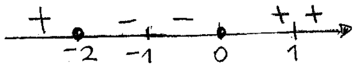
Università degli Studi di Trento – Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione – CdL in Filosofia
CORSO DI ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2010-2011 – Rovereto, 6 – 10 dicembre 2010

1) $\boxed{f(x) = (x^2+2x)\,e^{-x}} = \dfrac{(x^2+2x)}{e^x}$

i) $\operatorname{dom} f = \mathbb{R} = \,]-\infty,+\infty[$

ii) 

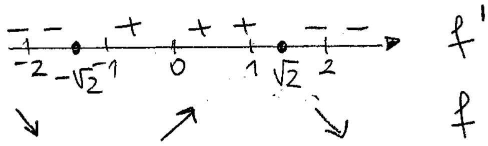
$\dfrac{x^2+2x}{e^x}$ $\left(e^x > 0 \ \forall x \in \mathbb{R}\,!\right)$

iii) $\lim_{x\to-\infty} f(x) = +\infty$ $\lim_{x\to+\infty} f(x) = 0$ ($y=0$ asintoto orizz. per f per $x \to +\infty$)

iv) f è somma e rapporto di funzioni continue, e il denominatore non si annulla mai; quindi f è continua su tutto $\mathbb{R}$.

v) $\operatorname{dom} f' = \mathbb{R}$ $f'(x) = (2x+2)\,e^{-x} - (x^2+2x)\,e^{-x} = (-x^2+2)\,e^{-x}$

$f'(x) = 0 \iff -x^2+2 = 0 \iff x_{1/2} = \pm\sqrt{2}$ pt. critici



$x = -\sqrt{2}$ pt. di min. loc. stretto per f

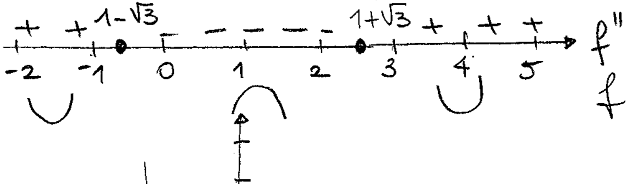
$f(-\sqrt{2}) = \dfrac{(2-2\sqrt{2})}{e^{-\sqrt{2}}}$

$x = \sqrt{2}$ pt. di max. loc. stretto per f

$f(\sqrt{2}) = \dfrac{2+2\sqrt{2}}{e^{\sqrt{2}}}$

vi) $\operatorname{dom} f'' = \mathbb{R}$ $f''(x) = -2x\,e^{-x} - (-x^2+2)\,e^{-x} = (x^2-2x-2)\,e^{-x}$

$f''(x) = 0 \iff x^2-2x-2 = 0 \iff x_{1/2} = \dfrac{2\pm\sqrt{4+8}}{2} = \dfrac{2\pm2\sqrt{3}}{2}$



$= 1\pm\sqrt{3}$

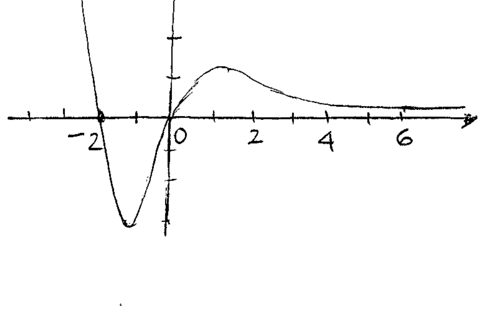


grafico qualitativo di f

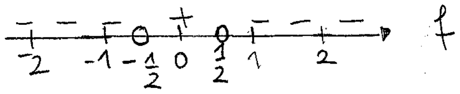
□

$$\boxed{f(x)=\frac{1}{1-4x^2}}$$

f è pari!!

i) $\mathrm{dom}\, f = \mathbb{R}\setminus\{-\frac{1}{2},\frac{1}{2}\} =]-\infty,-\frac{1}{2}[\cup]-\frac{1}{2},\frac{1}{2}[\cup]\frac{1}{2},+\infty[$

ii) segno di f



iii) $\lim_{x\to-\infty} f(x) = 0$ $\quad y=0$ asintoto orizzontale per f, per $x\to-\infty$

$\lim_{x\to-\frac{1}{2}^-} f(x) = -\infty$ $\quad x=-\frac{1}{2}$ asintoto verticale per f.

