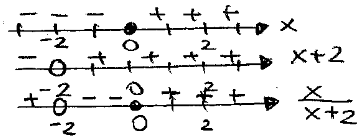
Università degli Studi di Trento – Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione – CdL in Filosofia
Corso di ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
PRIMA PROVA INTERMEDIA – a.a. 2010/11 – Rovereto, 3 novembre 2010

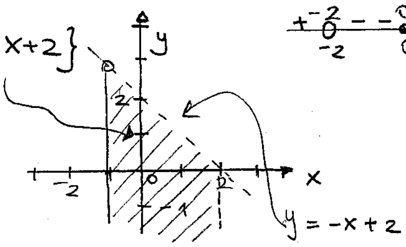
FILA (A)

a1) $A = [-1,3[$, $B = \{-2,2\}$ $\qquad A \cup B = \underline{\underline{\{-2\} \cup [-1,3[}}$; $\qquad A \cap B = \underline{\underline{\{2\}}}$.

a2) $\frac{x}{x+2} > 0$ $\qquad S = \underline{\underline{]-\infty,-2[\,\cup\,]0,+\infty[}}$.



a3) $E = \{(x,y) \in \mathbb{R}^2 : -1 \le x < 2,\ y < -x+2\}$

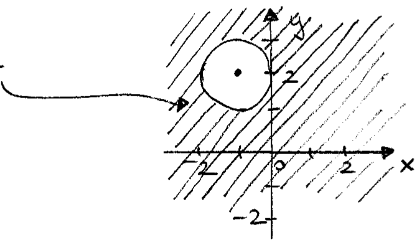


a4) $-x^2+3x+4 < 0 \iff x^2-3x-4 > 0$ $\qquad x^2-3x-4=0 \iff$

$x_{1/2} = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2} = \begin{cases} -1 \\ 4 \end{cases}$

$S = \underline{\underline{]-\infty,-1[\,\cup\,]4,+\infty[}}$.

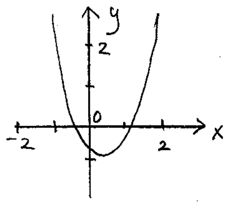
a5) $(x+1)^2 + (y-2)^2 \ge 1$



a6) "Lo studente Pinco Pallino non ha studiato per la prova intermedia o (lo studente) non supera la prova."

a7) $y = 2 - 1(x+1) \Rightarrow \underline{\underline{y = -x+1}}$.

a8)



(i) V (concavità della parabola verso l'alto) $\quad$ (ii) F $\quad$ (iii) V (ci sono due intersezioni della parabola con l'asse x) $\quad$ (iv) F

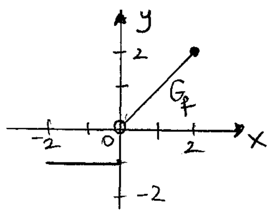
a9) $E = [-2,2]$ $\qquad F = \{3\}$

(i) V (poiché $0 \in E$ e $3 \in F$)

(ii) F (poiché $3 \notin E$)

(iii) F (è vero che $(2,3) \in E \times F$).

a10)



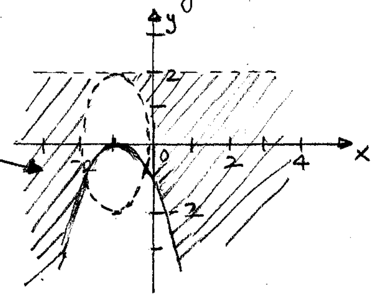
$f(-2) = \underline{-1}$ $\qquad f([-2,2]) = \underline{\underline{\{-1\} \cup]0,2]}}$.

☐

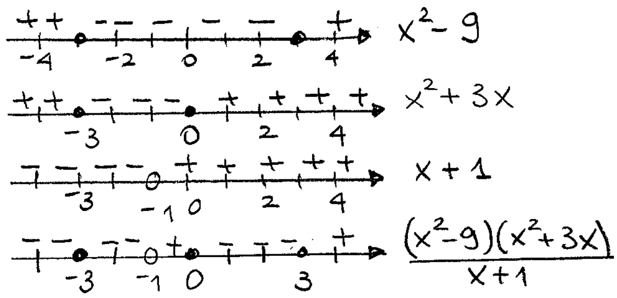
b1) i) "La crisi economica in Europa permane e il tasso di disoccupazione giovanile rimane invariato o diminuisce". □

ii) $\begin{cases} 4x^2+y^2+8x>0 \\ -x^2-2x-1 \le y < 2 \end{cases} \iff \begin{cases} 4(x^2+2x)+y^2>0 \\ -(x+1)^2 \le y < 2 \end{cases} \iff \begin{cases} 4(x+1)^2+y^2>4 \\ -(x+1)^2 \le y < 2 \end{cases}$

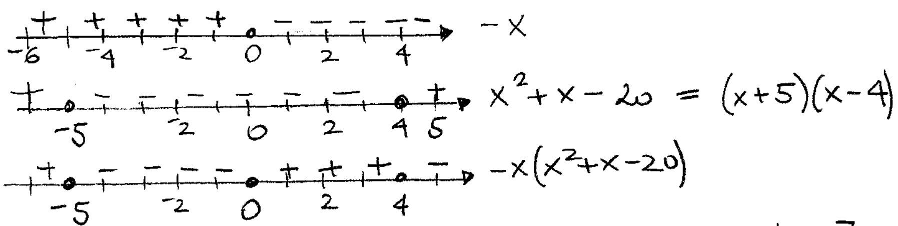
$\iff \begin{cases} (x+1)^2+\dfrac{y^2}{4}>1 \\ -(x+1)^2 \le y < 2 \end{cases}$



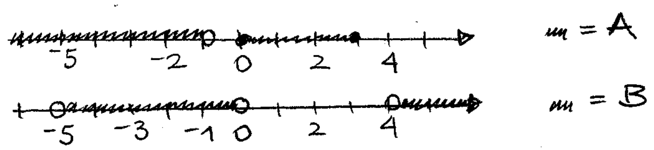
b2) i)



$A = \left]-\infty,-1\right[\cup [0,3]$



$B = \left]-5,0\right[\cup \left]4,+\infty\right[$

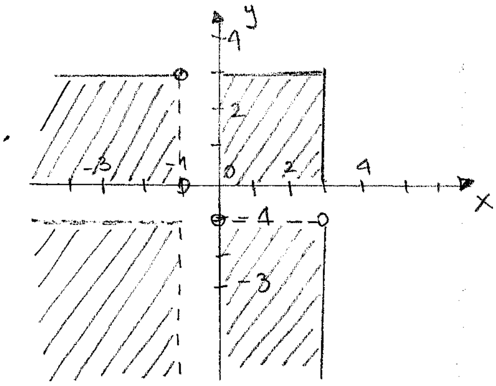


$A \setminus B = \left]-\infty,-5\right] \cup [0,3]$. □

ii) A non è limitato inferiormente e B non è limitato superiormente (e quindi nessuno dei due è un insieme limitato).

