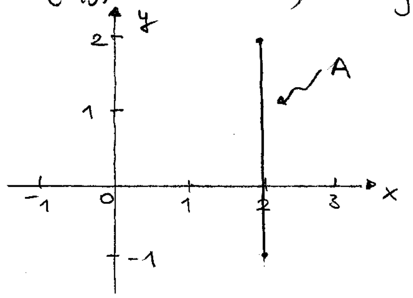
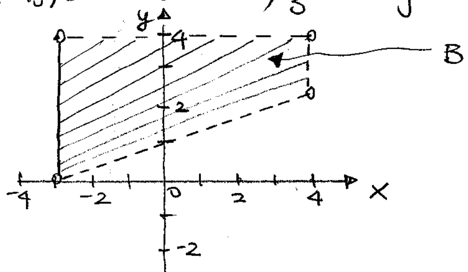
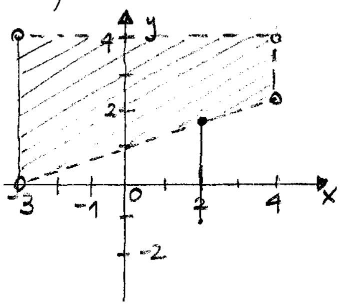
Università degli Studi di Trento - Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie delle Comunicazioni e CdL in Filosofia
Corso di ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
Verifica settimanale - a.a. 2010/11 - Rovereto, 4-8 ottobre 2010

1) i) $A=\{(x,y)\in\mathbb{R}^2 : x=2\ ,\ -1\le y\le 2\}$ $\qquad B=\{(x,y)\in\mathbb{R}^2 : -3\le x<4\ ,\ \frac{1}{3}x+1<y<4\}$.

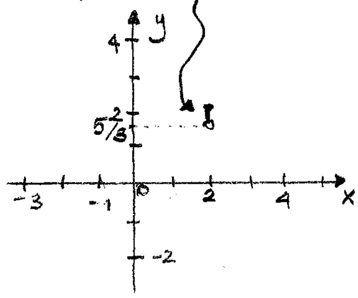




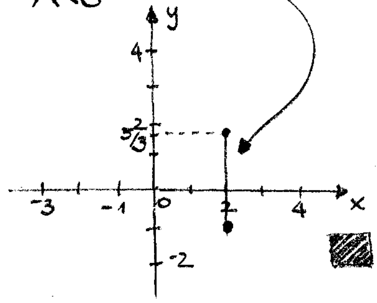
ii) $A\cup B$



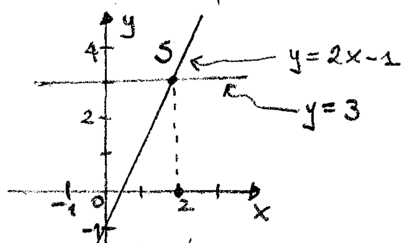
$A\cap B$



$A\setminus B$

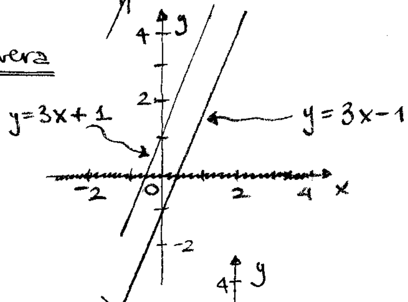


2) i) $2x-1=3 \iff 2x=4 \iff \underline{x=2}$



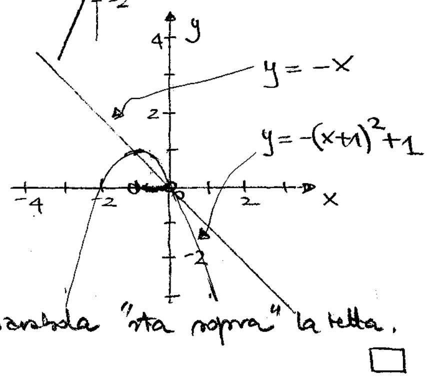
La soluzione è la coordinata x del punto d'intersezione S della retta $y=2x-1$ e $y=3$.

