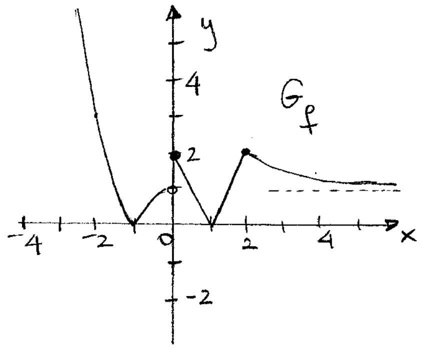
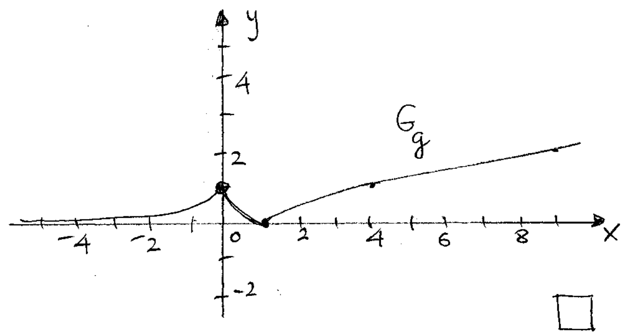
Università degli Studi di Trento - Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione - CdL in Filosofia
Corso di ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2010/2011 - Rovereto, 8 - 12 novembre 2010

1) i)





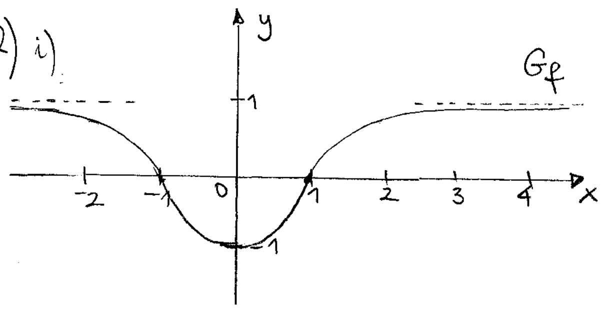
ii) $\max\limits_{[-2,2[} f = f(-2) = 3$ $x=-2$ pt. di massimo;

$\min\limits_{[-2,2[} f = 0$ $x=-1$, $x=1$ pt. di minimo.

iii) g: su $]-\infty, 0]$ g è crescente;
su $[0,1]$ g è decrescente;
su $[1,+\infty[$ g è crescente.

iv) $(f\circ g)(4) = f(g(4)) = f(1) = 0$
$(g\circ f)(-3) = g(f(-3)) = g(8) = |\sqrt{8}-1| = \sqrt{8}-1$.

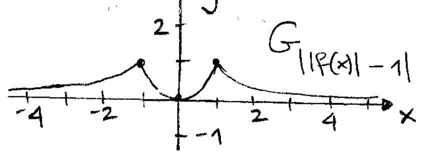
2) i)

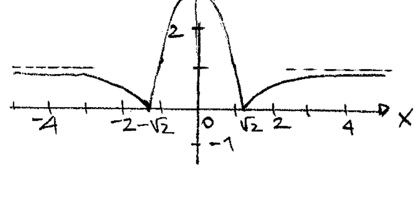


ii) f: su $]-\infty,0]$ f è decrescente;
su $[0,+\infty[$ f è crescente.

iii) Si ha $x\in[-1,1] \iff -x\in[-1,1]$ e $f(-x) = (-x)^4-1 = x^4-1 = f(x)$
$x\in]-\infty,-1[\iff -x\in]1,+\infty[$ e $f(-x) = -\frac{1}{(-x)^2}+1 = -\frac{1}{x^2}+1 = f(x)$

iv)





