Università degli Studi di Trento — Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione
ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2009/10 — Rovereto, 9 – 13 novembre 2009

1) $x^2 \log_{1/2} \frac{1}{4} - x \log_2 8 = \log \frac{1}{e} \iff x^2 \cdot 2 - x \cdot 3 = -1$

$\iff 2x^2 - 3x + 1 = 0 \iff x_{1/2} = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm 1}{4} \begin{array}{l} \nearrow \frac{1}{2} \\ \searrow 1 \end{array}$

$\Rightarrow \underline{\underline{S = \{\frac{1}{2}, 1\}}};$

$\frac{2^{x^2-x}}{4} = 1 \iff 2^{x^2-x-2} = 2^0 \iff x^2 - x - 2 = 0$

$\iff x_{1/2} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} \begin{array}{l} \nearrow -1 \\ \searrow 2 \end{array} \Rightarrow \underline{\underline{S = \{-1, 2\}}}.$

2) $\left(\frac{1}{4}\right)^{x^2-1} = 4^{-2} \iff 4^{-x^2+1} = 4^{-2} \iff x^2 - 1 = 2$

$\iff x^2 = 3 \Rightarrow \underline{\underline{S = \{-\sqrt{3}, \sqrt{3}\}}};$

$e^x = \frac{1}{e} \iff e^x = e^{-1} \iff x = -1 \Rightarrow \underline{\underline{S = \{-1\}}};$

$\log_3 x = -1 \iff x = \frac{1}{3} \Rightarrow \underline{\underline{S = \{\frac{1}{3}\}}};$

$\log_4 (x^2-1) \leq 1 \iff \log_4(x^2-1) \leq \log_4 4$

$\iff \begin{cases} x^2 - 1 > 0 \\ x^2 - 1 \leq 4 \end{cases} \iff \begin{cases} x^2 > 1 \\ x^2 \leq 5 \end{cases}$

$\Rightarrow \underline{\underline{S = [-\sqrt{5}, -1[\cup]1, \sqrt{5}]}};$

$3^{-|x|+1} > 27^x \iff 3^{-|x|+1} > 3^{3x} \iff -|x| + 1 > 3x$

$\iff 3x + |x| - 1 < 0$

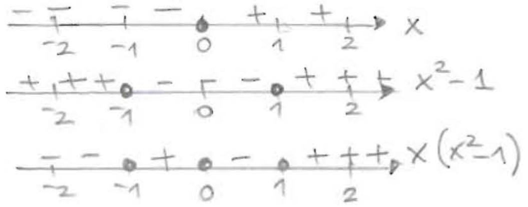
$\iff \begin{cases} x \geq 0 \\ 3x + x - 1 < 0 \end{cases}$ o $\begin{cases} x < 0 \\ 3x - x - 1 < 0 \end{cases}$

$\iff \begin{cases} x \geq 0 \\ 4x < 1 \end{cases}$ o $\begin{cases} x < 0 \\ 2x < 1 \end{cases}$

$\Rightarrow \underline{\underline{S =]-\infty, \frac{1}{4}[}};$

$2^{x^3-x} > 1 \iff 2^{x^3-x} > 2^0 \iff x^3 - x > 0 \iff x(x^2-1) > 0$

$\Rightarrow S = \left]-1,0\right[\cup \left]1,+\infty\right[$;



$$\log_{\frac{1}{3}}(|x|+1) > -2 \iff \log_{1/3}(|x|+1) > \log_{1/3} 9$$

$$\iff \begin{cases} |x|+1 > 0 & \text{(sempre vero!)} \\ |x|+1 < 9 \end{cases}$$

$$\iff |x| < 8 \Rightarrow \underline{\underline{S = \left]-8,8\right[}}.$$

$$\log|x| \le 0 \iff \begin{cases} |x|>0 \\ |x| \le 1 \end{cases} \iff \begin{cases} x \ne 0 \\ -1 \le x \le 1 \end{cases} \Rightarrow \underline{\underline{S = [-1,1]\setminus\{0\}}}.$$

