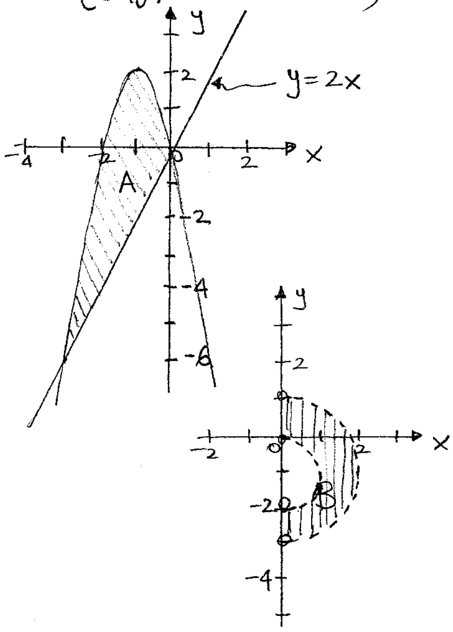
Università degli Studi di Trento – Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione – CdL in Filosofia
Corso di ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
Verifica settimanale – a.a. 2010/11 – Rovereto, 11–15 ottobre 2010

1) i) $A = \{(x,y) \in \mathbb{R}^2 : 2x \le y \le -2x^2 - 4x\}$

$B = \{(x,y) \in \mathbb{R}^2 : x \ge 0, \quad 1 < x^2 + (y+1)^2 < 4\}$



$y = -2x^2 - 4x$
$= -2(x^2 + 2x)$
$= -2(x+1)^2 + 2$

$-2x^2 - 4x = 0$
$-2x(x+2) = 0$
$\Leftrightarrow x = 0, \ x = -2$

Inoltre la retta $y = 2x$ interseca la parabola di eq. $y = -2x^2 - 4x$ nei pt. (x,y) t.c.

$\begin{cases} y = 2x \\ 2x = -2x^2 - 4x \end{cases} \Leftrightarrow \begin{array}{l} y = 2x \\ 2x^2 + 6x = 0 \end{array}$

$\Rightarrow S_1 = (0,0) \qquad S_2 = (-3,-6)$

$\square$

ii) rette orizzontali che non intersecano mai A : $y = k$ $k < -6$ o $k > 2$

B : $y = k$ $k \le -3$ o $k \ge 1$;

rette verticali che non intersecano mai A : $x = k$ $k < -3$ o $k > 0$

B : $x = k$ $k < 0$ o $k \ge 2$.

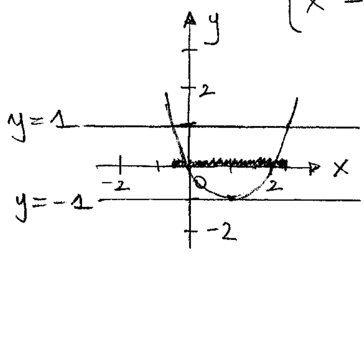
$\blacksquare$

2) i) $-1 \le x^2 - 2x \le 1 \Leftrightarrow \begin{cases} x^2 - 2x + 1 \ge 0 \\ x^2 - 2x - 1 \le 0 \end{cases}$

Ora $x^2 - 2x + 1 = 0 \Leftrightarrow (x-1)^2 = 0 \Leftrightarrow x = 1$

$x^2 - 2x - 1 = 0 \Leftrightarrow x_{1/2} = \dfrac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

Allora $\begin{cases} x^2 - 2x + 1 \ge 0 \\ x^2 - 2x - 1 \le 0 \end{cases} \Leftrightarrow \begin{cases} x \in \mathbb{R} \\ x \in [1-\sqrt{2}, 1+\sqrt{2}] \end{cases} \Rightarrow S = [1-\sqrt{2}, 1+\sqrt{2}]$



$y = x^2 - 2x$
$= (x-1)^2 - 1$

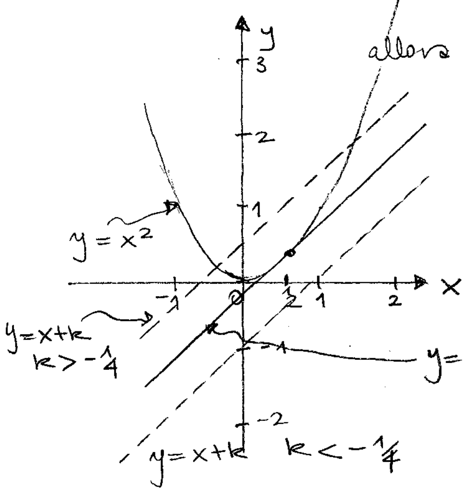
sono le coordinate x dei pt. (x,y) sulla parabola $y = x^2 - 2x$ che si trovano "sopra" la retta $y = -1$ e "sotto" la retta $y = 1$.