$\lim_{x\to-\frac{1}{2}^+} f(x) = +\infty$

$\lim_{x\to\frac{1}{2}^-} f(x) = +\infty$ $\quad x=\frac{1}{2}$ asintoto verticale per f

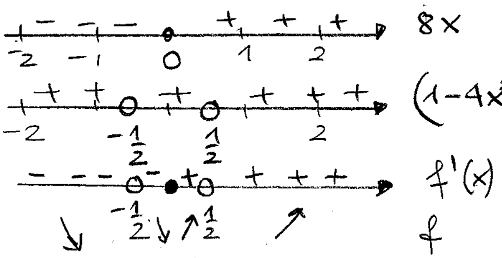
$\lim_{x\to\frac{1}{2}^+} f(x) = -\infty$

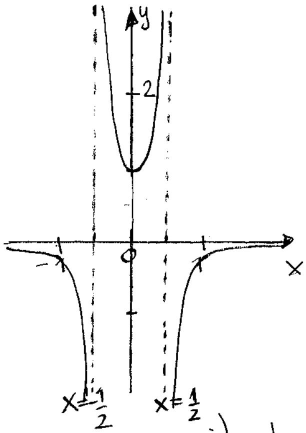
$\lim_{x\to+\infty} f(x) = 0$ $\quad y=0$ asintoto orizzontale per f, per $x\to+\infty$.

iv) Continuità di f su tutto il dominio $\mathbb{R}\setminus\{-\frac{1}{2},\frac{1}{2}\}$.

v) $\mathrm{dom}\, f' = \mathbb{R}\setminus\{-\frac{1}{2},\frac{1}{2}\}$ $\quad f'(x) = \frac{8x}{(1-4x^2)^2}$

$f'(x)=0 \iff x=0$
pt. critico





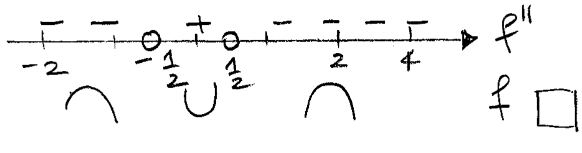
$x=0$ pt. di min loc. stretto per f

$f(0)=1$.

vi) $\mathrm{dom}\, f'' = \mathbb{R}\setminus\{-\frac{1}{2},\frac{1}{2}\}$ $\quad f''(x) = \frac{8(1-4x^2)^{\cancel{2}} + 8x\cdot 2\cancel{(1-4x^2)}8x}{(1-4x^2)^{\cancel{4}\,3}}$

$$= \frac{8-32x^2+128x^2}{(1-4x^2)^3} = \frac{96x^2+8}{(1-4x^2)^3}$$

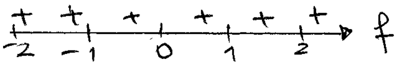
il segno di f'' coincide quindi con il segno di f poiché $96x^2+8>0$:



$$\boxed{f(x) = \log(1+e^x)}$$

i) $\operatorname{dom} f = \mathbb{R} =]-\infty, +\infty[$ poichè $1+e^x > 0 \ \forall x \in \mathbb{R}$

ii) segno di f



$1+e^x > 1 \ \forall x \in \mathbb{R}$ poichè $e^x > 0 \ \forall x \in \mathbb{R}$

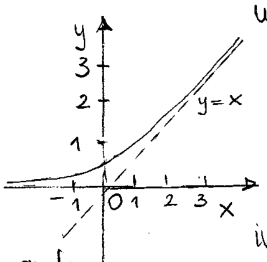


grafico qualitativo di f

iii) $\lim_{x \to -\infty} \log(1+e^x) = 0$ $y=0$ asintoto orizzontale per f, per $x \to -\infty$

$\lim_{x \to +\infty} \log(1+e^x) = +\infty$. Poichè $\lim_{x \to +\infty} \frac{\log(1+e^x)}{x} = 1$, $\lim_{x \to +\infty} [\log(1+e^x) - x] = 0$ si ha che $y = x$ è asint. obliquo per f, per $x \to +\infty$.

iv) f è continua su $\mathbb{R}$, poichè composizione di funzioni continue.

v) $\operatorname{dom} f' = \mathbb{R}$ $f'(x) = \frac{e^x}{1+e^x} > 0 \ \forall x \in \mathbb{R}$

Quindi f è crescente su tutto $\mathbb{R}$.

vi) $\operatorname{dom} f'' = \mathbb{R}$ $f''(x) = \frac{e^x(1+e^x) - e^x \cdot e^x}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2} > 0 \ \forall x \in \mathbb{R}$

f è convessa su $\mathbb{R}$.

