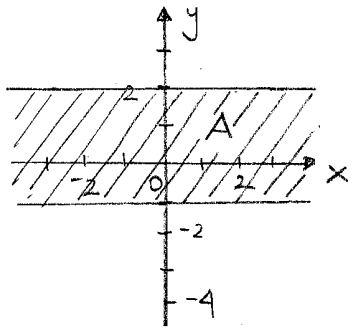
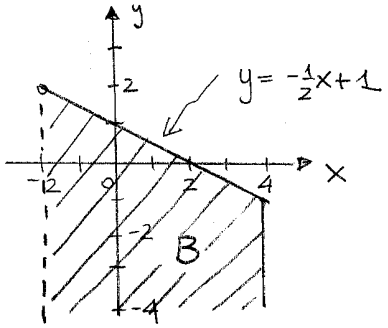
Università degli Studi di Trento — Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione
ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2009/10 — Rovereto, 28 settembre — 2 ottobre 2009

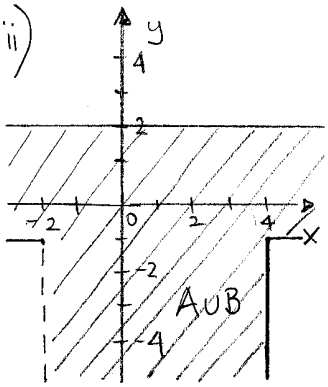
1) i) $A=\{(x,y)\in\mathbb{R}^2 : -1\le y\le 2\}$, $B=\{(x,y)\in\mathbb{R}^2 : -2<x\le 4,\ y\le -\frac{1}{2}x+1\}$

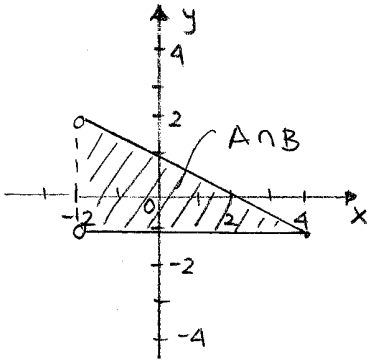


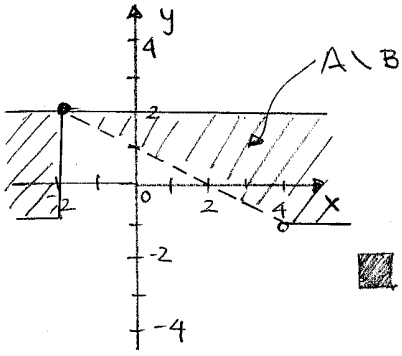


□

ii)