$\blacksquare$

3) i) $(\sqrt{x})^2 = x \quad \forall x \ge 0$;

$\sqrt[3]{x^3} = x \quad \forall x \in \mathbb{R}$;

$\sqrt{x^2} = |x| \quad \forall x \in \mathbb{R}$;

$3^{\log_3(x^2-1)} = x^2-1 \iff x^2-1>0 \iff x^2>1 \Rightarrow \underline{\underline{S = \left]-\infty,-1\right[\cup \left]1,+\infty\right[}}$;

$\log_2(2^{x-1}) = x-1 \quad \forall x \in \mathbb{R}$.

$\square$

ii) a) $e^{-x}\cdot e^{-x^2+1} > e^{2x} \iff e^{-x-x^2+1} > e^{2x} \iff -x-x^2+1 > 2x$

$$\iff x^2+3x-1<0 \Rightarrow \underline{\underline{S = \left]\frac{-3-\sqrt{13}}{2}, \frac{-3+\sqrt{13}}{2}\right[}}.$$

$$\left(\frac{1}{4}\right)^x \cdot 4^{-x^2+2x} \ge 1 \iff 4^{-x-x^2+2x} \ge 4^0 \iff -x^2+x \ge 0$$

$$\iff x^2-x \le 0 \Rightarrow \underline{\underline{S = [0,1]}}.$$

$$9^x < 3\cdot 3^{|x|} \iff 3^{2x} < 3^{1+|x|} \iff 2x < 1+|x|$$

$$\iff \begin{cases} x \ge 0 \\ x < 1 \end{cases} \text{ o } \begin{cases} x<0 \\ 3x<1 \end{cases} \Rightarrow \underline{\underline{S = \left]-\infty,1\right[}}.$$

b) $3^{x^2} - 3^{1+|x+1|} \ge 0 \iff 3^{x^2} \ge 3^{1+|x+1|} \iff x^2 \ge 1+|x+1|$

$$\iff \begin{cases} x+1 \ge 0 \\ x^2-(x+1)-1 \ge 0 \end{cases} \text{ o } \begin{cases} x+1 < 0 \\ x^2+(x+1)-1 \ge 0 \end{cases}$$

$$\iff \begin{cases} x \ge -1 \\ x^2-x-2 \ge 0 \end{cases} \text{ o } \begin{cases} x<-1 \\ x^2+x \ge 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x \geq -1 \\ x \leq -1 \text{ o } x \geq 2 \end{cases} \quad \text{o} \quad \begin{cases} x < -1 \\ x \leq -1 \text{ o } x \geq 0 \end{cases}$$

$$\Rightarrow \underline{\underline{S =]-\infty, -1] \cup [2, +\infty[\,.}}$$

$$\log_{1/4}(x^2+1) - \log_4 \tfrac{1}{4} < 0 \Leftrightarrow \log_{1/4}(x^2+1) + 1 < 0$$

$$\log_{1/4}(x^2+1) < -1$$

$$\log_{1/4}(x^2+1) < \log_{1/4} 4$$

$$\underset{\log_{1/4} x \downarrow}{\Leftrightarrow} \quad x^2+1 > 4 \Leftrightarrow x^2 > 3$$

$$\Rightarrow \underline{\underline{S =]-\infty, -\sqrt{3}[\cup]\sqrt{3}, +\infty[\,.}}$$

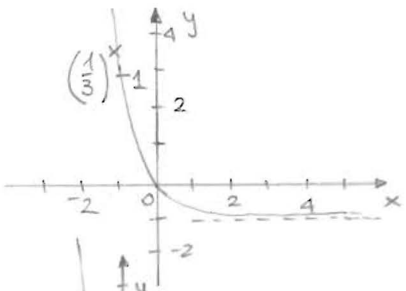
$$\log_3(x^2+x) < \log_3(2|x|) \Leftrightarrow \begin{cases} x^2+x > 0 \\ |x| > 0 \\ x^2+x < 2|x| \end{cases} \Leftrightarrow \begin{cases} x < -1 \text{ o } x > 0 \\ x \neq 0 \\ x^2+x < 2|x| \end{cases}$$

