Università degli Studi di Trento – Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione – CdL in Filosofia
ESAME SCRITTO DI ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2010 – 2011 – Rovereto, 10 gennaio 2011

FILA (A)

1) i)

$\mathcal{A}$	V	V	F	F
$\mathcal{B}$	V	F	V	F
non $\mathcal{B}$	F	V	F	V
$\mathcal{A}$ e (non $\mathcal{B}$)	F	V	F	F
non[$\mathcal{A}$ e (non $\mathcal{B}$)]	V	F	V	V
non $\mathcal{A}$	F	F	V	V
(non $\mathcal{A}$) o $\mathcal{B}$	V	F	V	V

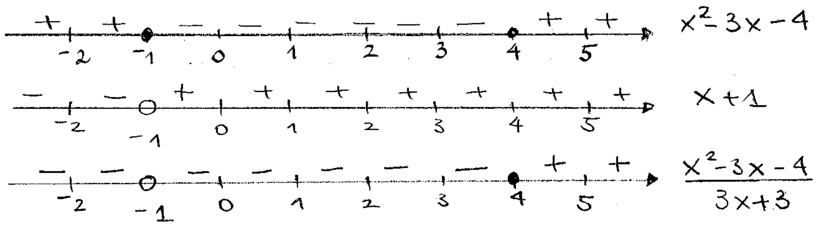
le due proposizioni sono equivalenti.

$\square$

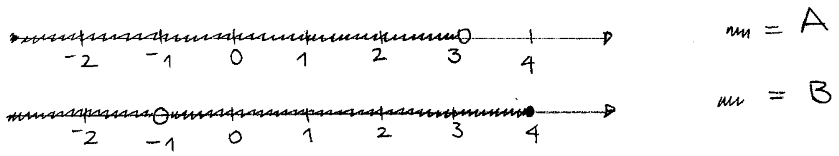
ii) a) $\begin{cases} x-2 \geq 0 \\ x(x-2)-3<0 \end{cases}$ o $\begin{cases} x-2<0 \\ -x^2+2x-3<0 \end{cases}$ $\Longleftrightarrow$ $\begin{cases} x \geq 2 \\ -1<x<3 \end{cases}$ o $\begin{cases} x<2 \\ \mathbb{R} \end{cases}$

$$A = \{x \in \mathbb{R} : x|x-2| < 3\} =]-\infty, 3[.$$

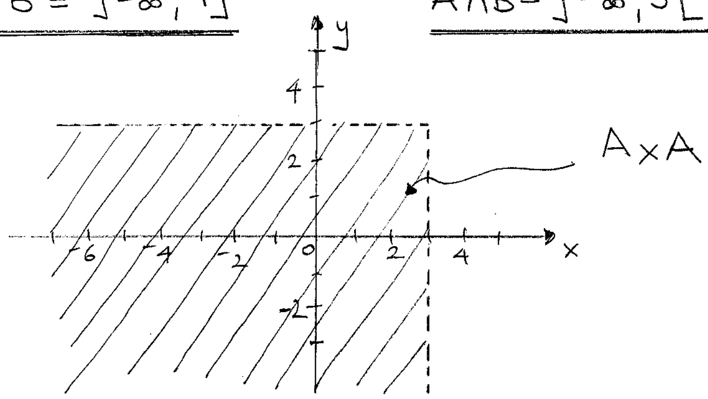
$$\frac{x^2-1}{3x+3} \leq 1 \iff \frac{x^2-1}{3x+3} - 1 \leq 0 \iff \frac{x^2-3x-4}{3x+3} \leq 0$$



$$B =]-\infty, 4] \setminus \{-1\}$$



b) $A \cup B =]-\infty, 4]$ $\qquad$ $A \cap B =]-\infty, 3[\setminus \{-1\}$.