$x^2 = x+k \iff x^2-x-k=0 \iff x_{1/2} = \dfrac{1 \pm \sqrt{1+4k}}{2}$

allora $\nexists$ soluzione dell'eq. se $1+k\cdot 4<0$, ossia $k<-\frac{1}{4}$

$\exists$ una soluzione dell'eq. se $1+4k=0$, ossia $k=-\frac{1}{4}$

Sono due soluzioni dell'eq. se $1+4k>0$, ossia $k>-\frac{1}{4}$.



La soluzione corrisponde alla coordinate x del pt. di intersezione (x,y) (o dei pt. di intersezione), se esiste (se esistono) della parabola di eq. $y=x^2$ con la retta di eq. $y=x+k$, al variare di $k\in\mathbb{R}$.

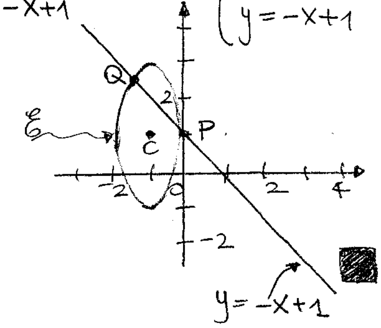
Si ha allora per $k=-\frac{1}{4}$, $x=\frac{1}{2}$; per $k<-\frac{1}{4}$ $\nexists$ soluzione;

per $k>-\frac{1}{4}$ $\quad x_{1/2} = \dfrac{1\pm\sqrt{1+4k}}{2}$ $\square$

ii) $\left.\begin{array}{ll} \mathcal{E}: & (x+1)^2 + \dfrac{(y-1)^2}{4} = 1 \\ r: & y=-x+1 \end{array}\right\} \Rightarrow \begin{cases} (x+1)^2 + \dfrac{(-x+\cancel{1}-\cancel{1})^2}{4} = 1 \\ y=-x+1 \end{cases}$

$\iff \begin{cases} x^2+2x+\cancel{1}+\dfrac{x^2}{4}=\cancel{1} \\ y=-x+1 \end{cases} \iff \begin{cases} 5x^2+8x=0 \\ y=-x+1 \end{cases} \iff \begin{cases} x(5x+8)=0 \\ y=-x+1 \end{cases}$

$\Rightarrow \underline{\underline{P=(0,1)}} \qquad \underline{\underline{Q=\left(-\frac{8}{5}, +\frac{13}{5}\right)}}$.

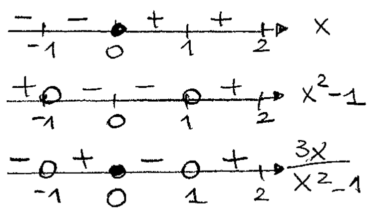


$d(P,Q) = \sqrt{\dfrac{64}{25}+\dfrac{64}{25}} = \underline{\underline{\frac{8}{5}\sqrt{2}}}$. ■

3) i) $\dfrac{x(x^2-4)}{x^2-1} > x \iff \dfrac{\cancel{x^3}-4x-\cancel{x^3}+x}{x^2-1}>0 \iff \dfrac{-3x}{x^2-1}>0$

$\iff \dfrac{3x}{x^2-1}<0$

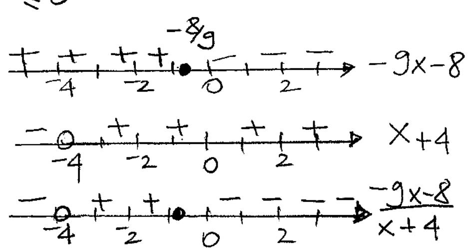
$\underline{\underline{S =]-\infty,-1[\cup]0,1[}}$.



$\dfrac{x^2-3x}{x+4} \le x+2 \iff \dfrac{\cancel{x^2}-3x-\cancel{x^2}-6x-8}{x+4} \le 0$

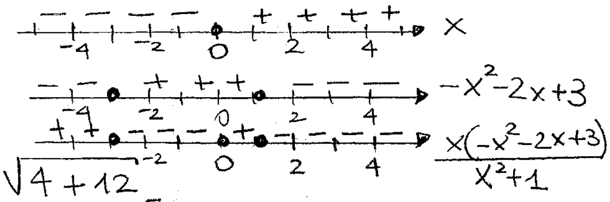
$\iff \dfrac{-9x-8}{x+4} \le 0$

$\underline{\underline{S =]-\infty,-4[\cup \left[-\frac{8}{9},+\infty\right[}}$.



