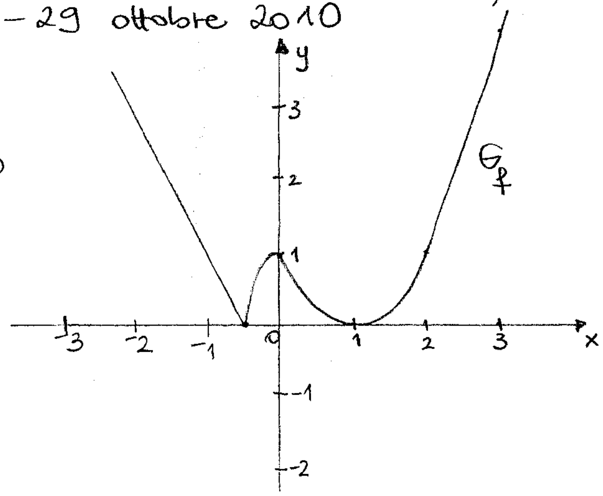
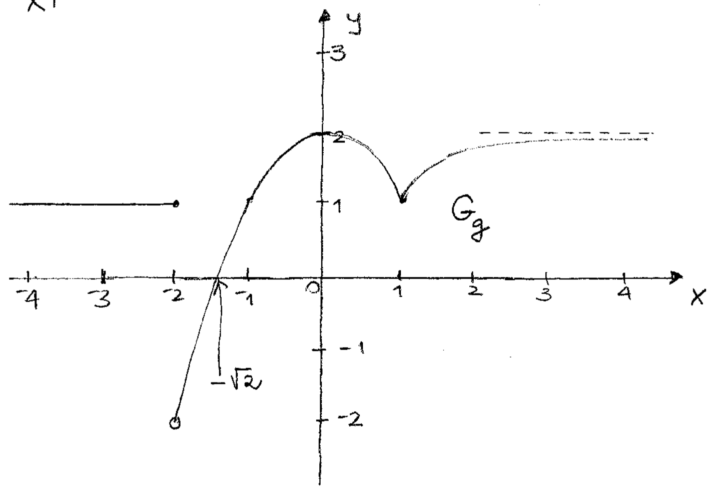
Università degli Studi di Trento — Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione — CdL in Filosofia
Corso di ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2010/2011 — Rovereto, 25 – 29 ottobre 2010

1) i) $f(x) = \begin{cases} -2x-1 & \text{se } x < -\frac{1}{2} \\ 8x^3+1 & \text{se } -\frac{1}{2} \le x < 0 \\ (x-1)^2 & \text{se } x \ge 0 \end{cases}$

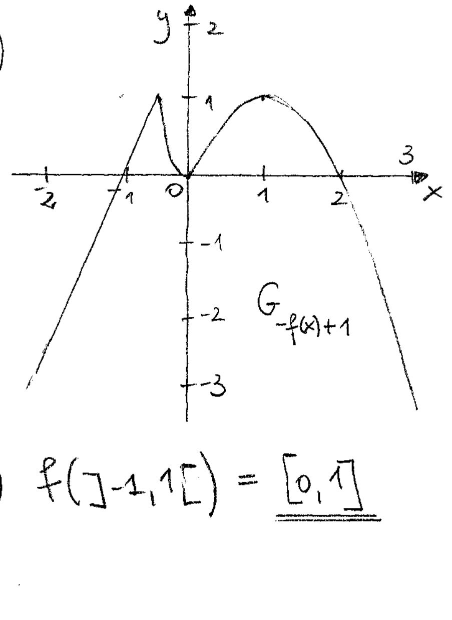


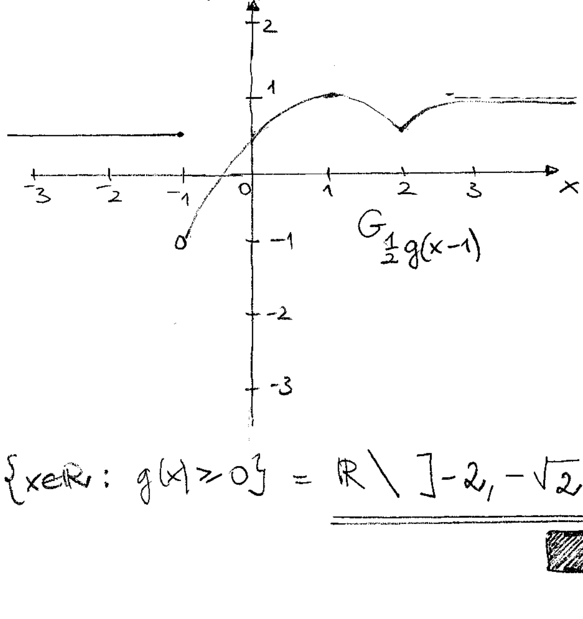
$g(x) = \begin{cases} 1 & \text{se } x \le -2 \\ -x^2+2 & \text{se } -2 < x < 1 \\ 2 - \frac{1}{x^4} & \text{se } x \ge 1 \end{cases}$