2) $x^2+y^2-2x+4y+4=0 \iff x^2-2x+y^2+4y+4=0$

$\iff (x-1)^2-1+(y+2)^2-\cancel{4}+\cancel{4}=0$

$\iff (x-1)^2+(y+2)^2=1 \qquad C_1=(1,-2) \quad r=1$

$x^2+16y^2-4x=0 \iff x^2-4x+16y^2=0$

$\iff (x-2)^2-4+16y^2=0 \iff (x-2)^2+16y^2=4$

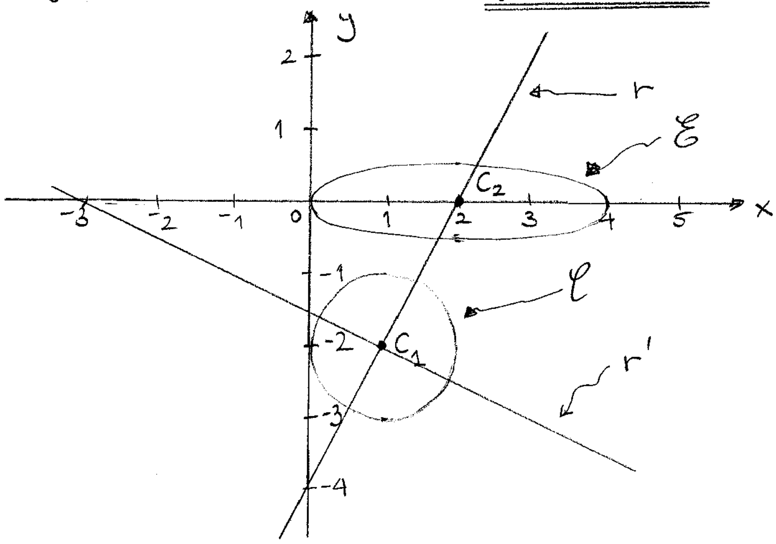
$\iff \dfrac{(x-2)^2}{4}+\dfrac{y^2}{\frac{1}{4}}=1 \qquad C_2=(2,0) \quad a=2, \; b=\frac{1}{2}$.

i) $C_1=(1,-2)$, $C_2=(2,0)$

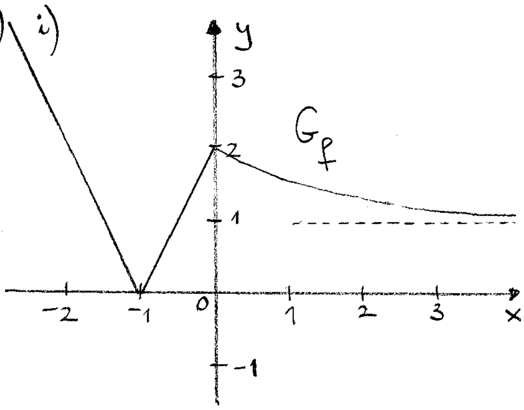
$\dfrac{y-y_1}{y_2-y_1}=\dfrac{x-x_1}{x_2-x_1} \Rightarrow \dfrac{y+2}{0+2}=\dfrac{x-1}{2-1}$

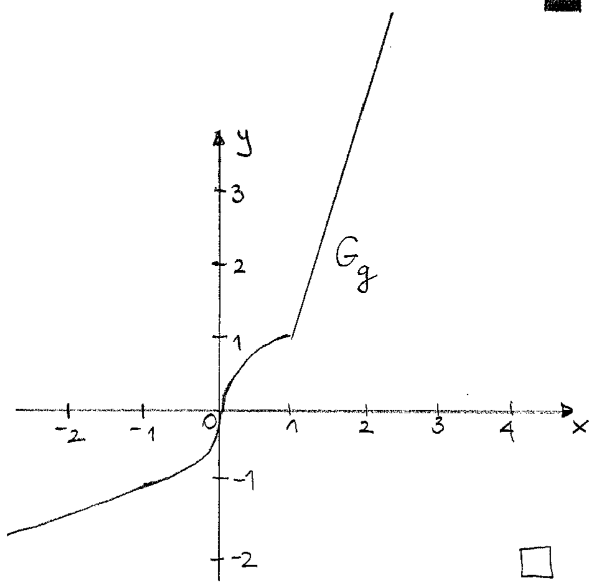
$\Rightarrow \underline{\underline{y=2x-4}}$.

ii) r' : $y=-2-\frac{1}{2}(x-1) \Rightarrow \underline{\underline{y=-\frac{1}{2}x-\frac{3}{2}}}$