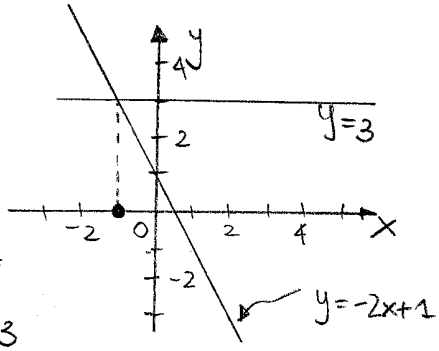




■

2) i) • $-2x+1=3 \iff -2x=2 \iff x=-1$

$\Rightarrow S=\{-1\}$

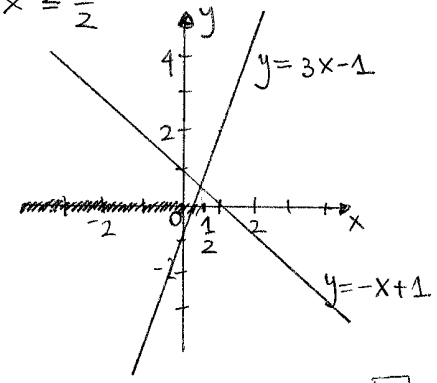


$x=-1$ è la coordinata x del pt. di intersezione delle rette $y=-2x+1$ e $y=3$

□

• $3x-1\le -x+1 \iff 4x\le 2 \iff x\le \frac{1}{2}$

$\Rightarrow S=]-\infty, \frac{1}{2}]$

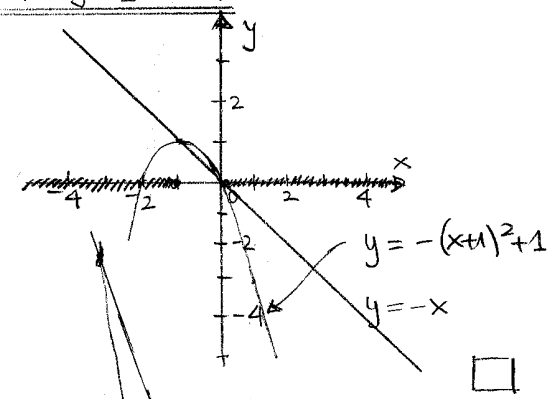


□

• $-(x+1)^2+1 \leq -x \iff -x^2-2x-1+1 \leq -x$

$\iff -x^2-x \leq 0 \iff x^2+x \geq 0$

$\Rightarrow S =]-\infty,-1] \cup [0,+\infty[$

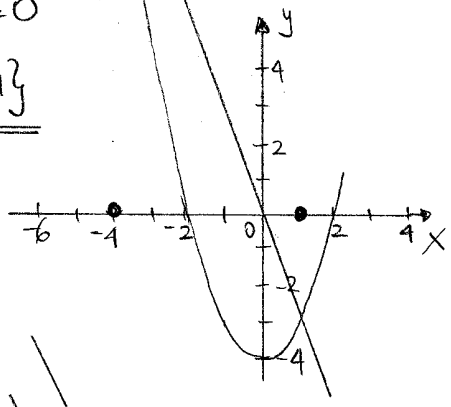


□

ii) • $x^2-4=-3x \iff x^2+3x-4=0$

$\iff (x+4)(x-1)=0$

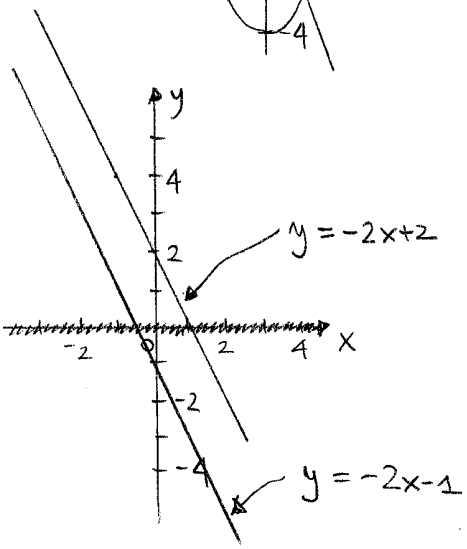
$\Rightarrow S = \{-4,1\}$



• $-2x-1 \leq -2x+2 \iff -1 \leq 2$

$\Rightarrow S = \mathbb{R}$

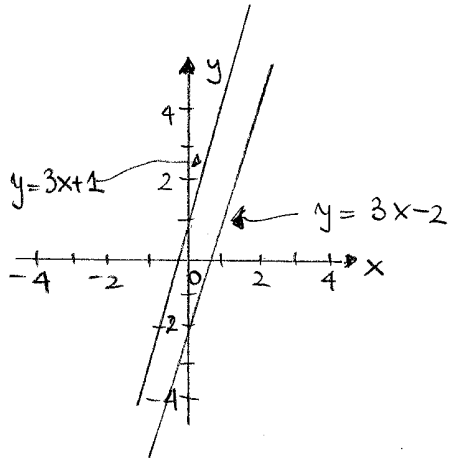
□



□

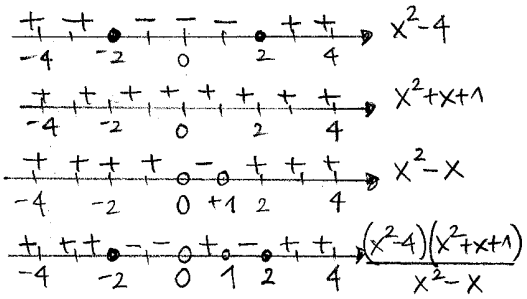
• $3x-2 \geq 3x+1 \iff -2 \geq 1$

$\Rightarrow S = \emptyset$