ii) Se $k<0$ $\nexists$ sono soluzioni dell'eq. $f(x)=k$;
se $k=0$ $\exists$ sono 2 soluzioni " ;
se $0<k<1$ $\exists$ sono 4 soluzioni " ;
se $k=1$ $\exists$ sono 3 soluzioni "
se $k>1$ $\exists$ sono 2 soluzioni dell'eq. $f(x)=k$.

iii)





iv) $f(]-1,1[) = \underline{\underline{[0,1]}}$ $\qquad$ $\{x \in \mathbb{R} : g(x) \ge 0\} = \underline{\underline{\mathbb{R} \setminus]-2, -\sqrt{2}[}}$.

2) i) $f(x) = \sqrt[3]{x-1} - 1$

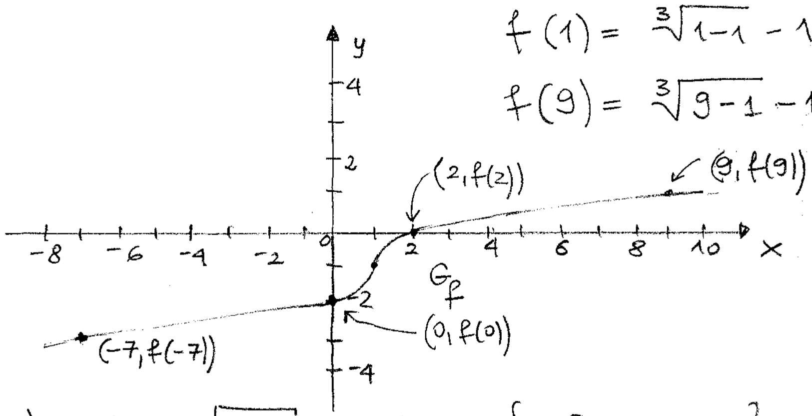
$\underline{\underline{\text{dom} f = \mathbb{R}}}$

$f(-7) = \sqrt[3]{-8} - 1 = -2 - 1 = -3$

$f(0) = \sqrt[3]{-1} - 1 = -1 - 1 = -2$

$f(1) = \sqrt[3]{1-1} - 1 = -1 \qquad f(2) = \sqrt[3]{1} - 1 = 0$

$f(9) = \sqrt[3]{9-1} - 1 = 2 - 1 = 1$



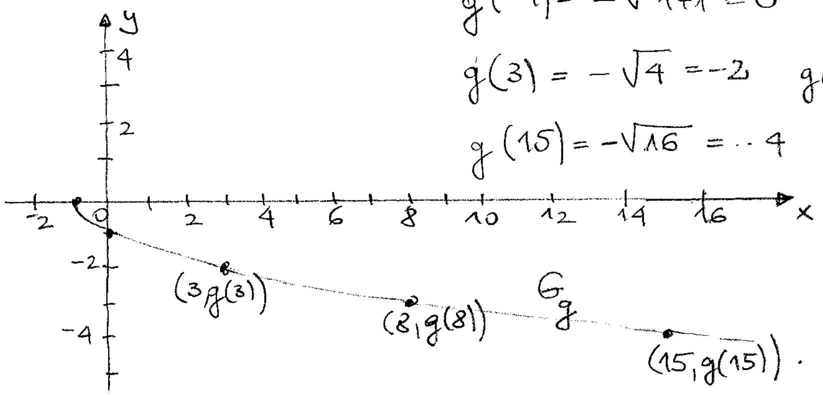
□

ii) $g(x) = -\sqrt{x+1}$ $\qquad$ $\text{dom}\, g = \{x \in \mathbb{R} : x+1 \geq 0\} = \underline{\underline{[-1, +\infty[}}$.