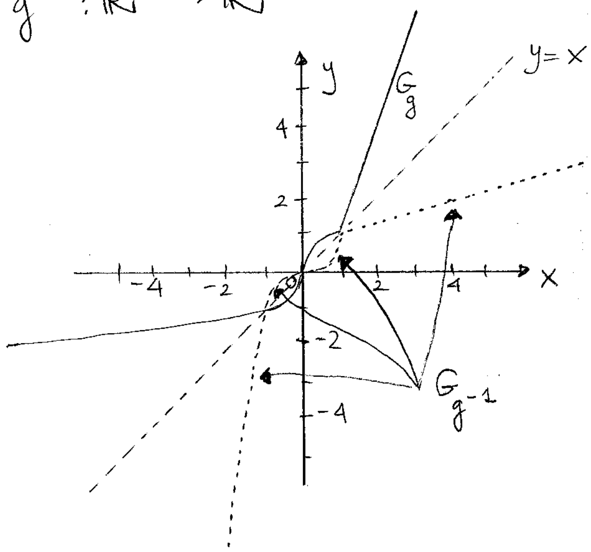
3) i)





ii) $\underline{\underline{f(\mathbb{R})=[0,+\infty[}}$.

iii) g è iniettiva (è strettamente crescente) e suriettiva ($g(\mathbb{R}) = \mathbb{R}$), quindi $\exists\ g^{-1} : \mathbb{R} \to \mathbb{R}$



iv) $\max\limits_{[-2,1]} f = \underline{\underline{2}}$ $\qquad x=-2,\ x=0$ pt. di massimo di f su $[-2,1]$;

$\min\limits_{[-2,1]} f = \underline{\underline{0}}$ $\qquad x=-1$ pt. di minimo di f su $[-2,1]$.

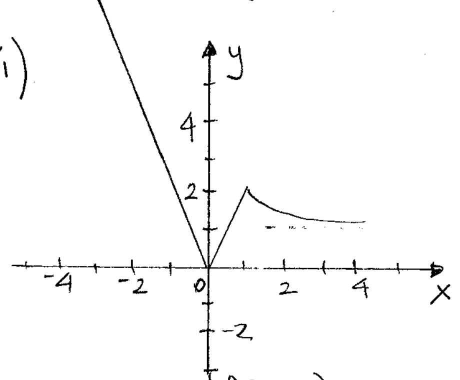
$x=1$ pt. di minimo locale di f su $[-2,1]$ (oltre ai pt. di min. e max di f individuati sopra).

v) $(f+g)(-1) = f(-1) + g(-1) = 0 - 1 = \underline{\underline{-1}}$;

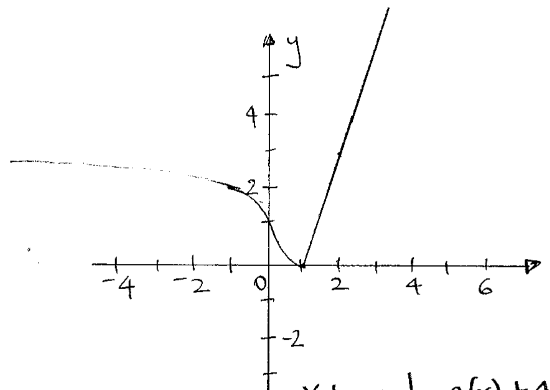
$(fg)(1) = f(1) \cdot g(1) = \frac{3}{2} \cdot 1 = \underline{\underline{\frac{3}{2}}}$;

$(g \circ f)(0) = g(f(0)) = g(2) = \underline{\underline{4}}$.

vi)



$x \longmapsto f(x-1)$



$x \longmapsto |-g(x)+1|$

4) i) $\underline{\underline{S = \left]-\infty, -\frac{1}{5}\right[}}$; $\qquad \underline{\underline{S = \left]3, +\infty\right[}}$ $\qquad$ (vedi SP1, Fila (B), Es. 1).

ii) $\displaystyle\int_0^4 |x^2-1|\,dx = \underline{\underline{\frac{56}{3}}}$; $\qquad \displaystyle\int_1^2 \left(\frac{2x^2}{x^3+1} + x^{-2} + 1\right) dx = \underline{\underline{\frac{2}{3}\log\frac{9}{2} + \frac{3}{2}}}$

(vedi SP1, Fila (B), Es. 2 i)).