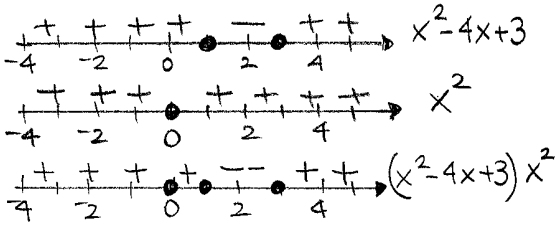
■

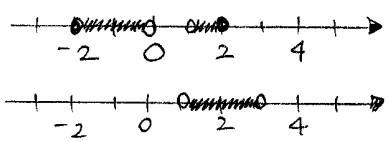
3) $A=\left\{x\in\mathbb{R}: \dfrac{(x^2-4)(x^2+x+1)}{x^2-x}\le 0\right\} = \underline{[-2,0[\cup]1,2]}$



$B=\left\{x\in\mathbb{R}: \dfrac{(x^2-4x+3)x^2}{3}<0\right\}$

$= \underline{]1,3[}$

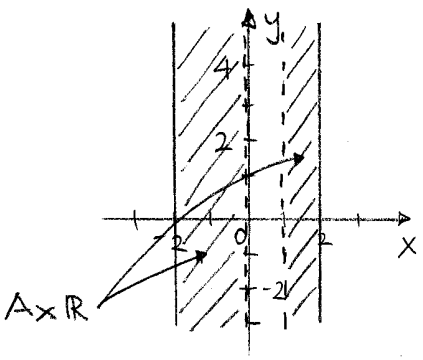


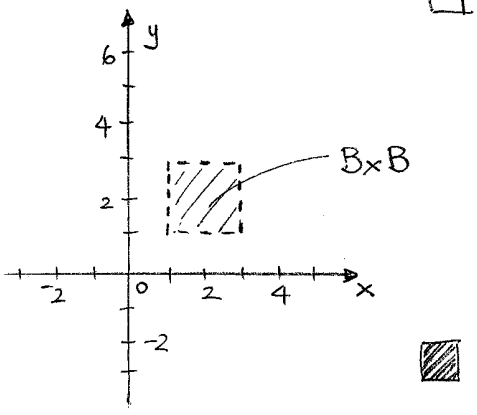
i) 

A non è un intervallo di $\mathbb{R}$; B è un intervallo di $\mathbb{R}$. □

ii) $A\cup B = \underline{[-2,0[\cup]1,3[}$; $A\cap B = \underline{]1,2]}$;

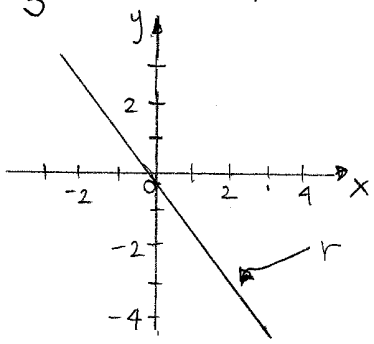
$A\setminus B = \underline{[-2,0[}$. □





4) i) $m=-\frac{4}{3}$ $P=(-1,1)$ $r:\ y=1-\frac{4}{3}(x+1)$

$\Rightarrow \underline{y=-\frac{4}{3}x-\frac{1}{3}}$



□

ii) rette perpendicolari alla retta r : $y = \frac{3}{4}x + q \quad q \in \mathbb{R}$

rette parallele alla retta r : $y = -\frac{4}{3}x + q' \quad q' \in \mathbb{R}$.

