in [0, +\infty[\quad (g \circ f)(x) = g((x-1)^3) = \sqrt[4]{(x-1)^3 + 1} - 1.$$

Poiché $g([-1, +\infty[) = [-1, +\infty[\subset \text{dom} f = \mathbb{R}$ si ha

$$\forall x \in [-1, +\infty[\quad (f \circ g)(x) = f(g(x)) = \left(\sqrt[4]{x+1} - 2\right)^3.$$

4) $f : [-1, 2] \to [0, 5]$ $\qquad f(x) = \begin{cases} \sqrt{x+1} & \text{se } -1 \le x \le 0 \\ x^2 + 1 & \text{se } 0 < x \le 2 \end{cases}$



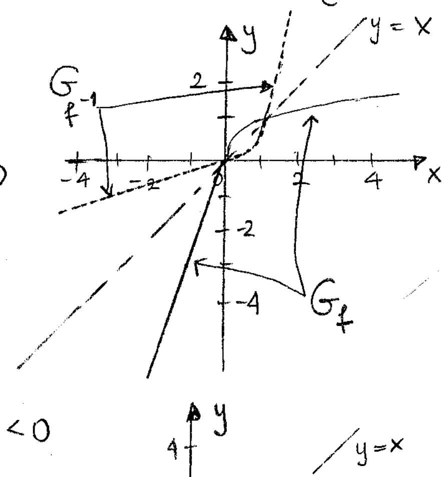
$f^{-1} : [0, 5] \to [-1, 2]$

Analiticamente

$$f^{-1}(x) = \begin{cases} x^2 - 1 & \text{se } 0 \le x \le 1 \\ \sqrt{x-1} & \text{se } 1 < x \le 5 \end{cases}$$

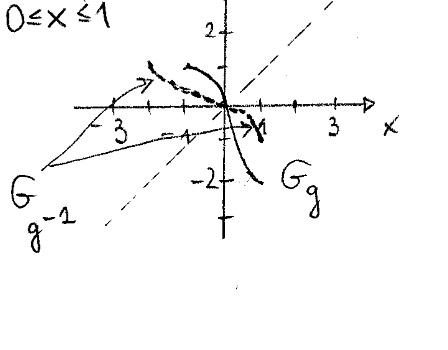
5) $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} 3x & \text{se } x \le 0 \\ \sqrt[3]{x} & \text{se } x > 0 \end{cases}$$



$f^{-1} : \mathbb{R} \to \mathbb{R}$

$$g(x) = \begin{cases} -\sqrt[3]{x} & \text{se } -1 \le x < 0 \\ 2x(x-2) & \text{se } 0 \le x \le 1 \end{cases}$$



$g : [-1, 1] \to [-2, 1]$

$g^{-1} : [-2, 1] \to [-1, 1]$