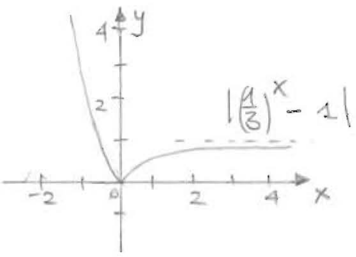
$$\Leftrightarrow \begin{cases} x < -1 \\ x < 0 \\ x^2+x < -2x \end{cases} \quad \text{o} \quad \begin{cases} x > 0 \\ x > 0 \\ x^2+x < 2x \end{cases}$$

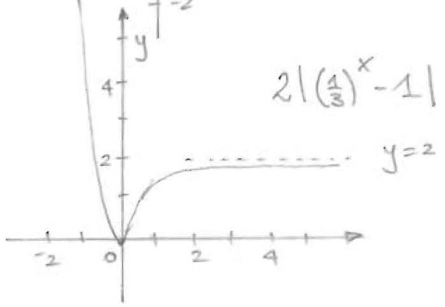
$$\Rightarrow \underline{\underline{S =]-3, -1[\cup]0, 1[\,.}}$$

4) $|2(\frac{1}{3})^x - 2|$ insieme di def. $= \underline{\underline{\mathbb{R}}}$

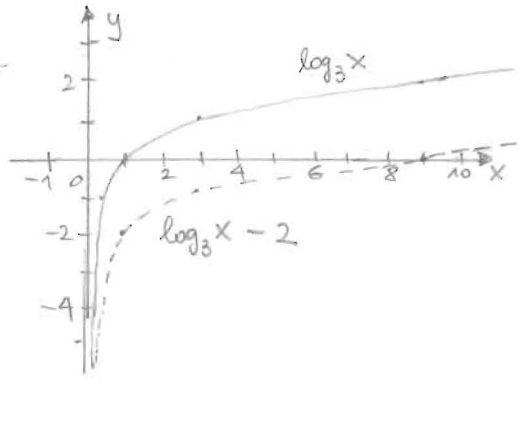
$= 2|(\frac{1}{3})^x - 1|$

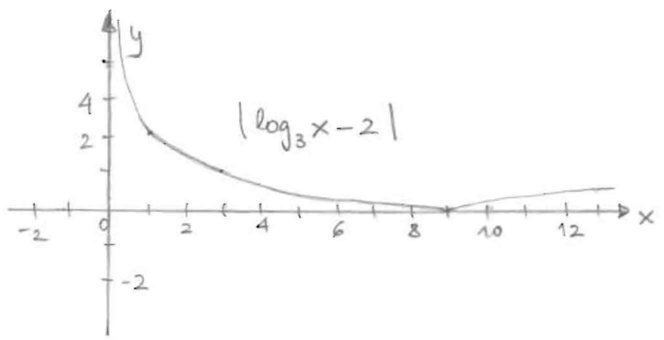




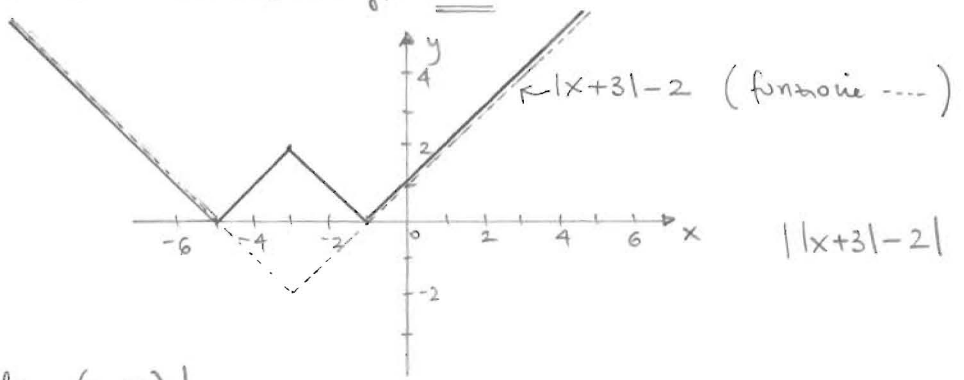


$|\log_3 x - 2|$ insieme di def. $= \underline{\underline{]0, +\infty[}}$



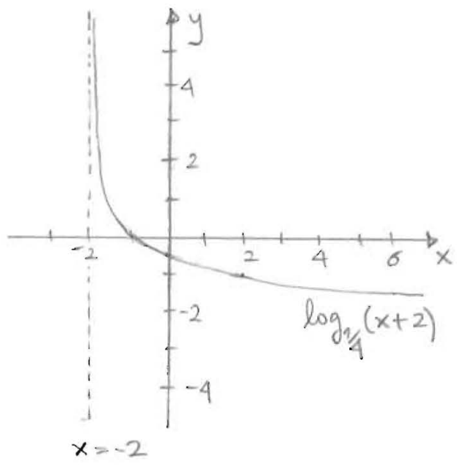


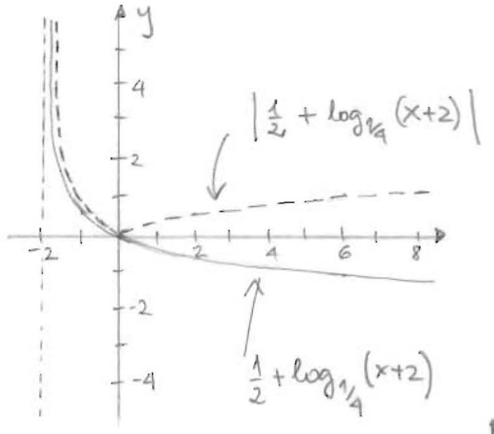