■

5) i) $y = -x^2 + 3x - \frac{1}{4}$. $y = -(x^2 - 3x) - \frac{1}{4}$

$y = -(x - \frac{3}{2})^2 - \frac{1}{4} + \frac{9}{4}$

$\underline{y = -(x - \frac{3}{2})^2 + 2}$ $\rightsquigarrow V_1 = (\frac{3}{2}, 2)$

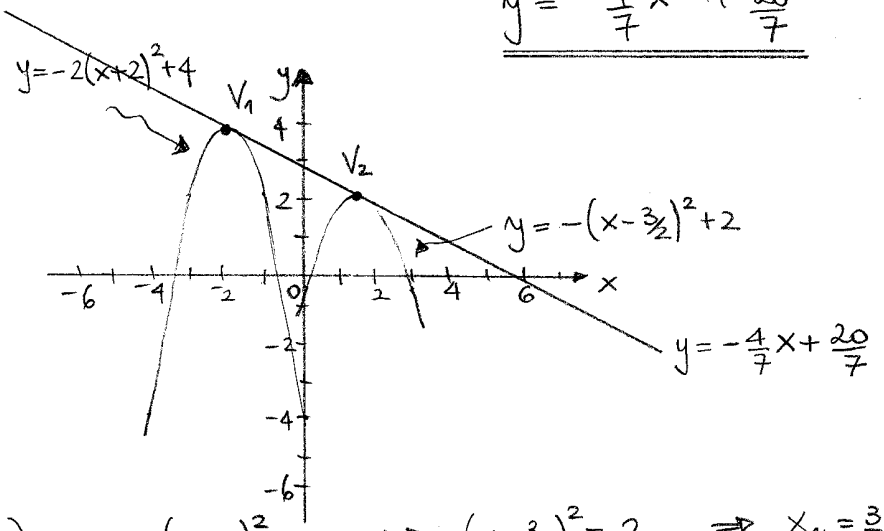
$y = -2x^2 - 8x - 4$ $y = -2(x^2 + 4x) - 4$

$\underline{y = -2(x+2)^2 + 4}$ $\rightsquigarrow V_2 = (-2, 4)$

□

ii) $\frac{y-2}{4-2} = \frac{x - \frac{3}{2}}{-2 - \frac{3}{2}} \iff y - 2 = -2 \cdot \frac{2}{7}(x - \frac{3}{2})$

$\underline{y = -\frac{4}{7}x + \frac{20}{7}}$



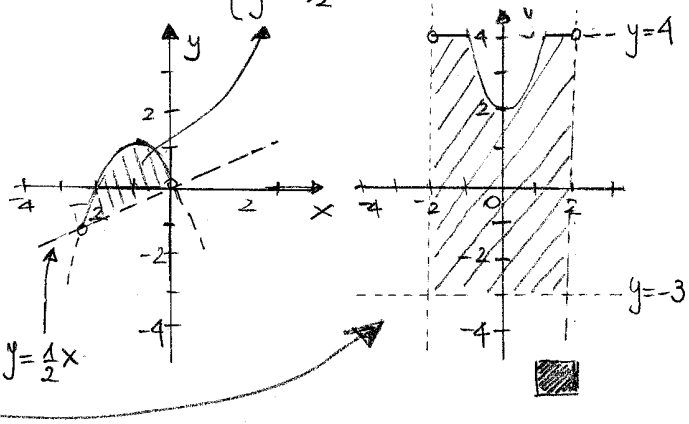
□

iii) $0 = -(x - \frac{3}{2})^2 + 2 \iff (x - \frac{3}{2})^2 = 2 \implies \underline{x_{1/2} = \frac{3}{2} \pm \sqrt{2}}$.

$0 = -2(x+2)^2 + 4 \iff (x+2)^2 = 2 \implies \underline{x_{1/2} = -2 \pm \sqrt{2}}$. ■

6) $\begin{cases} x^2 + 2x + y \leq 0 \\ y - \frac{1}{2}x > 0 \end{cases} \iff \begin{cases} y \leq -(x^2 + 2x) \\ y > \frac{1}{2}x \end{cases} \iff \begin{cases} y \leq -(x+1)^2 + 1 \\ y > \frac{1}{2}x \end{cases}$

$\begin{cases} x^2 < 4 \\ y \leq 2x^2 + 2 \\ -3 < y \leq 4 \end{cases} \iff \begin{cases} -2 < x < 2 \\ y \leq 2x^2 + 2 \\ -3 < y \leq 4 \end{cases}$



■