Università di Trento - Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione - CdL in Filosofia
CORSO DI ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2010-2011 — Rovereto, 13 - 15 dicembre 2010

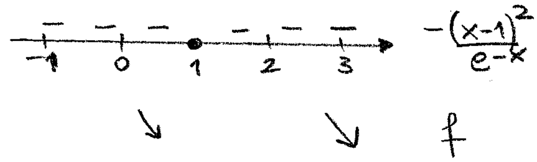
1) i) $f(x) = \frac{x^2+1}{e^x}$ $\quad \left(= (x^2+1)e^{-x}\right)$

- $\mathrm{dom} f = \mathbb{R} =]-\infty, +\infty[$
- segno di f: sempre positivo, poichè $x^2+1>0$, $e^x>0$ $\forall x \in \mathbb{R}$.
- $\lim_{x\to-\infty} f(x) = +\infty$, $\lim_{x\to+\infty} f(x) = 0$.

  $y=0$ è asintoto orizzontale per f, per $x\to+\infty$.
- f è continua su tutta $\mathbb{R}$ (rapporto tra funzioni continue, ed e^x non si annulla mai).
- $\mathrm{dom} f' = \mathbb{R}$

$$f'(x) = 2xe^{-x} + (x^2+1)e^{-x}\cdot(-1)$$
$$= (-x^2+2x-1)e^{-x} = -(x-1)^2 e^{-x}$$

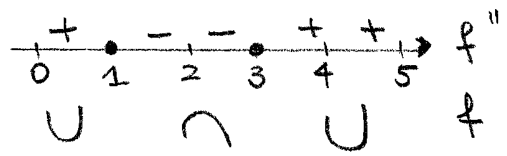
$f'(x)=0 \iff x=1$ pt. critico $\qquad f(1) = \frac{2}{e} < 1$



x è un pt. con tangente orizzontale, ma non è né un pt. di max. loc, né un punto di min. loc.

- $\mathrm{dom} f'' = \mathbb{R}$

$$f''(x) = -2(x-1)e^{-x} + (x-1)^2 e^{-x}$$
$$= (x-1)e^{-x}\left[-2+x-1\right]$$
$$= (x-1)(x-3)e^{-x}$$



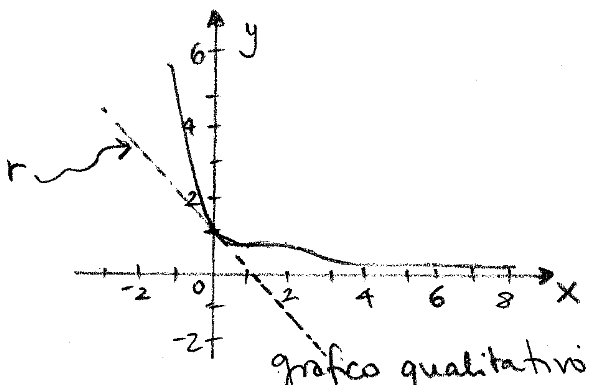


grafico qualitativo di f

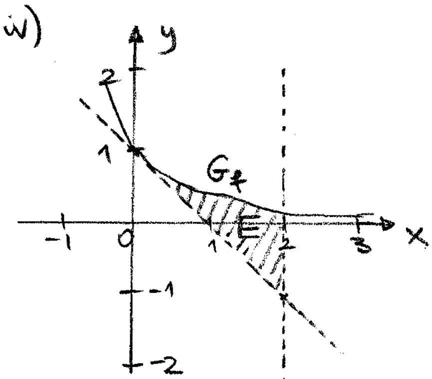
ii) $y = -x+1$ eq. retta tg. al grafico di f nel pt. $(0,1)$.

iii) $F(x) = -e^{-x}(x^2+2x+3)$: F è derivabile in $\mathbb{R}$, e

$$F'(x) = e^{-x}(x^2+2x+3) - e^{-x}(2x+2) = e^{-x}(x^2+1) = f(x) \quad \forall x \in \mathbb{R}.$$

Quindi F è una primitiva di f. □

iv)



$$\text{area}\, E = \int_0^2 \left[f(x) - (-x+1)\right] dx =$$

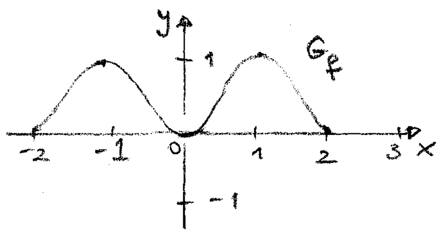
$$= \int_0^2 f(x)\,dx + \left[\frac{x^2}{2} - x\right]_0^2 =$$

$$= F(2) - F(0) + \cancel{\left[\frac{4}{2} - 2\right]} =$$

$$= \left[-e^{-2}(4+4+3)\right] - \left[-e^{0}(3)\right] =$$

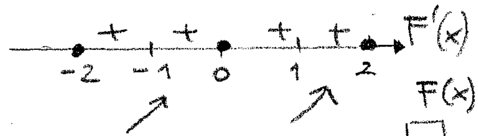
$$= \underline{\underline{3 - \frac{11}{e^2}}}.$$