$\cancel{3x}-1\le \cancel{3x}+1 \iff -1\le 1$ sempre vera

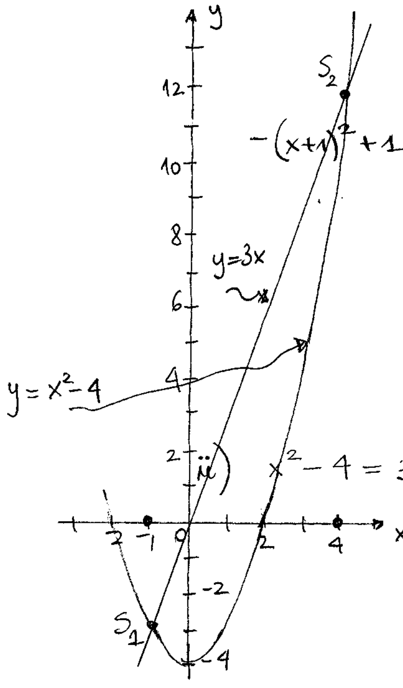


La retta $y=3x-1$ è parallela alla retta $y=3x+1$. Inoltre, la retta $y=3x-1$ sta sempre "sotto" la retta $y=3x+1$.

$-(x+1)^2+1>-x \iff -x^2-2x-1+1>-x$
$\iff x^2+x<0$
$\iff \underline{\underline{x\in\,]-1,0[}}$



La soluzione corrisponde alle coordinate x dei punti (x,y) appartenenti alla parabola $y=-(x+1)^2+1$ per i quali $y=-(x+1)^2+1$ è maggiore di $y=-x$, che intuitivamente significa le x per cui la parabola "sta sopra" la retta. □

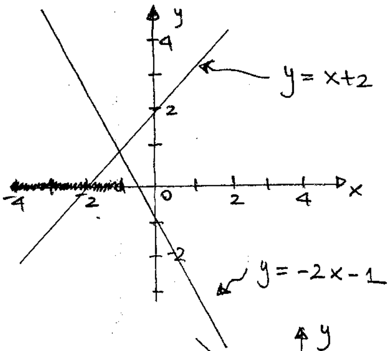


ii) $x^2-4=3x \iff x^2-3x-4=0$

$\iff x_{1/2}=\dfrac{+3\pm\sqrt{9+16}}{2}=\dfrac{+3\pm5}{2}=\begin{cases}+4\\-1\end{cases}$ $\qquad \underline{\underline{x=-1,\ x=4}}$

La soluzione corrisponde alle coordinate x dei pt. d'intersezione S_1, S_2 della parabola $y=x^2-4$ e la retta $y=3x$.

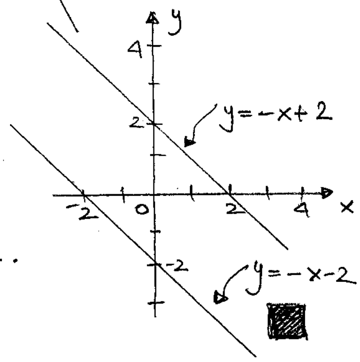
$-2x-1 \geq x+2 \iff 3x \leq -3 \iff \underline{\underline{x \leq -1}}$



La soluzione corrisponde alle coordinate x dei pt. (x,y) appartenenti alla retta $y=-2x-1$ e per cui $y=-2x-1$ è maggiore di $y=x+2$.

$-x-2 \geq -x+2 \iff -2 \geq 2$

L'insieme delle soluzioni è vuoto. Infatti, la retta di eq. $y=-x-2$ è parallela alla retta di eq. $y=-x+2$, ma la retta $y=-x-2$ sta sempre "sotto" la retta $y=-x+2$.