$\square$

$$\boxed{f(x) = \log\left(\frac{x^2+1}{x^2+2}\right)}$$

f è pari !!

i) $\operatorname{dom} f = \mathbb{R} =]-\infty, +\infty[$ poichè $\frac{x^2+1}{x^2+2} > 0 \ \forall x \in \mathbb{R}$

ii) segno di f : $0 < \frac{x^2+1}{x^2+2} < 1$ e quindi $f(x) < 0 \ \forall x \in \mathbb{R}$

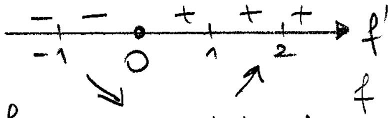
iii) $\lim_{x \to -\infty} f(x) = 0$ $y=0$ asintoto orizzontale per f, per $x \to -\infty$

$\lim_{x \to +\infty} f(x) = 0$ $y=0$ asintoto orizzontale per f, per $x \to +\infty$.

iv) composizione di funzioni continue su $\mathbb{R}$!

v) $\operatorname{dom} f' = \mathbb{R}$ $f'(x) = \frac{\cancel{x^2+2}}{x^2+1} \cdot \frac{2x(x^2+2) - (x^2+1)2x}{(x^2+2)^{\cancel{2}}}$

$= \frac{2x}{(x^2+1)(x^2+2)}$



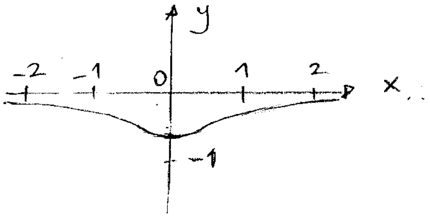
$f'(x) = 0 \iff x = 0$ pt. critico per f

$x=0$ pt. di min. loc. stretto per f

$f(0) = \log\left(\frac{1}{2}\right) = -\log 2$

vi) $\text{dom} f'' = \mathbb{R}$ $\qquad f''(x) = \dfrac{-6x^4-6x^2+4}{(x^2+1)^2(x^2+2)^2}$ $\qquad f''(x)=0 \iff x^2 = \dfrac{-3\pm\sqrt{33}}{6}$

$x_1 \sim \pm 0{,}7$



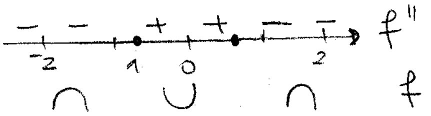


grafico qualitativo di f.

∎

2) i) $\sum_{n=1}^{8}\left(\dfrac{2}{2n+1}-\dfrac{3}{2n}\right)$; $\quad \sum_{n=1}^{7}\dfrac{2^n}{3^n}$; $\quad \sum_{n=0}^{100}(-1)^{n+1}x^n$.

ii) a) $\sum_{j=1}^{6}\text{area}(R_j) = \sum_{j=1}^{6} 2 = \underline{\underline{12}}$ $\qquad \text{area}(R_j) = \dfrac{1}{\cancel{j}}\cdot 2\cancel{j} = 2 \quad \forall j \in \{1,\dots,6\}$

☐

b) $\sum_{m=1}^{5}\dfrac{m-1}{2m} = \dfrac{1}{2}\left(0+\dfrac{1}{2}+\dfrac{2}{3}+\dfrac{3}{4}+\dfrac{4}{5}\right) = \dfrac{1}{2}\cdot\dfrac{30+40+45+48}{60}$

$= \underline{\underline{\dfrac{163}{120}}}$.