2)



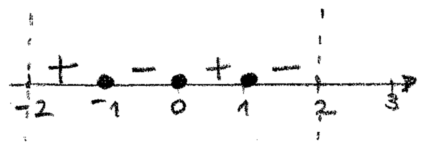
i) Osserviamo che $f(x) \geq 0 \ \forall x \in [-2,2]$ e $f(x) = 0 \Leftrightarrow x \in \{-2, 0, 2\}$; si ha che F è strettamente crescente su $[-2,2]$

$(F'(x) = f(x) \quad \forall x \in\,]-2,2[$



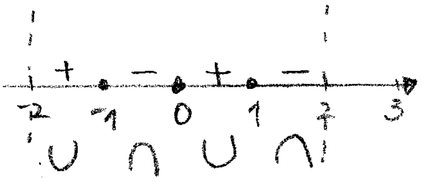
□

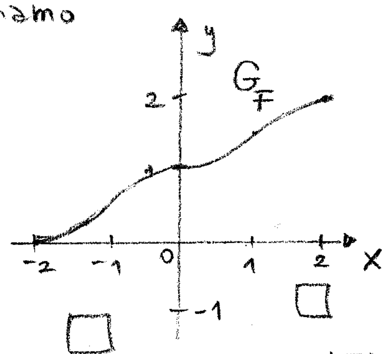
ii)



□

iii) Da $F'(x) = f(x) \ \forall x \in\,]-2,2[$ segue che $F''(x) = f'(x) \ \forall x \in\,]-2,2[$ poiché f è derivabile su $]-2,2[$. Dunque il segno di F'' coincide con il segno di $f'(x)$ e quindi abbiamo





□ □

iv) F non è nè pari, nè dispari.

v) $F(2) = \int_0^2 f(t)\,dt = 2\,\text{area}(E) = 2.$

3) i) $\int_{-1}^{0}(-3x+1)^4\,dx = \left[\frac{(-3x+1)^5}{5}\cdot\frac{1}{-3}\right]_{-1}^{0} = -\frac{1}{15}+\frac{1}{15}\cdot 4^5 = \frac{1023}{15}$

$= \frac{341}{5}$

$\int_0^1 \left[(e^x+2)^4 e^x + \frac{1}{x-2}\right]dx = \left[\frac{(e^x+2)^5}{5} + \log|x-2|\right]_0^1 =$

$= \frac{(e+2)^5}{5} - \frac{3^5}{5} - \log 2 .$

$\int_{-1}^0 \frac{x}{x^2+1}\,dx = \left[\frac{1}{2}\log(x^2+1)\right]_{-1}^0 = -\frac{1}{2}\log 2$ □

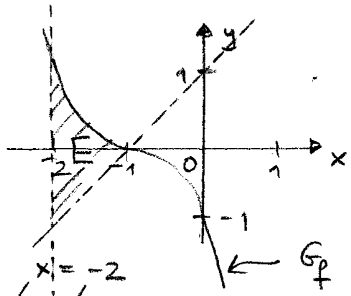
ii) $\int_1^2 \frac{x^3+2}{x}\,dx = \int_1^2 \left(x^2+\frac{2}{x}\right)dx = \left[\frac{x^3}{3}+2\log|x|\right]_1^2 = \frac{8}{3}+2\log 2-\frac{1}{3} =$

$= \frac{7}{3}+\log 4 .$

$\int_0^1 3x^2e^{x^3}\,dx = \left[e^{x^3}\right]_0^1 = e-1 .$

$\int_{-2}^2 |4-x^2|\,dx = \int_{-2}^2 (4-x^2)\,dx = \left[4x-\frac{x^3}{3}\right]_{-2}^2 = 2\left(8-\frac{8}{3}\right) = \frac{32}{3} .$ ■

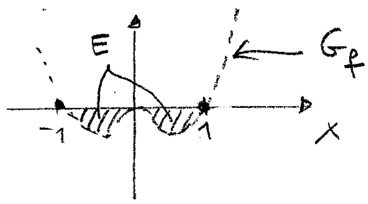
$4-x^2 \geq 0$
in $[-2,2]$



4) i) $f(x) = -(x+1)^3 \qquad y = x+1 \qquad x = -2$

$\text{area}(E) = \int_{-2}^{-1}\left[-(x+1)^3-(x+1)\right]dx =$

$= \left[-\frac{(x+1)^4}{4}-\frac{x^2}{2}-x\right]_{-2}^{-1} = \left(-\frac{1}{2}+1\right)+\frac{1}{4}+\cancel{2}-\cancel{2} = \frac{3}{4}$ □



ii) $f(x) = x^4-x^2 \qquad y=0$

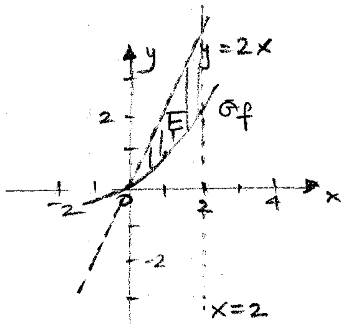
$= x^2(x^2-1)$

$\text{area}(E) = -\int_{-1}^{1} f(x)\,dx = -\int_{-1}^{1}(x^4-x^2)\,dx =$

$= -\left[\frac{x^5}{5}-\frac{x^3}{3}\right]_{-1}^{1} = -\left[\frac{1}{5}-\frac{1}{3}\right]+\left[-\frac{1}{5}+\frac{1}{3}\right] = \frac{2}{3}-\frac{2}{5} = \frac{4}{15} .$