$\max A = 3$ $\quad \nexists \min B$. □

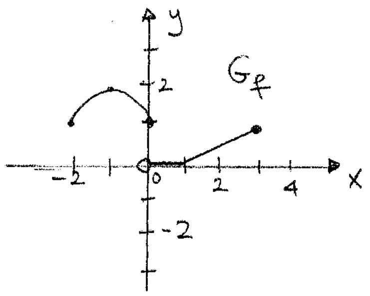
iii) $A \times A$

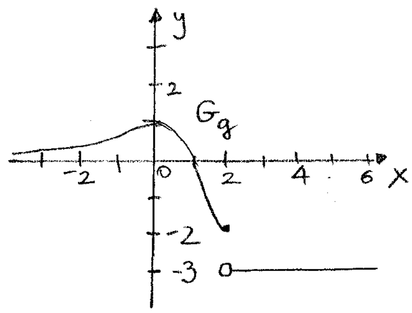


iv) $x = k \quad -1 \leq k < 0 \quad o \quad k > 3.$

v) a) V b) F c) V d) F la scrittura.

b3)





i) se $k<0$, $\nexists$ono soluz. dell'eq. $f(x)=k$;
se $k=0$, $\exists$ono infinite " ;
se $0<k<1$, $\exists$ una soluzione " ;
se $k=1$, $\exists$ono 3 soluzioni " ;
se $1<k<2$, $\exists$ono 2 soluzioni " ;
se $k=2$, $\exists$ una soluzione " ;
se $k>2$, $\nexists$ono soluzioni dell'eq. $f(x)=k$.

ii) $g(\mathbb{R}) = \{-3\} \cup [-2,1] \not\subseteq \text{dom} f$ e quindi si deve restringere il dominio di g affinché si possa considerare la composizione $f \circ g$; si osserva subito che invece $g(]-\infty, 2]) = [-2,1] \subseteq \text{dom} f$!! e quindi
$f \circ g$ è ben definita per $x \in]-\infty, 2]$.
La composizione $g \circ f$ è invece definita $\forall x \in [-2,3]$.

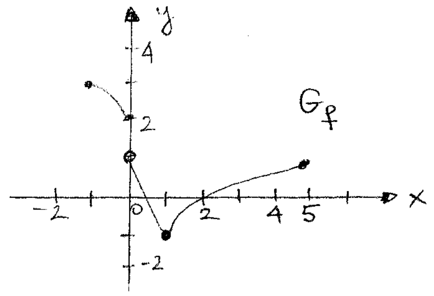
iii) $(f+g)(0) = f(0)+g(0) = 1+1 = \underline{\underline{2}}$;

$(fg)(3) = f(3)\cdot g(3) = 1\cdot(-3) = \underline{\underline{-3}}$;

$(g\circ f)(1) = g(f(1)) = g(0) = \underline{\underline{1}}$;

$(f\circ g)(3) \nexists$ poiché $g(3)=-3$ e $f(-3) \nexists$.

b4) i)



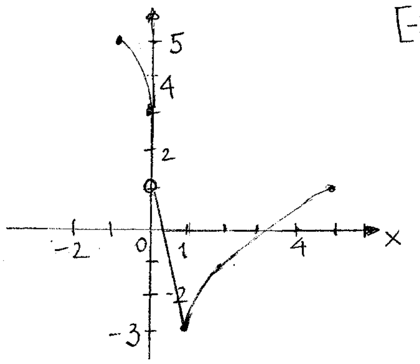
$f([-1,5]) = \underline{\underline{[-1,1] \cup [2,3]}}$.

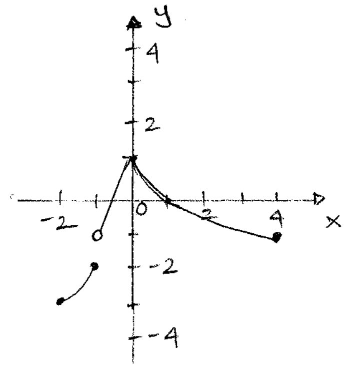
ii) $\min_{[-1,5]} f = -1$ $\quad x=1$ pt. di minimo ; $\quad \max_{[-1,5]} f = 3$ $\quad x=-1$ pt. di max.

iii) Basta prendere $x_1 = \frac{1}{2} \neq x_2 = 2$ e $f(\frac{1}{2}) = f(2) = 0$!

$I = [1,5]$ e $f|_I$ è iniettiva.

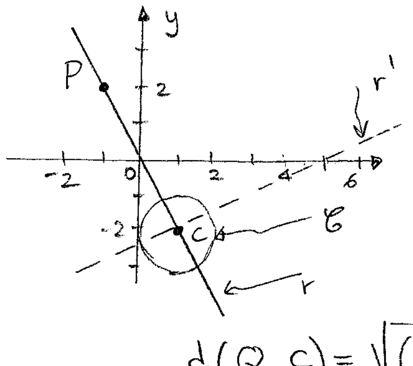
iv) $\underset{[-1,5]}{\overset{\cap}{x}} \longmapsto 2f(x)-1$ $\qquad$ $\underset{[-2,4]}{\overset{\cap}{x}} \longmapsto -f(x+1)$





b5) i) $x^2+y^2-2x+4y+4=0$ $\qquad (x-1)^2-1+(y+2)^2-4+4=0$

$(x-1)^2+(y+2)^2=1 \qquad C=(1,-2) \quad r=1$



$\underline{y=-2x}$.

ii) $y=-2+\frac{1}{2}(x-1) \Rightarrow \underline{\underline{y=\frac{1}{2}x-\frac{5}{2}}}$.

iii) $Q=(5,0)$

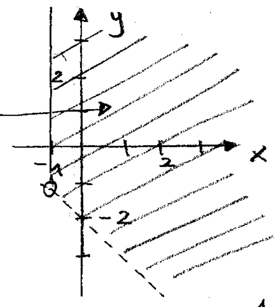
$d(Q,C) = \sqrt{(5-1)^2+(-2-0)^2} = \sqrt{16+4} = \underline{\underline{\sqrt{20}}} = 2\sqrt{5}$