$||x+3|-2|$ insieme di def. $= \mathbb{R}$



$||x+3|-2|$ funzione ——

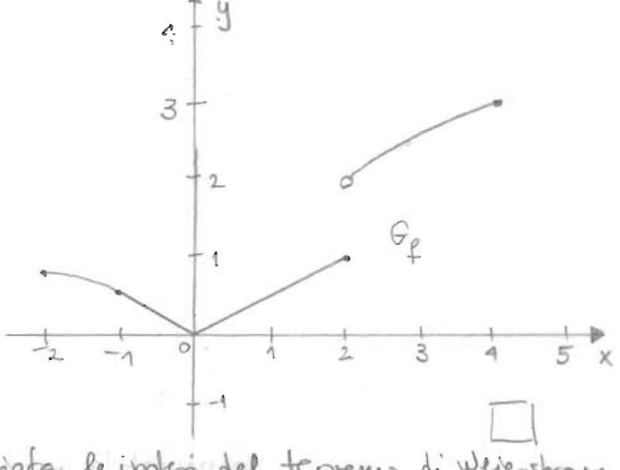
$\left|\frac{1}{2} + \log_{1/4}(x+2)\right|$





5) i) $f: [-2,4] \to \mathbb{R}$

$$f(x) = \begin{cases} -2^x + 1 & \text{se } -2 \le x \le -1 \\ \frac{1}{2}|x| & \text{se } -1 < x \le 2 \\ -\log_{\frac{1}{2}} x + 1 & \text{se } 2 < x \le 4 \end{cases}$$



ii) $f([-2,4]) = [0,1] \cup]2,3]$.

f è continua su $[-2,2]$ e quindi soddisfa le ipotesi del teorema di Weierstrass, mentre non è continua nell'intervallo $[-2,4]$, e quindi non soddisfa le ip. del T.W.

$\min_{[-2,2]} f = 0$, $x=0$ pt. di min; $\max_{[-2,2]} f = 1$, $x=2$ pt. di max;

$\min_{[-2,4]} f = 0$, $x=0$ pt. di min; $\max_{[-2,4]} f = 3$, $x=4$ pt. di max.