□

iii) $f(x) = 2^x - 1 \qquad y = 2x \qquad x = 2$

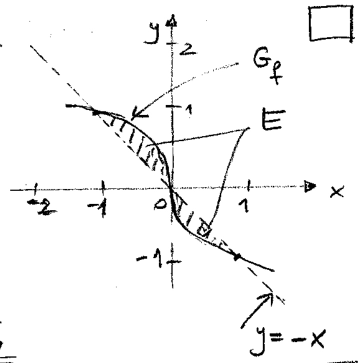


$$\text{area}(E) = \int_0^2 (2x - 2^x + 1)\,dx$$

$$= \left[x^2 - \frac{2^x}{\log 2} + x\right]_0^2 =$$

$$= 4 - \frac{4}{\log 2} + 2 + \frac{1}{\log 2} = 6 - \frac{3}{\log 2}.$$

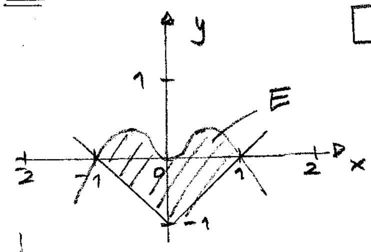
iv) $f(x) = -\sqrt[3]{x} \qquad y = -x$



$$\text{area}(E) = 2 \cdot \int_{-1}^0 \left(-\sqrt[3]{x} + x\right) dx =$$

$$= 2\left[-\frac{x^{4/3}}{4/3} + \frac{x^2}{2}\right]_{-1}^0 = 2\left(\frac{3}{4} - \frac{1}{2}\right) = \frac{1}{2}$$

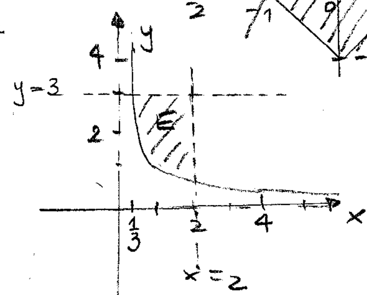
v) $f(x) = x^2 - x^4 \qquad g(x) = |x| - 1$



$$\text{area}(E) = \frac{4}{15} + \cancel{\frac{2 \cdot 1}{2}} = \frac{19}{15}$$

vedi ii)

vi) $f(x) = \frac{1}{x} \qquad x = 2 \qquad y = 3$

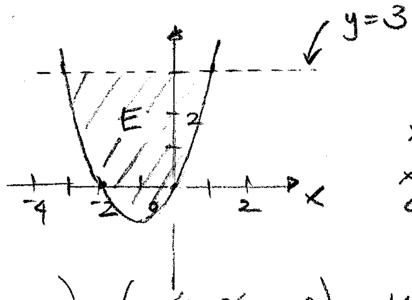


$$\text{area}(E) = \int_{1/3}^2 \left(3 - \frac{1}{x}\right) dx =$$

$$= \Big[3.x - \log|x|\Big]_{1/3}^2 = (6 - \log 2) - (1 + \log 3) =$$

$$= 5 - \log 6.$$

5) i) $f(x) = x^2 + 2x \qquad y = 3$

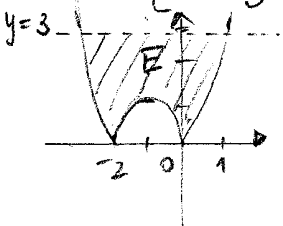


$x^2 + 2x = 3 \iff$
$x^2 + 2x - 3 = 0$
$\iff (x+3)(x-1) = 0$

$$\text{area}(E) = \int_{-3}^1 \left[3 - x^2 - 2x\right] dx =$$

$$= \left[3x - \frac{x^3}{3} - x^2\right]_{-3}^1 = \left(3 - \frac{1}{3} - 1\right) - \left(\cancel{-9} + \cancel{9} - 9\right) = 11 - \frac{1}{3} = \frac{32}{3}$$

ii)



$$\text{area}(E) = \frac{32}{3} + 2\int_{-2}^0 (x^2 + 2x)\,dx = \frac{32}{3} + 2\left[\frac{x^3}{3} + x^2\right]_{-2}^0 =$$

$$= \frac{32}{3} - 2\left(-\frac{8}{3} + 4\right) = 8.$$