$g(-1) = -\sqrt{-1+1} = 0 \qquad g(0) = -1$

$g(3) = -\sqrt{4} = -2 \qquad g(8) = -\sqrt{9} = -3$

$g(15) = -\sqrt{16} = -4$



□

iii) $\min\limits_{[0,1]} f = \underline{\underline{-2}}$ $\qquad$ $\underline{\underline{x = 0 \text{ pt. di min.}}}$

$\max\limits_{[0,1]} f = \underline{\underline{-1}}$ $\qquad$ $\underline{\underline{x = 1 \text{ pt. di max.}}}$

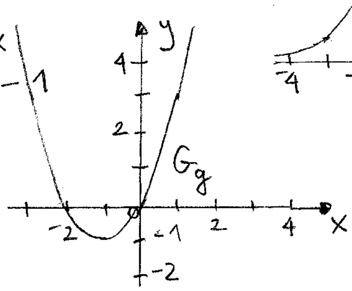
$\min\limits_{[0,1]} g = g(1) = \underline{\underline{-\sqrt{2}}}$ $\qquad$ $\underline{\underline{x = 1 \text{ pt. di min.}}}$

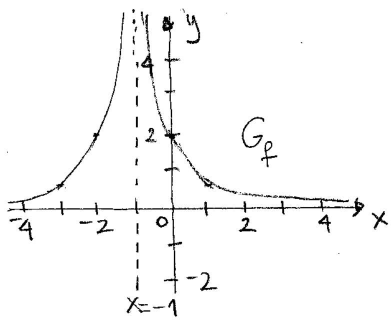
$\max\limits_{[0,1]} g = g(0) = \underline{\underline{-1}}$ $\qquad$ $\underline{\underline{x = 0 \text{ pt. di max.}}}$

■

3) i) $f : \mathbb{R} \setminus \{-1\} \to \mathbb{R}$, $\qquad f(x) = \dfrac{2}{(x+1)^2}$

$g : \mathbb{R} \to \mathbb{R} \qquad g(x) = x^2 + 2x$
$= (x+1)^2 - 1$





$f(\mathbb{R}\setminus\{-1\}) = \underline{\underline{]0,+\infty[}}$ $\quad g(\mathbb{R}) = \underline{\underline{[-1,+\infty[}}$ $\square$

ii) $(f+g)(1) = f(1)+g(1) = \frac{2}{(1+1)^2} + 1+2 = \frac{1}{2} + 3 = \underline{\underline{\frac{7}{2}}}$;

$(fg)(0) = f(0)\cdot g(0) = \frac{2}{(0+1)^2}\cdot(0+0) = 2\cdot 0 = \underline{\underline{0}}$;

$\left(\frac{g}{f}\right)(-2) = \frac{g(-2)}{f(-2)} = \frac{0}{2} = \underline{\underline{0}}$;

$\left(\frac{f}{g}\right)(-2)$ $\nexists$ poichè $g(-2)=0$. $\square$

iii) $(g\circ f)$: poichè $\operatorname{dom} f = \mathbb{R}\setminus\{-1\}$ e

$f(\mathbb{R}\setminus\{-1\}) =]0,+\infty[\subset \mathbb{R} = \operatorname{dom} g$

$g\circ f : \underline{\underline{\mathbb{R}\setminus\{-1\}}} \to \mathbb{R}$ è ben definita.

$(f\circ g)$: poichè $\operatorname{dom} g = \mathbb{R}$ e $g(\mathbb{R}) = [-1,+\infty[$

con $g(-1) = -1$ e f non è definita in -1,

allora dobbiamo restringere il dominio di g; si ha che

$f\circ g : \underline{\underline{\mathbb{R}\setminus\{-1\}}} \to \mathbb{R}$ è ben definita. $\square$

$$(g\circ f)(x) \underset{x\in\mathbb{R}\setminus\{-1\}}{=} g(f(x)) = g\left(\frac{2}{(x+1)^2}\right) = \underline{\underline{\frac{4}{(x+1)^4} + \frac{4}{(x+1)^2}}}.$$