$\sum_{k=2}^{39}\left(\dfrac{1}{k}-\dfrac{1}{k+1}\right) = \left(\dfrac{1}{2}-\cancel{\dfrac{1}{3}}\right)+\left(\cancel{\dfrac{1}{3}}-\dfrac{1}{4}\right)+\dots+\left(\dfrac{1}{38}-\cancel{\dfrac{1}{39}}\right)+\left(\cancel{\dfrac{1}{39}}-\dfrac{1}{40}\right) =$

$= \dfrac{1}{2}-\dfrac{1}{40} = \underline{\underline{\dfrac{19}{40}}}$

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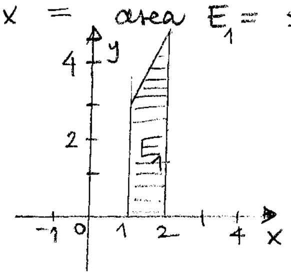
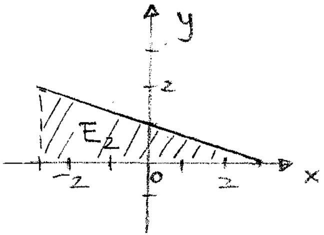
3) $\int_1^2 (2x+1)\,dx = \text{area } E_1 = \dfrac{5+3}{2}\cdot 1 = \underline{\underline{4}}$;

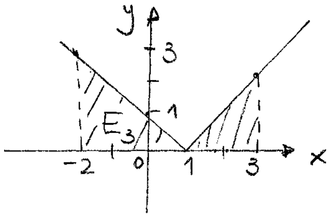




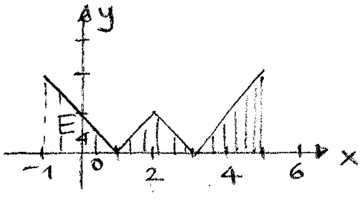
$\int_{-3}^{3}\left(-\dfrac{x}{3}+1\right)dx = \text{area } E_2 = \dfrac{6\cdot\cancel{2}}{\cancel{2}} = \underline{\underline{6}}$;

$\int_{-2}^{3}|x-1|\,dx = \text{area } E_3 =$

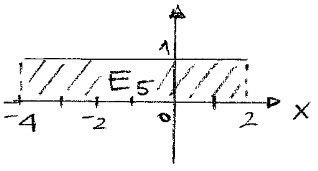
$= \dfrac{3\cdot 3}{2}+\dfrac{2\cdot 2}{2} = \underline{\underline{\dfrac{13}{2}}}$



$\int_{-1}^{5}\big||x-2|-1\big|\,dx = \text{area } E_4 = \underline{\underline{5}}$



$\int_{-4}^{2} dx = \text{area } E_5 = \underline{\underline{6}}$



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4) i) $\int_{-1}^{1}(2e^x - x^2)\,dx = \left[2e^x - \frac{x^3}{3}\right]_{-1}^{1} = \left(2e-\frac{1}{3}\right)-\left(2e^{-1}+\frac{1}{3}\right)$

$= \underline{\underline{2\left(e-\frac{1}{e}-\frac{1}{3}\right)}}$.

$\int_{0}^{1}(x+2^x)\,dx = \left[\frac{x^2}{2}+\frac{2^x}{\log 2}\right]_{0}^{1} = \left(\frac{1}{2}+\frac{2}{\log 2}\right)-\left(0+\frac{1}{\log 2}\right) = \underline{\underline{\frac{1}{2}+\frac{1}{\log 2}}}$.

$\int_{2}^{4}\left(x^{-2}-\frac{1}{x}\right)dx = \left[-x^{-1}-\log x\right]_{2}^{4} = \left(-\frac{1}{4}-\log 4\right)-\left(-\frac{1}{2}-\log 2\right) = \underline{\underline{\frac{1}{4}-\log 2}};$

$\square$

ii) $\int_{1}^{2}\left(\sqrt[4]{x}-3\sqrt{x}\right)dx = \int_{1}^{2}\left(x^{1/4}-3x^{1/2}\right)dx = \left[\frac{x^{5/4}}{5/4}-3\frac{x^{3/2}}{3/2}\right]_{1}^{2} =$

$= \left(\frac{4}{5}\cdot 2^{5/4}-2\cdot 2^{3/2}\right)-\left(\frac{4}{5}-2\right) = \underline{\underline{\frac{4}{5}\cdot 2^{5/4}-2\cdot 2^{3/2}+\frac{6}{5}}}$.