5) vedi SP1, Fila (B), Es. 5.

6) vedi SP1, Fila (B), Es. 6.

FILA (B)

1) i)

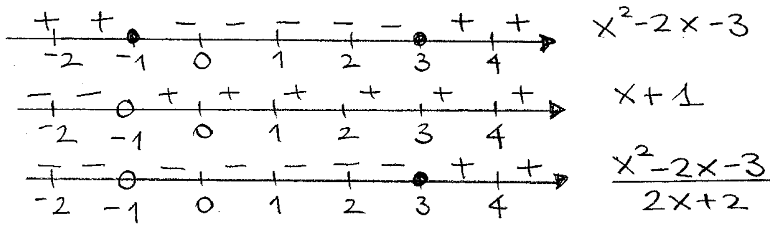
$\mathcal{A}$	V	V	F	F
$\mathcal{B}$	V	F	V	F
non $\mathcal{B}$	F	V	F	V
$\mathcal{A}$ o (non $\mathcal{B}$)	V	V	F	V
non [$\mathcal{A}$ o (non $\mathcal{B}$)]	F	F	V	F
non $\mathcal{A}$	F	F	V	V
(non $\mathcal{A}$) e $\mathcal{B}$	F	F	V	F

le due proposizioni sono equivalenti.

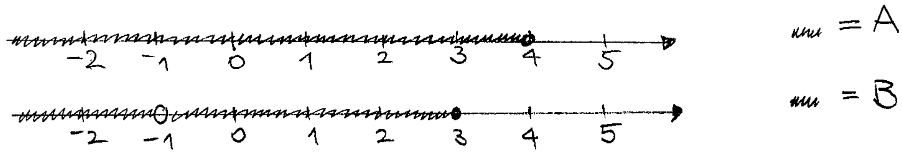
ii) a) $\begin{cases} x-3 \geq 0 \\ x(x-3)-4 \leq 0 \end{cases}$ o $\begin{cases} x-3<0 \\ -x^2+3x-4 \leq 0 \end{cases} \iff \begin{cases} x \geq 3 \\ -1 \leq x \leq 4 \end{cases}$ o $\begin{cases} x<3 \\ \mathbb{R} \end{cases}$

$$A = \{x \in \mathbb{R} : x|x-3| \leq 4\} = \underline{\underline{]-\infty, 4]}}.$$

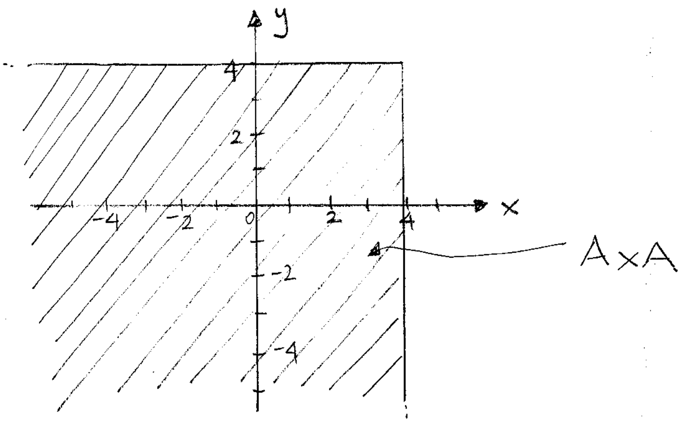
$$\frac{x^2-1}{2x+2} \leq 1 \iff \frac{x^2-1}{2x+2} - 1 \leq 0 \iff \frac{x^2-2x-3}{2x+2} \leq 0$$



$$\underline{\underline{B =]-\infty, 3] \setminus \{-1\}}}.$$



b) $\underline{\underline{A \cup B =]-\infty, 4]}}$ $(=A)$ $\quad$ $\underline{\underline{A \cap B =]-\infty, 3] \setminus \{-1\}}}$ $(=B)$