FILA (B)

a1) $A =]-\infty,-1[$ $\quad B = \{-2,3\}$ $\quad A\cap B = \underline{\underline{\{-2\}}}$ $\quad A\setminus B = \underline{\underline{]-\infty,-2[\,\cup\,]-2,-1[}}.$

a2) $\dfrac{x^2}{x+3} < 0$ $\quad S = \underline{\underline{]-\infty,-3[}}.$

a3) $E = \{(x,y)\in\mathbb{R}^2 : y > -x-2 \;,\; x \geq -1\}$

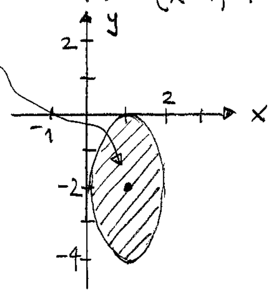


a4) $-x^2+x+2 > 0 \iff x^2-x-2<0$

$S = \underline{\underline{]-1,2[}}.$

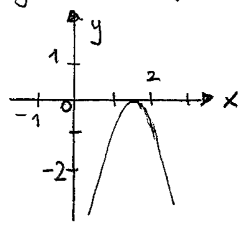
$x^2-x-2=0 \iff x_{1/2} = \dfrac{1\pm\sqrt{1+8}}{2} = \dfrac{1\pm 3}{2} \begin{cases} -1 \\ 2 \end{cases}$

a5) $4(x-1)^2+(y+2)^2 \leq 4 \iff (x-1)^2 + \dfrac{(y+2)^2}{4} \leq 1$



a6) "La città di Rovereto non investe nell'offerta culturale per studenti universitari o per immigrati."

a7) $y = -1+2(x-2) \Rightarrow \underline{\underline{y = 2x-5}}.$

a8) 

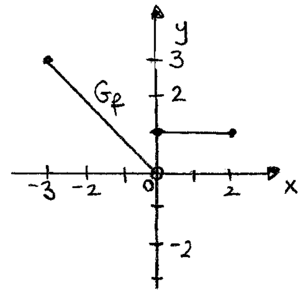
(i) F $\quad$ (ii) V (concavità della parabola verso il basso) $\quad$ (iii) V (c'è un'intersezione della parabola con l'asse x) $\quad$ (iv) F

a9) $E =]-\infty,-1[\,\cup\,]1,+\infty[$ $\quad F = \{-2\}$

(i) V (poiché $-2\in E$, $-2\in F$)

(ii) V (poiché $-2\in E$)

(iii) F (poiché $-2\in E\cap F$).

a10) 

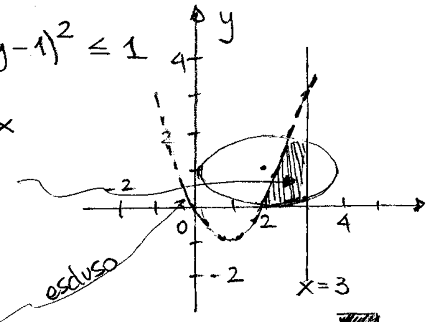
$f(0) = \underline{\underline{1}}$ $\quad f([-3,2]) = \underline{\underline{]0,3]}}.$

□

b1) i) Lo studente Tizio Caio supera la prima prova intermedia di AM e non può partecipare alla seconda prova intermedia."

ii)
$$\begin{cases} x^2+4y^2-4x-8y+4\le 0 \\ y-x^2+2x<0 \\ x\le 3 \end{cases} \iff \begin{cases} (x-2)^2-\cancel{4}+4(y^2-2y)+\cancel{4}\le 0 \\ y<x^2-2x \\ x\le 3 \end{cases}$$

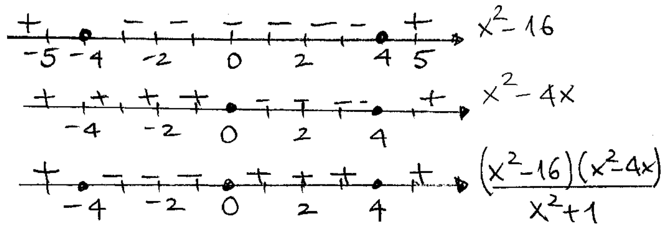
$$\iff \begin{cases} (x-2)^2+4(y-1)^2\le 4 \\ y<x^2-2x \\ x\le 3 \end{cases} \iff \begin{cases} \left(\frac{x-2}{4}\right)^2+(y-1)^2\le 1 \\ y<x^2-2x \\ x\le 3 \end{cases}$$

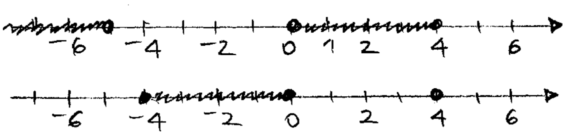


$y=x^2-2x$
$=(x-1)^2-1$

b2) i) $A=\{x\in\mathbb{R}: -x(x^2+x-20)\ge 0\} = \underline{\underline{]-\infty,-5]\cup[0,4]}}$.

$B=\left\{x\in\mathbb{R}: \frac{(x^2-16)(x^2-4x)}{x^2+1}\le 0\right\} = \underline{\underline{[-4,0]\cup\{4\}}}$.





= A

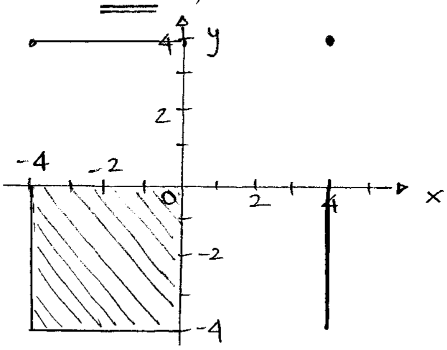
= B

$A\cup B = \underline{\underline{]-\infty,-5]\cup[-4,4]}}$.

ii) A non è limitato inferiormente; B è limitato.