3) $5|x+2| > 15 \iff |x+2| > 3$

$\iff x+2 < -3 \quad o \quad x+2 > 3$

$\iff x \in \underline{\underline{]-\infty, -5[\cup]1, +\infty[}}$. □

$-|-2x| + x^2 = 3 \iff -|2x| + x^2 = 3 \iff \begin{cases} x \geq 0 \\ x^2 - 2x - 3 = 0 \end{cases} o \begin{cases} x < 0 \\ x^2 + 2x - 3 = 0 \end{cases}$

$\iff \begin{cases} x \geq 0 \\ x_{1/2} = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} \begin{smallmatrix} -1 \\ 3 \end{smallmatrix} \end{cases} o \begin{cases} x < 0 \\ x_{1/2} = \frac{-2 \pm \sqrt{4+12}}{2} \end{cases} = \begin{smallmatrix} -3 \\ 1 \end{smallmatrix}$

$\Rightarrow x \in \underline{\underline{\{-3, 3\}}}$. □

$|x^2 - 5| \leq 4 \iff -4 \leq x^2 - 5 \leq 4 \iff 1 \leq x^2 \leq 9$

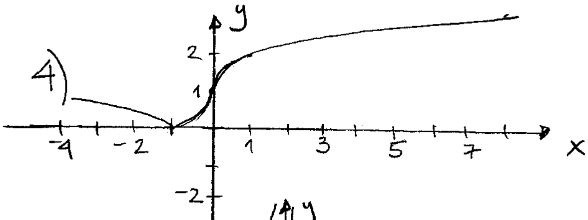
$\Rightarrow x \in \underline{\underline{[-3, -1] \cup [1, 3]}}$. □

$|x^2 + 3x| \leq 0 \iff x^2 + 3x = 0 \Rightarrow x \in \underline{\underline{\{-3, 0\}}}$. □

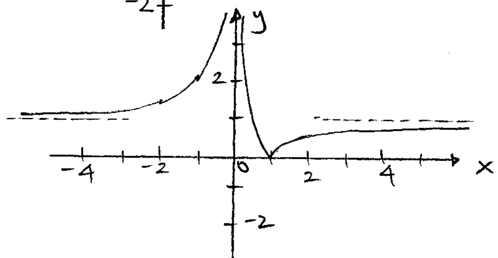
$||x+1| - 1| > 0 \iff |x+1| - 1 \neq 0 \iff |x+1| \neq 1$

$\iff x+1 \neq -1 \quad o \quad x+1 \neq 1 \Rightarrow \underline{\underline{x \in \mathbb{R} \setminus \{-2, 0\}}}$ ■

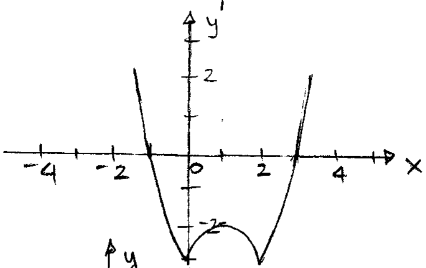
4)



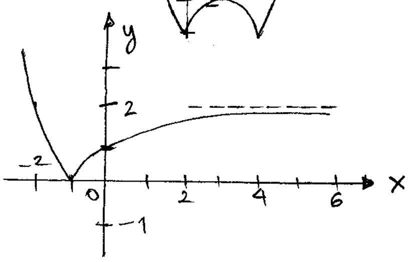
$G_{|1+\sqrt[3]{x}|}$ $\quad \mathrm{dom}\left(|1+\sqrt[3]{x}|\right) = \mathbb{R}$;



$G_{|\frac{1}{x}-1|}$ $\quad \mathrm{dom}\left(\left|\frac{1}{x}-1\right|\right) = \mathbb{R} \setminus \{0\}$;



$G_{|x^2-2|x|-3}$ $\quad \mathrm{dom}\left(|x^2 - 2|x| - 3\right) = \mathbb{R}$.



$G_{|(\frac{1}{2})^x - 2|}$ $\quad \mathrm{dom}\left(\left|\left(\frac{1}{2}\right)^x - 2\right|\right) = \mathbb{R}$.