2) $x^2+y^2-4x-2y+4=0 \iff x^2-4x+y^2-2y+4=0$

$$\iff (x-2)^2-\cancel{4}+(y-1)^2-1+\cancel{4}=0$$

$$\iff (x-2)^2+(y-1)^2=1 \qquad C_1=(2,1) \quad r=1$$

$16x^2+y^2-4y=0 \iff 16x^2+(y-2)^2-4=0$

$$\iff 16x^2+(y-2)^2=4$$

$$\iff \frac{x^2}{1/4}+\frac{(y-2)^2}{4}=1 \qquad C_2=(0,2) \quad a=\tfrac{1}{2}, \; b=2.$$

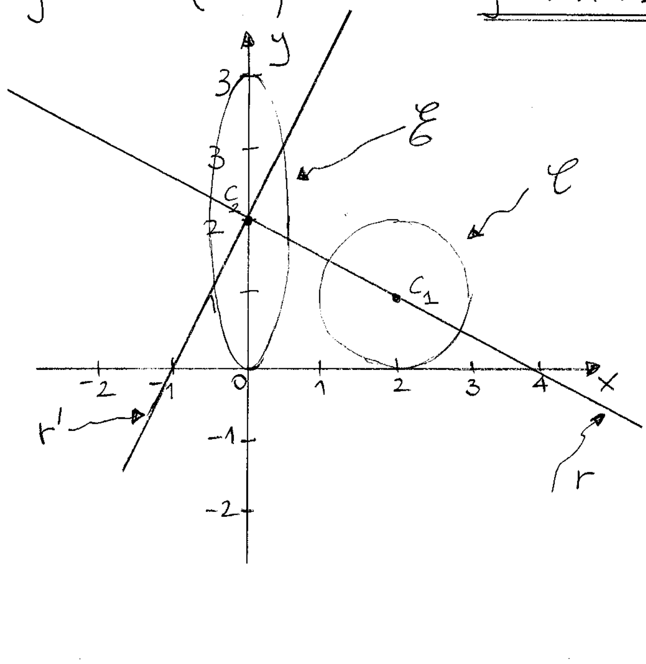
i) $C_1=(2,1)$, $C_2=(0,2)$

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1} \Rightarrow \frac{y-1}{2-1}=\frac{x-2}{0-2}$$

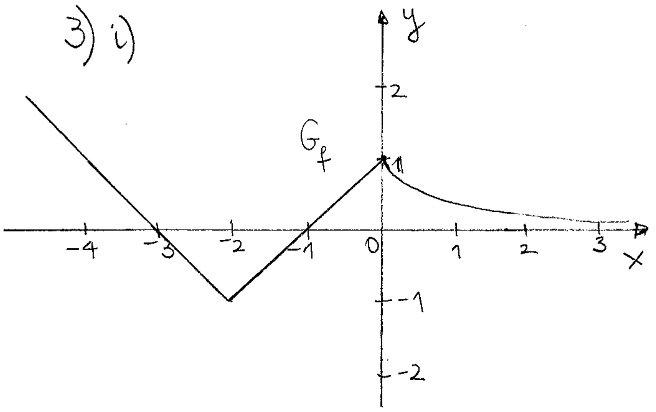
$$\Rightarrow \underline{\underline{y=-\frac{x}{2}+2}}.$$

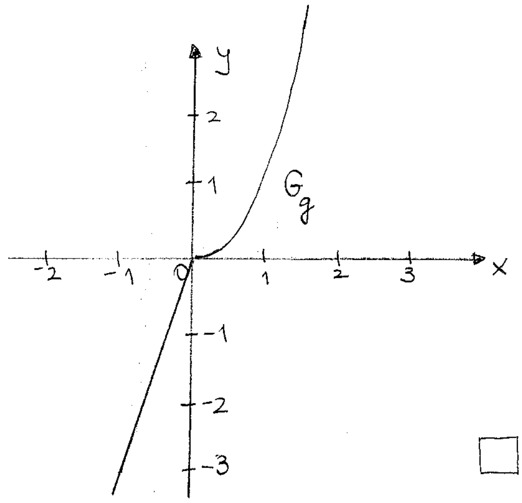
ii) r' : $y=2+2(x-0) \Rightarrow \underline{\underline{y=2x+2}}$

iii)