$\max A = \underline{\underline{4}}$, $\min B = \underline{\underline{-4}}$.

iii)

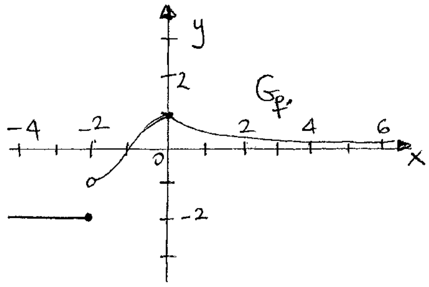


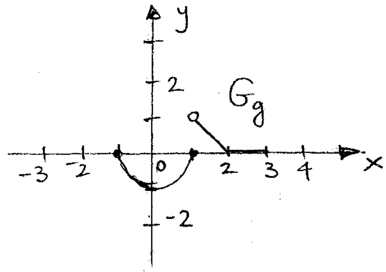
iv) $y=k$ $\quad \underline{\underline{k\in]-\infty,-4[\cup]0,4[\cup]4,+\infty[}}$

v) a) F b) F

c) V d) F.

b3)





i) se $k \in]-\infty, -2[\cup]-2, -1] \cup]1, +\infty[$ $\nexists$ sono soluzioni dell'eq. $f(x) = k$;

se $k = -2$ $\exists$ sono infinite soluzioni dell'eq. $f(x) = k$;

se $-1 < k \leq 0$ $\exists$ una soluzione " ;

se $0 < k < 1$ $\exists$ sono 2 soluzioni " ;

se $k = 1$ $\exists$ una soluzione dell'eq. $f(x) = k$. ☐

ii) $f \circ g$ è definita $\underline{\forall x \in [-1, 3]}$ poiché $g([-1,3]) = [-1, 1[\subset \mathbb{R}$.
ed $\mathbb{R}$ è il dominio di f.

$g \circ f$ è definita solo se $\underline{x \in]-2, +\infty[}$, poiché
$f(]-\infty, -2]) = \{-2\} \notin \operatorname{dom} g$. ☐

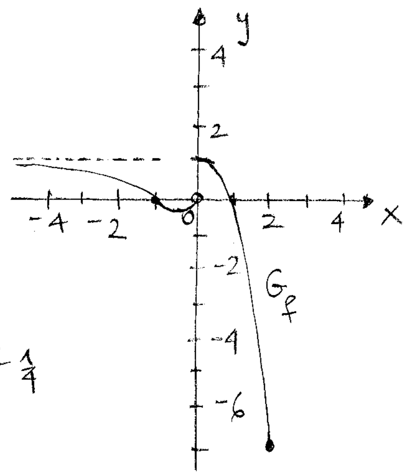
iii) $(f - g)(0) = f(0) - g(0) = 1 - (-1) = \underline{2}$;

$\left(\dfrac{f}{g}\right)(3)$ non è definito, poiché $g(3) = 0$;

$(g \circ f)(-1) = g(f(-1)) = g(0) = \underline{-1}$;

$(f \circ g)(2) = f(g(2)) = f(0) = \underline{1}$. ■

b4) i)



$y = x^2 + x = \left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$

$f(]-\infty, 2]) = \underline{[-7, 1]}$. ☐

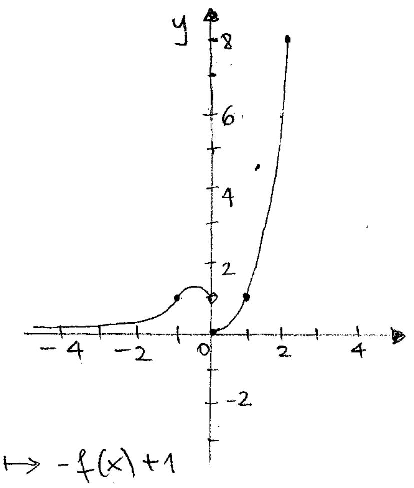
ii) $\min\limits_{]-\infty,2]} f = -7$ $x = 2$ pt. di minimo;

$\max\limits_{]-\infty,2]} f = 1$ $x = 0$ pt. di massimo. ☐

iii) $x_1 = -1 \neq x_2 = 1$ e $f(-1) = f(1) = 0$.

$I =]-\infty, -\frac{1}{2}]$ e $f|_I$ è iniettiva ☐

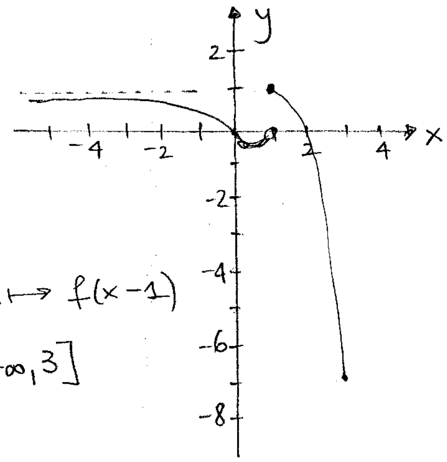
iv)



$x \mapsto -f(x)+1$

$\Uparrow$

$]-\infty,2]$



$x \mapsto f(x-1)$

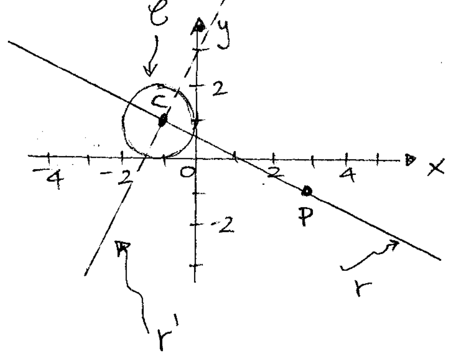
$\Uparrow$

$]-\infty,3]$

b5) i) $x^2+y^2+2x-2y+1=0$

$(x+1)^2-1+(y-1)^2-\cancel{1}+\cancel{1}=0$

$(x+1)^2+(y-1)^2=1 \qquad C=(-1,1) \quad r=1$



$y=-1-\frac{1}{2}(x-3)$

$\underline{\underline{y=-\frac{1}{2}x+\frac{1}{2}}}$.

ii) r' : $y=1+2(x+1)$

$\underline{\underline{y=2x+3}}$.

iii) $Q=\left(-\frac{3}{2},0\right)$

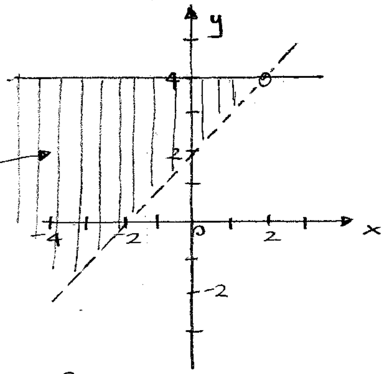
$d(Q,C)=\sqrt{\left(-\frac{3}{2}+1\right)^2+1}=\sqrt{\frac{1}{4}+1}=\underline{\underline{\frac{\sqrt{5}}{2}}}$.