■

5) i) $3x^2+2x|x|=1 \iff \begin{cases} x\geq 0 \\ 3x^2+2x^2=1 \end{cases}$ o $\begin{cases} x<0 \\ 3x^2-2x^2=1 \end{cases}$

$\iff \begin{cases} x\geq 0 \\ x^2=1/5 \end{cases}$ o $\begin{cases} x<0 \\ x^2=1 \end{cases}$

$\Rightarrow \quad x\in \underline{\underline{\left\{-1, \frac{1}{\sqrt{5}}\right\}}}$;

$\frac{x+3}{|x|-1}>2 \iff \frac{x+3-2|x|+2}{|x|-1}>0 \iff \frac{x-2|x|+5}{|x|-1}>0$

$\iff \begin{cases} x\geq 0 \\ \frac{-x+5}{x-1}>0 \end{cases}$ o $\begin{cases} x<0 \\ \frac{3x+5}{-x-1}>0 \end{cases}$

$\iff \begin{cases} x\geq 0 \\ x\in]1,5[\end{cases}$ o $\begin{cases} x<0 \\ x\in]-5/3,-1[\end{cases} \Rightarrow \underline{\underline{S=]-\frac{5}{3},-1[\cup]1,5[}}$;

$|-x|\left(x^2-2|x|\right)>0 \iff |x|\left(x^2-2|x|\right)>0$

$\iff \begin{cases} x>0 \\ x^2-2x>0 \end{cases}$ o $\begin{cases} x<0 \\ x^2+2x>0 \end{cases} \Rightarrow \underline{\underline{S=]-\infty,-2[\cup]2,+\infty[}}$.

$\square$

ii) $2^{|x+1|}<2 \iff 2^{|x+1|}<2^1 \underset{2^x\nearrow}{\iff} |x+1|<1 \iff -1<x+1<1$

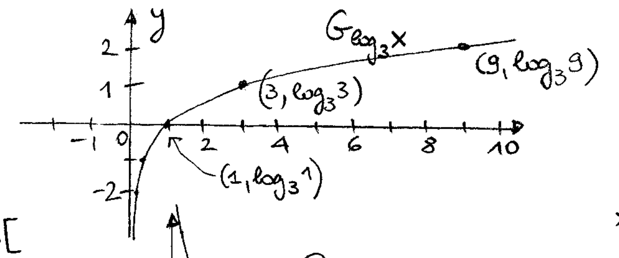
$\Rightarrow \quad S=\underline{\underline{]-2,0[}}$;

$\left(\frac{1}{16}\right)^{-|x|+1}=1 \iff -|x|+1=0 \iff |x|=1 \Rightarrow \underline{\underline{S=\{-1,1\}}}$;

$4^{|x^2-2|}<1 \iff 4^{|x^2-2|}<4^0 \underset{4^x\nearrow}{\iff} |x^2-2|<0 \qquad \underline{\underline{S=\emptyset}}$;

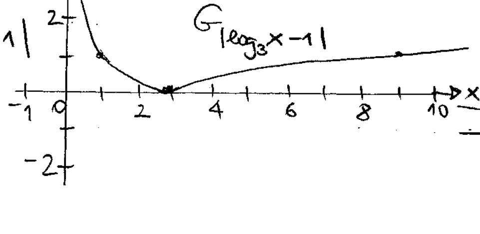
$4^{|x^2-2|}>1 \iff |x^2-2|>0 \iff x^2-2\neq 0 \Rightarrow \underline{\underline{S=\mathbb{R}\setminus\{-\sqrt{2},\sqrt{2}\}}}$.

6) i) $\log_3\frac{1}{9}=-2$; $\log_3\frac{4}{3}=-1$; $\log_3 1=0$; $\log_3 3=1$; $\log_3 9=2$



$\square$

ii) $]0,+\infty[\ni x\mapsto|\log_3 x-1|$



$\mathbb{R}\setminus\{0\} \ni x\mapsto|1-\log_3|x||$