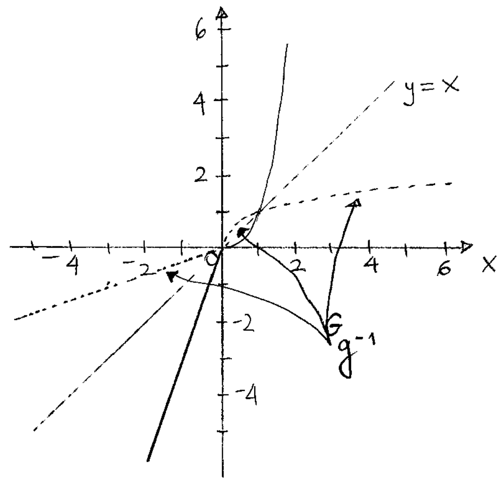
3) i)





ii) $\underline{\underline{f(\mathbb{R}) = [-1, +\infty[}}$.

iii) g è iniettiva (è strettamente crescente) e suriettiva $(g(\mathbb{R}) = \mathbb{R})$, quindi $\exists\, g^{-1} : \mathbb{R} \to \mathbb{R}$



iv) $\min\limits_{[-3,1]} f = \underline{\underline{-1}}$ $\quad x = -2$ pt. di minimo di f su $[-3,1]$;

$\max\limits_{[-3,1]} f = \underline{\underline{1}}$ $\quad x = 0$ pt. di massimo di f su $[-3,1]$.

$x = -3$ pt. di max locale di f su $[-3,1]$;
$x = 1$ pt. di min. locale di f su $[-3,1]$.

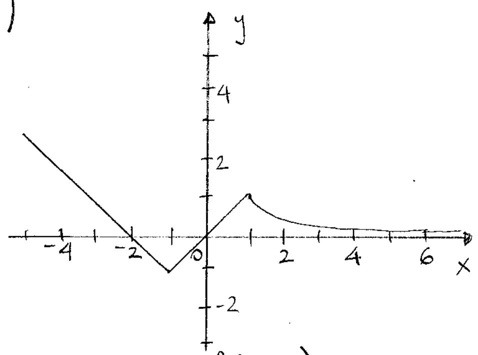
(oltre ai pt. di min. e max di f individuati sopra).

v) $(f+g)(-1) = f(-1) + g(-1) = 0 + (-3) = \underline{\underline{-3}}$;

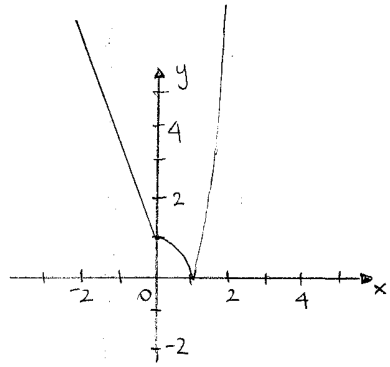
$(fg)(1) = f(1) \cdot g(1) = \frac{1}{2} \cdot 1 = \underline{\underline{\frac{1}{2}}}$;

$(g \circ f)(0) = g(f(0)) = g(1) = \underline{\underline{1}}$.

vi)



$x \longmapsto f(x-1)$



$x \longmapsto |-g(x)+1|$

4) i) $S = \left[-\frac{1}{5}, +\infty\right[$; $S = \left]-3,-2\right[\cup \left]2,3\right]$ (vedi SP1, Fila (A), Es. 1).

ii) $\displaystyle\int_{-3}^{0} |x^2-1|\,dx = \frac{22}{3}$; $\displaystyle\int_{1}^{2} \left(\frac{3x^3}{x^4+1} + x^{-1} + 1\right) dx = \frac{3}{4}\log 17 + \frac{1}{4}\log 2 + 1$

(vedi SP1, Fila (A), Es. 2 i)).

5) vedi SP1, Fila (A), Es. 5.

6) Vedi SP1, Fila (A), Es. 6.