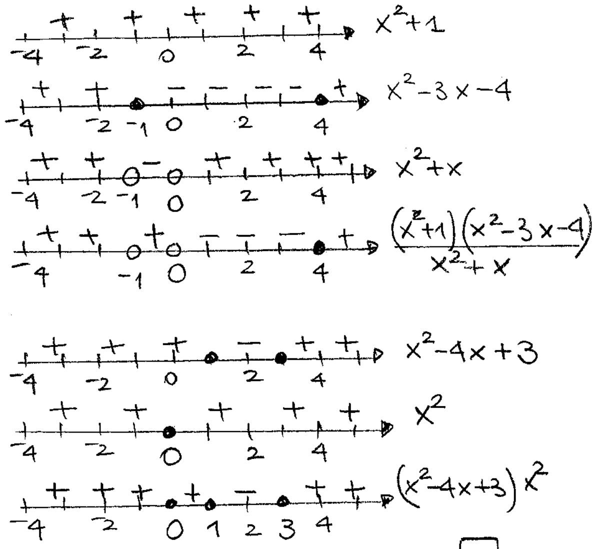


3) i) $A=\left\{x\in\mathbb{R} : \dfrac{(x^2+1)(x^2-3x-4)}{x^2+x} < 0\right\} = \underline{\underline{]0,4[}}$.

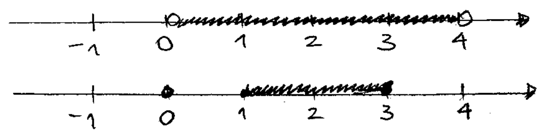
$B=\left\{x\in\mathbb{R} : \dfrac{(x^2-4x+3)\,x^2}{-3} \geq 0\right\}$

$=\left\{x\in\mathbb{R} : \dfrac{(x^2-4x+3)\,x^2}{3} \leq 0\right\}$

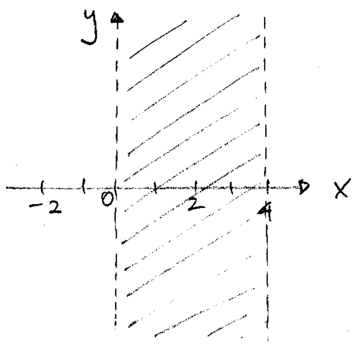
$= \underline{\underline{\{0\} \cup [1,3]}}$



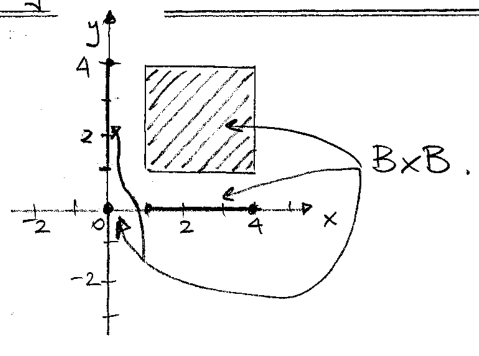
A è un intervallo, B non è un intervallo.



ii) $\underline{\underline{A\cup B = [0,4[}}$, $\underline{\underline{A\cap B = [1,3]}}$ $\underline{\underline{A\setminus B =]0,1[\cup]3,4[}}$.

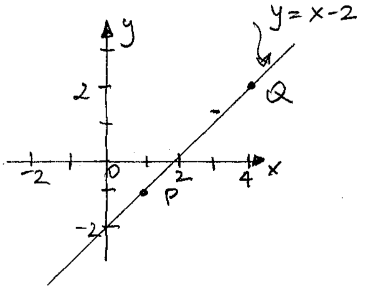


$A\times\mathbb{R}$



4) $P=(1,-1)$ $\qquad \dfrac{y-2}{-1-2} = \dfrac{x-4}{1-4} \iff y-2 = x-4$

$Q=(4,2)$ $\qquad \underline{\underline{y = x-2}}$



ii) eq. delle rette perpendicolari ad r :

$$y = -x + q \qquad q \in \mathbb{R} .$$

eq. delle rette parallele ad r :

$$y = x + q' \qquad q' \in \mathbb{R} .$$

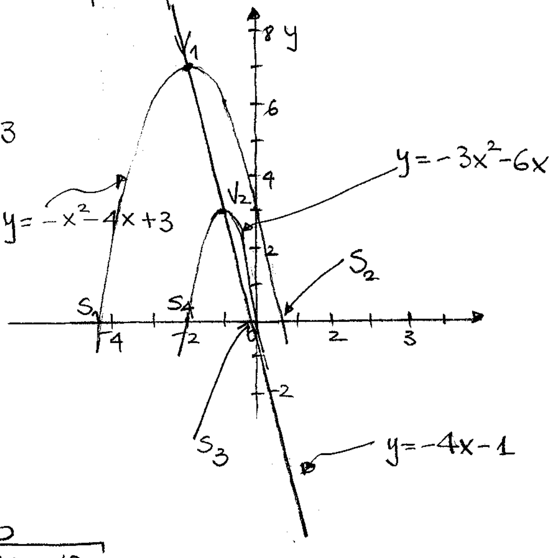
5) i)