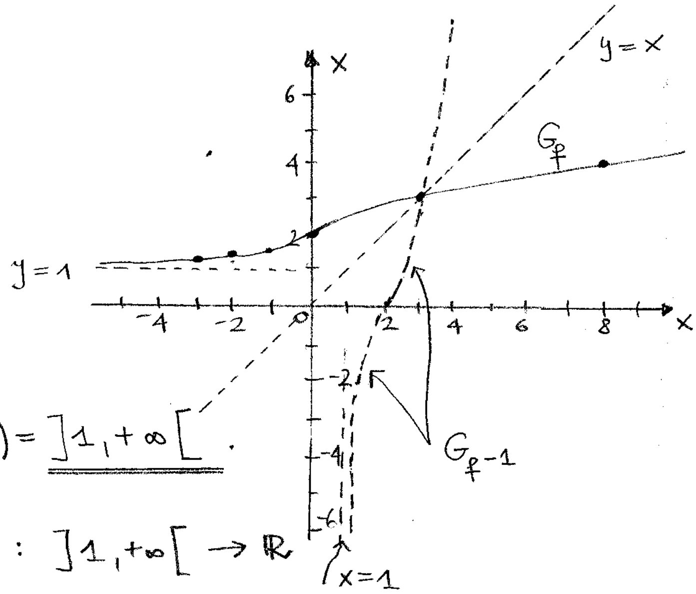
$$(f\circ g)(x) \underset{\substack{x\in\mathbb{R}\setminus\{-1\} \\ \text{così } g(x)\neq -1}}{=} f(g(x)) = f(x^2+2x) = \frac{2}{(x^2+2x+1)^2} = \frac{2}{((x+1)^2)^2} = \underline{\underline{\frac{2}{(x+1)^4}}}.$$

$\blacksquare$

4) $f:\mathbb{R}\to\mathbb{R}$ $\quad f(x) = \begin{cases} -\frac{1}{x-1}+1 & \text{se } x<0 \\ \sqrt{x+1}+1 & \text{se } x\geq 0. \end{cases}$

i) $f(-3) = \frac{1}{4}+1 = \frac{5}{4}$ $\quad f(-2) = \frac{4}{3}$ $\quad f(-1) = -\frac{1}{-2}+1 = \frac{3}{2}$

$f(0) = 2$ $\quad f(3) = \sqrt{3+1}+1 = 3$ $\quad f(8) = \sqrt{8+1}+1 = 4$.

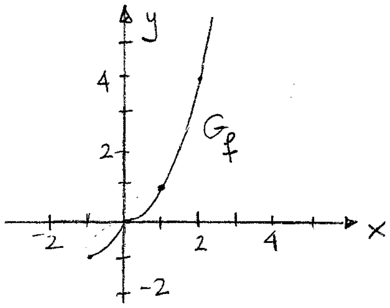


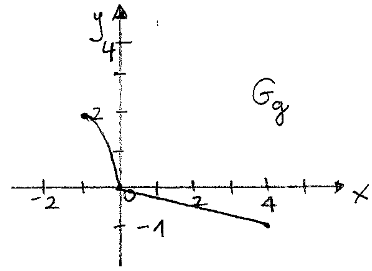
ii) $f(\mathbb{R}) = \underline{\underline{]1, +\infty[}}$.

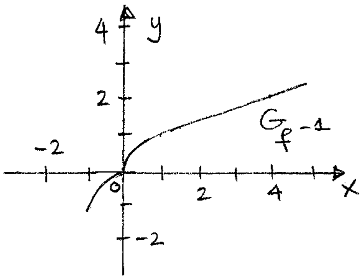
iii) $f^{-1} : \,]1, +\infty[\, \to \mathbb{R}$

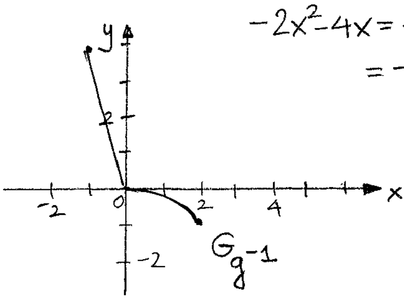
5) $f : [-1, +\infty[\, \to [-1, +\infty[$ $\qquad$ $g : [-1, 4] \to [-1, 2]$

$f(x) = \begin{cases} \sqrt[3]{x} & \text{se } -1 \le x \le 0 \\ x^2 & \text{se } x > 0 \end{cases}$ $\qquad$ $g(x) = \begin{cases} -2x^2 - 4x & \text{se } -1 \le x < 0 \\ -\frac{1}{4}x & \text{se } 0 \le x \le 4 \end{cases}$









$$-2x^2 - 4x = -2(x^2 + 2x) = -2(x+1)^2 + 2$$

ii) $\min\limits_{[-1,4]} g = -1 \qquad x = 4$ pt. di min.

$\max\limits_{[-1,4]} g = 2 \qquad x = -1$ pt. di max.

iii) $\min\limits_{[-1,2]} g^{-1} = -1 \qquad x = 2$ pt. di min.

$\max\limits_{[-1,2]} g^{-1} = 4 \qquad x = -1$ pt. di max.