FILA C

a1) $A = [1,+\infty[$ $\quad B = \{-1,2\}$ $\quad B \setminus A = \underline{\underline{\{-1\}}}$ $\quad A \cup B = \underline{\underline{\{-1\} \cup [1,+\infty[}}$.

a2) $\dfrac{x^2}{x+2} \geq 0$ $\quad \underline{\underline{S =]-2,+\infty[}}$.

a3) $E = \{(x,y) \in \mathbb{R}^2 : \ x+2 < y \leq 4\}$

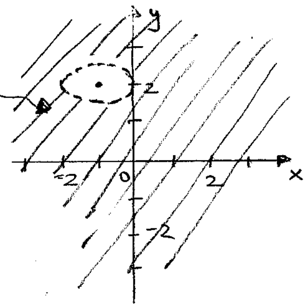


a4) $-x^2+x+6 \geq 0 \iff x^2-x-6 \leq 0$

$\underline{\underline{S = [-2,3]}}$.

$x^2-x-6=0 \iff x_{1/2} = \dfrac{1 \pm \sqrt{1+24}}{2} = \dfrac{1 \pm 5}{2} \begin{cases} -2 \\ 3 \end{cases}$

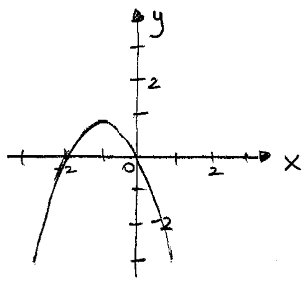
a5) $(x+1)^2 + 4(y-2)^2 > 1 \iff (x+1)^2 + \dfrac{(y-2)^2}{\frac{1}{4}} > 1$



a6) "La Provincia di Trento non organizza durante l'estate corsi d'Italiano per immigrati adulti e attività sportive per ragazzi."

a7) $y = -2 + 2(x-1) \implies \underline{y = 2x-4}$.

a8)



(i) V (poiché l'origine degli assi coordinate appartiene alla parabola);

(ii) V (concavità della parabola verso il basso);

(iii) V (ci sono due intersezioni della parabola con l'asse x);

(iv) F (l'asse di simmetria della parabola non coincide con l'asse y).

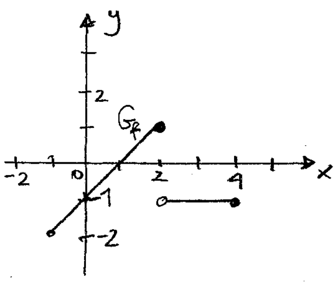
a9) $E = \{2\}$ $\quad F =]-1,1[$

(i) V (non ci sono elementi in comune ad E ed F);

(ii) V (poiché $2 \in E$ e $0 \in F$);

(iii) F ($2 \notin F$).

a10)



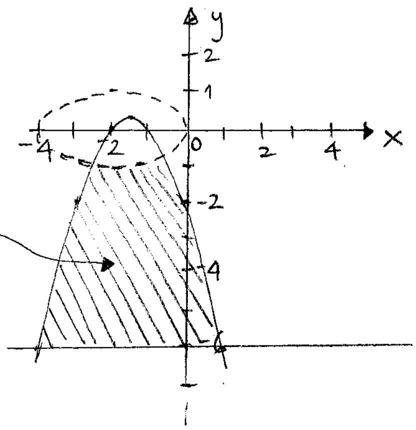
$f(0) = \underline{\underline{-1}}$ $\quad f([-1,4]) = \underline{\underline{[-2,1]}}$.

□

b1) i) "L'inflazione aumenta e il potere d'acquisto rimane invariato o aumenta".

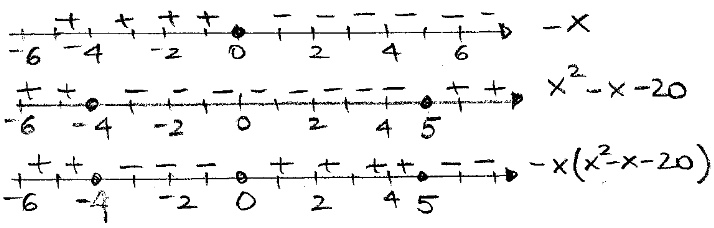
ii)
$$\begin{cases} x^2+4y^2+4x>0 \\ y+x^2+3x\le -2 \\ y\ge -6 \end{cases} \iff \begin{cases} (x+2)^2-4+4y^2>0 \\ y\le -(x^2+3x+2) \\ y\ge -6 \end{cases} \iff$$

$$\iff \begin{cases} \frac{(x+2)^2}{4}+y^2>1 \\ y\le -\left(x+\frac{3}{2}\right)^2+\frac{1}{4} \\ y\ge -6 \end{cases}$$

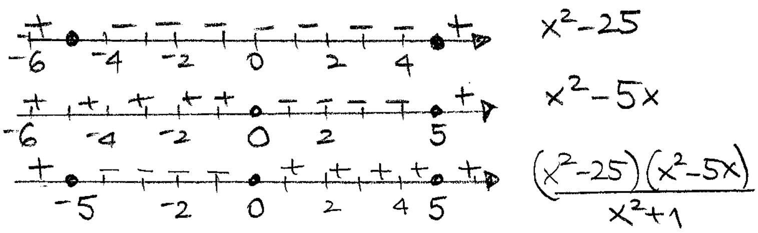


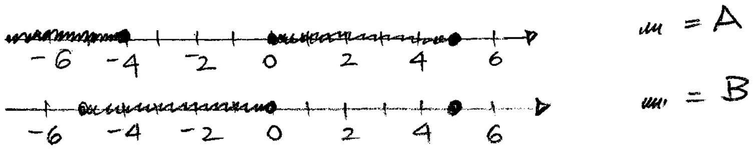
b2) i) $A=\{x\in\mathbb{R}: -x(x^2-x-20)\ge 0\}$

$=\{x\in\mathbb{R}: -x(x-5)(x+4)\ge 0\} = \underline{\underline{]-\infty,-4]\cup[0,5]}}$.



$B=\left\{x\in\mathbb{R}: \dfrac{(x^2-25)(x^2-5x)}{x^2+1}\le 0\right\} = \underline{\underline{[-5,0]\cup\{5\}}}$.