$\square$

ii) $\frac{-x^3+3x-2x^2}{x^2+1}<0$



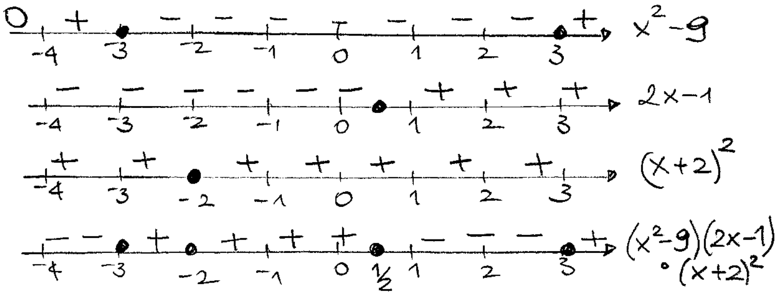
$\Leftrightarrow \quad \frac{x(-x^2-2x+3)}{x^2+1}<0$

$-x^2-2x+3=0 \Leftrightarrow x_{1/2}=\frac{2\pm\sqrt{4+12}}{-2}=$

$=\frac{2\pm 4}{-2}= \begin{cases} 1 \\ -3 \end{cases}$

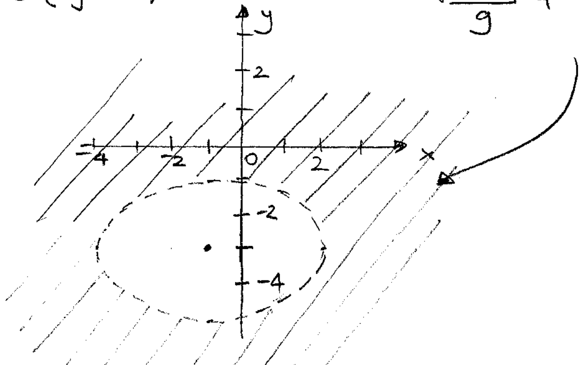
$S=]-3,0[\cup]1,+\infty[.$

$(x^2-9)(2x-1)(x+2)^2<0$



$S=]-\infty,-3[\cup]\frac{1}{2},3[.$

4) i) $4(x+1)^2+9(y+3)^2>36 \Leftrightarrow \frac{(x+1)^2}{9}+\frac{(y+3)^2}{4}>1$



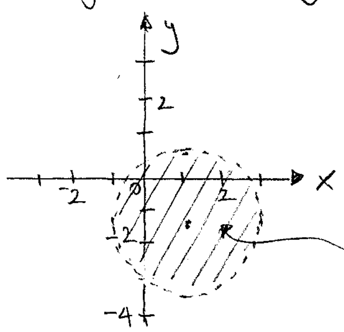
ii) $4x^2+4y^2-8x+12y<3 \Leftrightarrow 4x^2-8x+4y^2+12y<3$

$4(x^2-2x)+4(y^2+3y)<3$

$4(x-1)^2-4+4(y+\frac{3}{2})^2-9<3$

$4(x-1)^2+4(y+\frac{3}{2})^2<16$

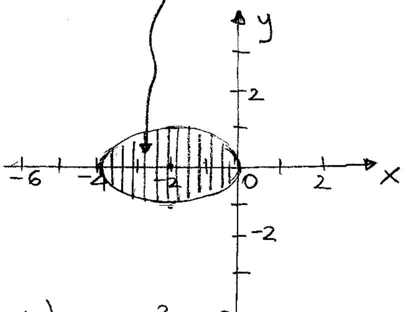
$(x-1)^2+(y+\frac{3}{2})^2<4$



iii) $x^2+4y^2+4x \le 0 \iff x^2+4x+4y^2 \le 0$

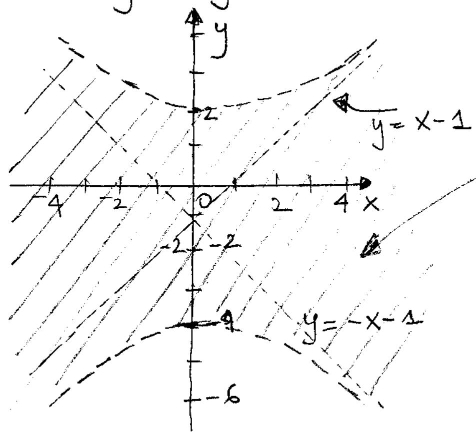
$\iff (x+2)^2+4y^2 \le 4$

$\iff \dfrac{(x+2)^2}{4} + y^2 \le 1$

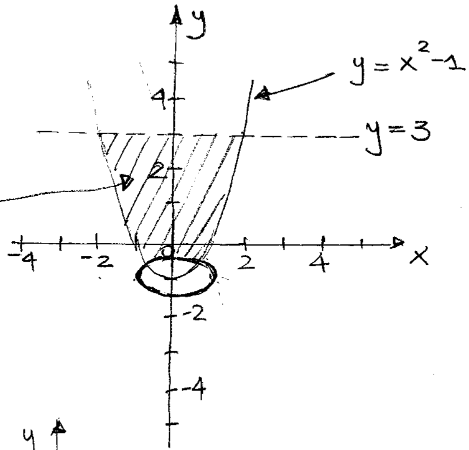


iv) $-x^2+y^2+2y-8<0 \iff -x^2+(y+1)^2-9<0$

$-\dfrac{x^2}{9} + \dfrac{(y+1)^2}{9} < 1$



5) ⓐ $\begin{cases} y \ge x^2-1 \\ y < 3 \\ x^2 + \dfrac{(y+1)^2}{1/4} \ge 1 \end{cases}$



ⓑ $\begin{cases} 1 < x^2+y^2 \le 4 \\ xy \le 0 \end{cases}$