$$y = -x^2 - 4x + 3 = -(x^2+4x) + 3 = -(x+2)^2 + 7$$

$$V_1 = (-2, 7)$$

$$y = -3x^2 - 6x = -3(x^2 + 2x) = -3(x+1)^2 + 3$$

$$V_2 = (-1, 3)$$



ii)

$$\frac{y-7}{3-7} = \frac{x+2}{-1+2} \iff \frac{y-7}{-4} = \frac{x+2}{1}$$

$$\iff y - 7 = -4x - 8$$

$$\underline{\underline{y = -4x - 1}}$$

iii)

$$\underline{\underline{y=0}} \iff -x^2-4x+3=0 \iff x^2+4x-3=0$$

$$\iff x_{1/2} = \frac{-4 \pm \sqrt{16+12}}{2} = \frac{-4 \pm \sqrt{28}}{2}$$

$$x_{1/2} = \frac{-4 \pm 2\sqrt{7}}{2} = -2 \pm \sqrt{7}$$

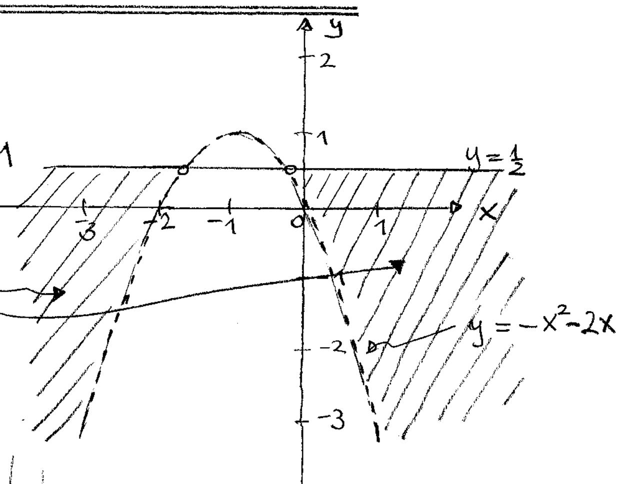
$$\underline{\underline{y=0}} \iff -3x^2-6x=0 \iff \underline{x_1=0,\ x_2=-2} .$$

$$\underline{\underline{S_1 = (-2-\sqrt{7}, 0) \qquad S_2 = (-2+\sqrt{7}, 0) \qquad S_3 = (0,0) \qquad S_4 = (-2, 0)}}$$

6)

$$x^2 + 2x + y > 0 \iff y > -x^2 - 2x$$

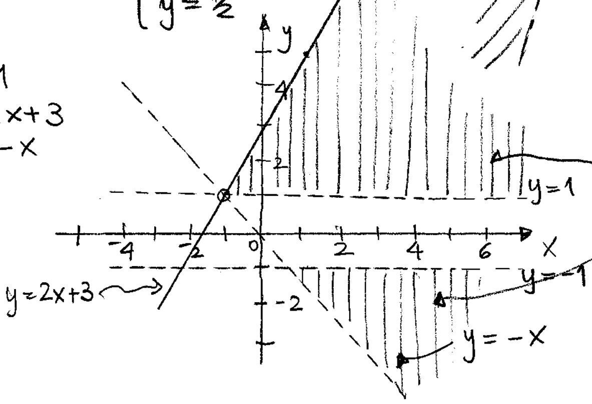
$$y > -(x+1)^2 + 1$$



Le coppie (x,y) che soddisfano

$$\begin{cases} x^2+2x+y > 0 \\ y \le \frac{1}{2} \end{cases}$$

$$\begin{cases} y^2 > 1 \\ y \le 2x+3 \\ y > -x \end{cases}$$



Le coppie (x,y) che soddisfano il sistema

$$\begin{cases} y^2 > 1 \\ y \le 2x+3 \\ y > -x . \end{cases}$$