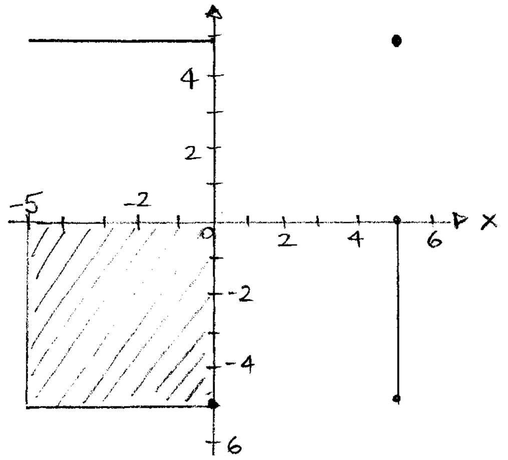


$A\cap B = \underline{\underline{[-5,-4]\cup\{0,5\}}}$.

ii) A non è limitato inferiormente, mentre B è limitato.

$\max A = \underline{5}$, $\min B = \underline{-5}$.

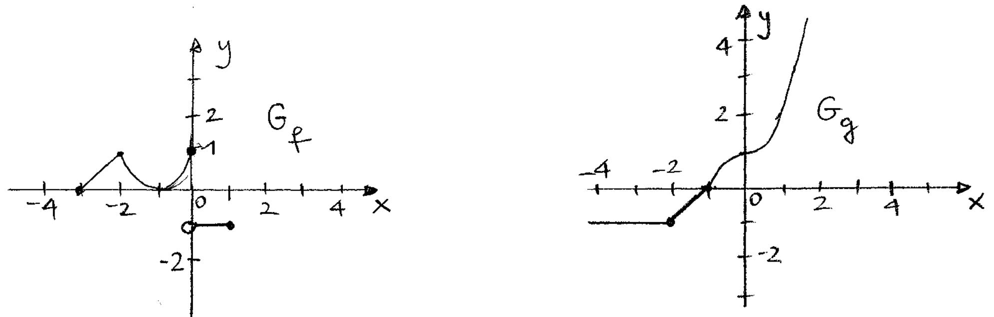
iii)



iv) $y = k$ con $k \in]-\infty, -5[\cup]0, 5[\cup]5, +\infty[$.

v) a) F b) V c) F d) V.

b3)



i) se $k \in]-\infty, -1[\cup]-1, 0[\cup]1, +\infty[$ $\nexists$ sono soluz. dell'eq. $f(x) = k$;

se $k = -1$ $\exists$ sono infinite soluzioni dell'eq. $f(x) = k$;

se $k = 0$ $\exists$ sono 2 soluzioni "

se $0 < k < 1$ $\exists$ sono 3 soluzioni "

se $k = 1$ $\exists$ sono 2 soluzioni "

ii) $f \circ g$ è ben definita su $\underline{]-\infty, 0]}$ poiché

$$g(]-\infty, 0]) = [-1, 1] \subset \mathrm{dom}\, f .$$

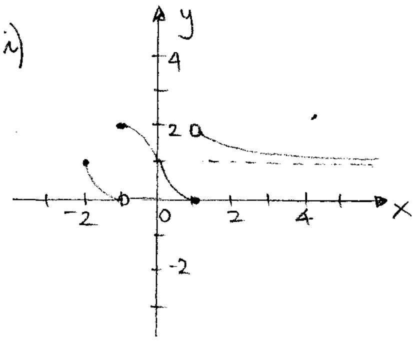
$g \circ f$ è ben definita su $[-3, 1]$ poiché

$$f([-3, 1]) \subseteq \mathbb{R} = \mathrm{dom}\, g .$$

iii) $(f-g)(-1) = f(-1) - g(-1) = \underline{0}$; $(fg)(-2) = f(-2)g(-2) = \underline{-1}$;

$(g \circ f)(-1) = g(f(-1)) = g(0) = \underline{1}$; $(f \circ g)(1)$ $\nexists$ poiché $g(1) = 2 \notin \mathrm{dom} f$.

b4) i)



$f([-2,+\infty[) = [0,2]$.

ii) $\min_{[-2,+\infty[} f = 0 \qquad x = 1$ pt. di minimo.

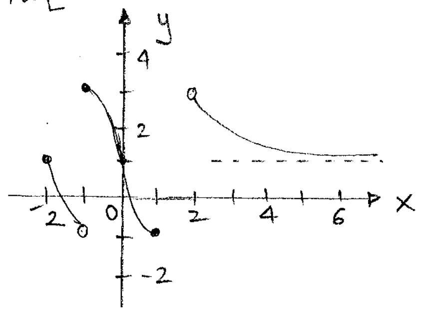
$\max_{[-2,+\infty[} f = 2 \qquad x = -1$ pt. di massimo.

iii) $x_1 = -2 \neq x_2 = 0 \quad$ e $\quad f(-2) = f(0)$

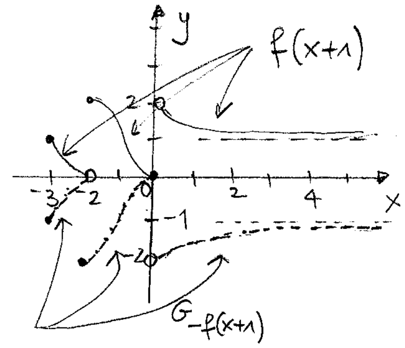
$I = [0,+\infty[$ e $f|_I$ è iniettiva.

iv)

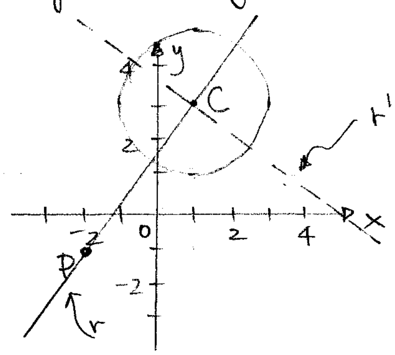
$\underset{[-2,+\infty[}{\overset{\cap}{x}} \longmapsto 2f(x) - 1$



$\underset{[-3,+\infty[}{\overset{\cap}{x}} \longmapsto -f(x+1)$



b5) i) $x^2 + y^2 - 2x - 6y + 6 = 0$



$(x-1)^2 - 1 + (y-3)^2 - 9 + 6 = 0$

$(x-1)^2 + (y-3)^2 = 4 \qquad C = (1,3);\ r = 2$

$y = 3 + \frac{4}{3}(x-1)$

$\underline{\underline{y = \frac{4}{3}x + \frac{5}{3}}}$.