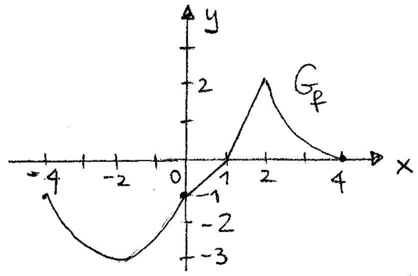
$\int_{1}^{e}\left(\frac{x^3+4x^4}{x^3}\right)dx = \int_{1}^{e}(1+4x)\,dx = \left[x+2x^2\right]_{1}^{e} = \left(e+2e^2\right)-(1+2)$

$= \underline{\underline{2e^2+e-3}};$

$\int_{-2}^{-1}\left(e^x-1\right)dx = \left[e^x-x\right]_{-2}^{-1} = \left(e^{-1}+1\right)-\left(e^{-2}+2\right) = \underline{\underline{\frac{1}{e}-\frac{1}{e^2}-1}}$.

$\blacksquare$

5)



$\boxed{\int_{-4}^{4} f(x)\,dx \;\underline{\underline{<0}}}$. Infatti

$\int_{-4}^{1} f(x)\,dx = -\text{area } E_1$

dove $E_1 = \left\{(x,y)\in\mathbb{R}^2 :\; -4\le x\le 1,\; f(x)\le y\le 0\right\}$

e $\int_{1}^{4} f(x)\,dx = \text{area } E_2$ dove $E_2 = \left\{(x,y)\in\mathbb{R}^2 :\; 1\le x\le 4,\; 0\le y\le f(x)\right\}$

Abbiamo $\int_{-4}^{4} f(x)\,dx = \int_{-4}^{1} f(x)\,dx + \int_{1}^{4} f(x)\,dx = -\text{area } E_1 + \text{area } E_2 < 0$

essendo area $E_1 >$ area E_2.

$\boxed{\int_{0}^{4} f(x)\,dx > 0}$. Infatti $\int_{0}^{1} f(x)\,dx = -\text{area } E_3$ dove $E_3 = \left\{(x,y)\in\mathbb{R}^2 : 0\le x\le 1,\; f(x)\le y\le 0\right\}$.

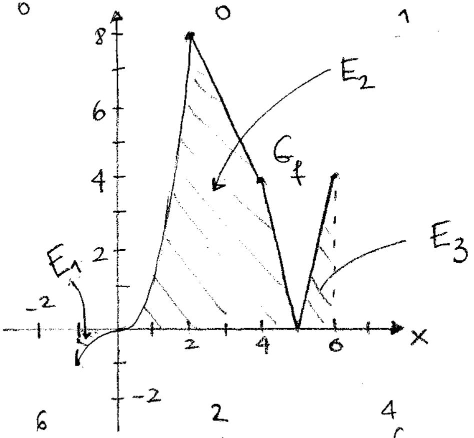
$\blacksquare$

Essendo area $E_3 <$ area E_2 si ha allora

$$\int_0^4 f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^4 f(x)\,dx = -\text{area } E_3 + \text{area } E_2 > 0 .$$

■

6) i)



ii)
$$\int_{-1}^{6} f(x)\,dx = \int_{-1}^{2} x^3\,dx + \int_{2}^{4} (-2x+12)\,dx + \int_{4}^{6} 4|x-5|\,dx =$$

$$= \left[\frac{x^4}{4}\right]_{-1}^{2} + \left[-x^2+12x\right]_{2}^{4} + \frac{1\cdot 4}{2} + \frac{1\cdot 4}{2} =$$

$$= \left(4-\frac{1}{4}\right) + \left(-16+48\right) - \left(-4+24\right) + 4 =$$

$$= \underline{\underline{20-\frac{1}{4}}} .$$

iii)
$$\int_{-1}^{6} f(x)\,dx = -\text{area } E_1 + \text{area } E_2 + \text{area } E_3 .$$

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