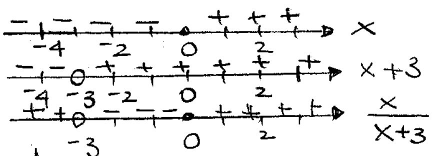
ii) r': $\quad y = 3 - \frac{3}{4}(x-1) \qquad \underline{\underline{y = -\frac{3}{4}x + \frac{15}{4}}}$

iii) $Q = (5,0) \qquad d(Q,C) = \sqrt{(5-1)^2 + 3^2} = \sqrt{16+9} = \underline{5}$

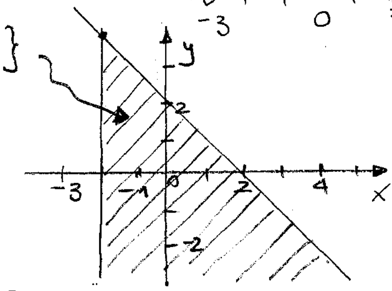
FILA (D)

a1) $A = \{-3, 2\}$ $B = [-2, +\infty[$ $A \cup B = \underline{\{-3\} \cup [-2, +\infty[}$ $B \setminus A = \underline{[-2, 2[\cup]2, +\infty[}$.

a2) $\dfrac{x}{x+3} < 0$ $S = \underline{]-3, 0[}$



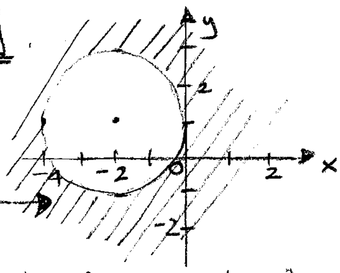
a3) $E = \{(x,y) \in \mathbb{R}^2 : \ -2 \le x \ , \ y \le -x+2\}$



a4) $-x^2+3x+4 \ge 0 \iff x^2-3x-4 \le 0$

$\iff (x-4)(x+1) \le 0$

$S = \underline{[-1, 4]}$.

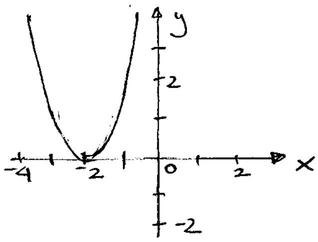


a5) $(x+2)^2 + (y-1)^2 \ge 4$

a6) "Lo studente Tizio Caio non ha frequentato il corso di Analisi Matematica o non ha studiato con impegno per la prova intermedia".

a7) $y = 1 - 2(x+2) \Rightarrow \underline{y = -2x - 3}$.

a8)



(i) (V) (concavità della parabola verso l'alto);

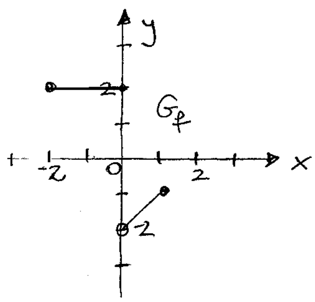
(ii) (V) (l'origine degli assi coordinati non appartiene alla parabola);

(iii) (F) (c'è un'intersezione della parabola con l'asse x);

(iv) (F) (l'asse di simmetria della parabola non coincide con l'asse y).

a9) $E = \{0\}$, $F = \{-2\}$ (i) (V) (ii) (F) (iii) (V).

a10)



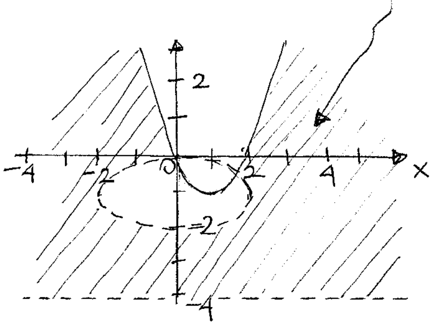
$f(1) = \underline{-1}$ $f([-2, 1]) = \underline{]-2, -1] \cup \{2\}}$

□

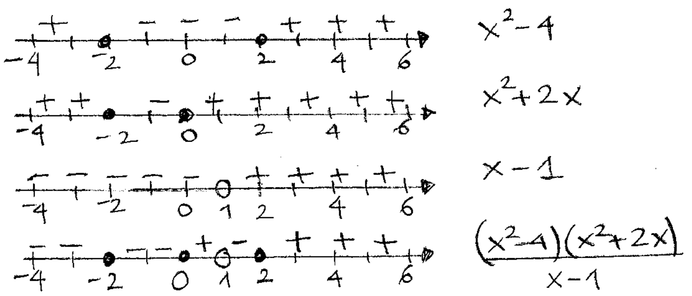
b1): "Lo studente Mario Bianchi supera l'esame di Analisi Matematica nella sessione d'esame invernale e il prossimo semestre non frequenta il corso di Psicometria".

ii) $\begin{cases} x^2+4y^2+8y>0 \\ -4<y\leq x^2-2x \end{cases} \iff \begin{cases} x^2+4(y^2+2y)>0 \\ -4<y\leq x^2-2x \end{cases}$

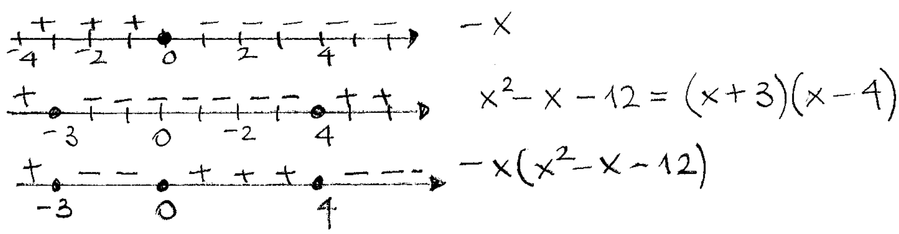
$\iff \begin{cases} x^2+4(y+1)^2>4 \\ -4<y\leq x^2-2x \end{cases} \iff \begin{cases} \frac{x^2}{4}+(y+1)^2>1 \\ -4<y\leq (x-1)^2-1 \end{cases}$



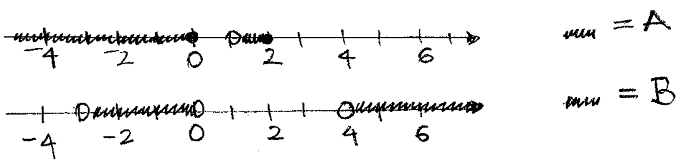
b2) i)



$A = \,]-\infty,0]\cup\,]1,2]$.



$B = \,]-3,0[\,\cup\,]4,+\infty[$,

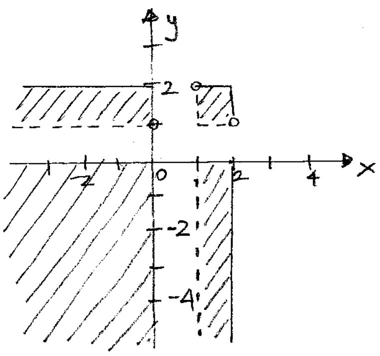


$A\cap B = \,]-3,0[$.

ii) A non è limitato inferiormente, mentre B non è limitato superiormente.

$\max A = \underline{\underline{2}}$, $\nexists$ $\underline{\underline{\min B}}$. ☐

iii)

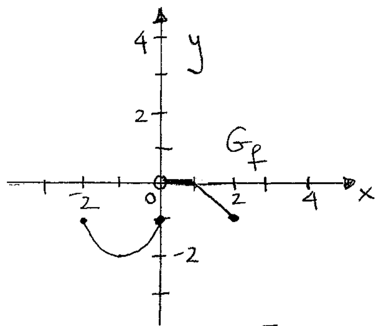


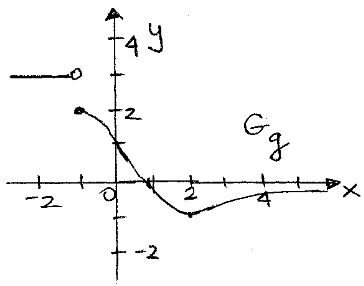
$A \times A$ ☐

iv) $x = k$, $0 < k \leq 1$ o $k > 2$. ☐

v) a) F b) F c) V d) V ■

b3)





i) se $k \in]-\infty, -2[\cup]0, +\infty[$ $\nexists$ sono soluzioni dell'eq. $f(x) = k$;

se $k = -2$ $\exists$ una soluzione dell'eq. $f(x) = k$;

se $-2 < k < -1$ $\exists$ sono 2 soluz. " ;

se $k = -1$ $\exists$ sono 3 soluzioni " ;

se $-1 < k < 0$ $\exists$ una soluzione " ;

se $k = 0$ $\exists$ sono infinite soluzioni dell'eq. $f(x) = k$. ☐

ii) $f \circ g$ è definita $\forall\, x \in \underline{\underline{[-1, +\infty[}}$ poiché $g([-1, +\infty[) = [-1, 2] \subset \operatorname{dom} f$, mentre $g(x) = 3$ $\forall\, x < -1$ e quindi per tali x $\nexists$ $f(g(x))$.

$g \circ f$ è definita $\forall\, x \in [-2, 2]$, poiché $f([-2, 2]) = [-2, 0]$ è contenuta in $\mathbb{R} = \operatorname{dom} g$. ☐

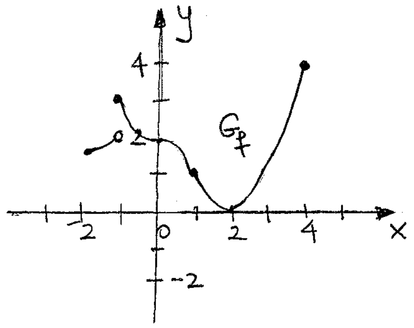
iii) $(f+g)(0) = f(0)+g(0) = -1+1 = \underline{\underline{0}}$;

$(fg)(-1) = f(-1)\,g(-1) = (-2)\cdot 2 = \underline{\underline{-4}}$;

$(g\circ f)(-1) = g(f(-1)) = g(-2) = \underline{\underline{3}}$,

$(f\circ g)(-2) \;\nexists$ poiché $g(-2)=3$ e $f(3)\;\nexists$.

b4) i)



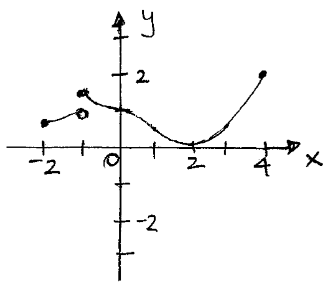
$f([-2,4]) = \underline{\underline{[0,4]}}$.

ii) $\min\limits_{[-2,4]} f = 0$ $\quad x=2$ pt. di minimo; $\quad \max\limits_{[-2,4]} f = 4$ $\quad x=4$ pt. di massimo.

iii) Basta prendere $x_1 = 1 \neq x_2 = 3$ e $f(1)=f(3)=1$!

$I = [-1,2]$ e $f|_I$ è iniettiva.

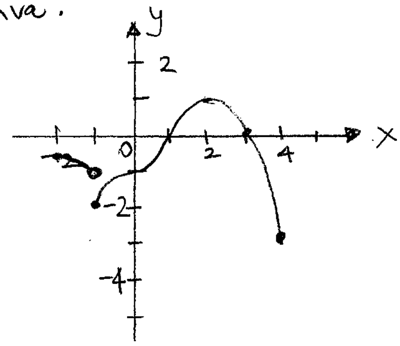
iv)



$x \longmapsto \frac{1}{2}f(x)$

$\Uparrow$

$[-2,4]$



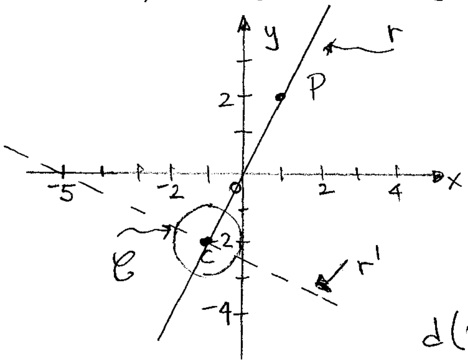
$x \longmapsto -f(x)+1$

$\Uparrow$

$[-2,4]$

b5) i) $x^2+y^2+2x+4y+4=0$ $\qquad (x+1)^2-1+(y+2)^2=0$

$(x+1)^2+(y+2)^2=1 \qquad C=(-1,-2)\quad r=1$



$\underline{y=2x}$.

ii) $y = -2-\frac{1}{2}(x+1) \;\Rightarrow\; \underline{\underline{y = -\frac{1}{2}x - \frac{5}{2}}}$.

iii) $Q = (-5,0)$

$d(Q,C) = \sqrt{(-5+1)^2+(-2-0)^2} = \sqrt{16+4} = \sqrt{20} = \underline{2\sqrt{5}}.$