Università degli Studi di Trento - Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione - CdL in Filosofia
Verifica settimanale di ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2011/2012 - Rovereto, 10 - 14 Ottobre 2011

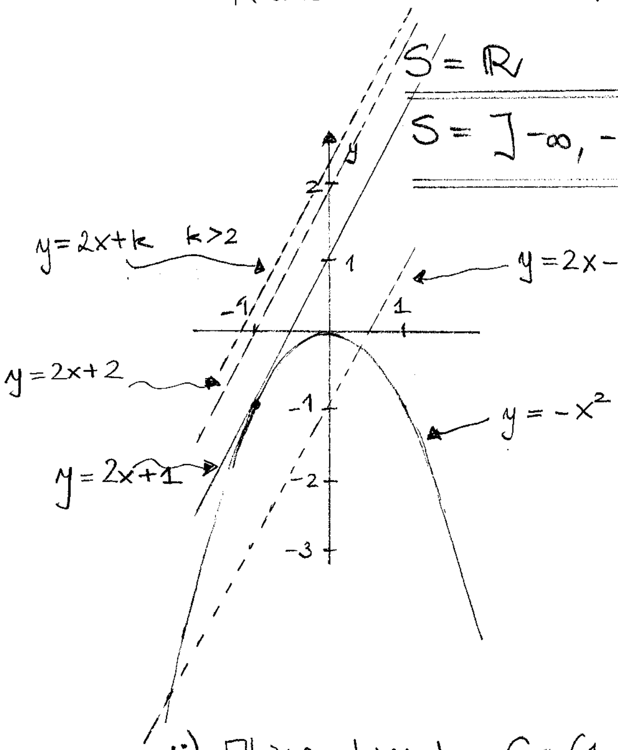
1) i) $-x^2 \le 2x+k \iff 0 \le x^2+2x+k$

Notiamo che $x^2+2x+k=0 \iff x_{1/2} = \frac{-2 \pm\sqrt{4-4k}}{2} = -1 \pm \sqrt{1-k}$

Allora $0 \le x^2+2x+k$ ha come insieme delle soluzioni

$S = \mathbb{R}$ se $k \ge 1$

$S = \left]-\infty, -1-\sqrt{1-k}\right] \cup \left[-1+\sqrt{1-k}, +\infty\right[$ se $k<1$.



Interpret. geom.: la soluzione corrisponde alle coordinate x dei pt. (x,y) appartenenti alla retta $y=2x+k$ per i quali $y=-x^2$ è minore di $y=2x+k$ che intuitivamente significa le x per cui la retta $y=2x+k$ "sta sopra o interseca" la parabola $y=-x^2$.

ii) Ellisse di centro $C=(1,-1)$ e semiassi $a=4$ e $b=1$:

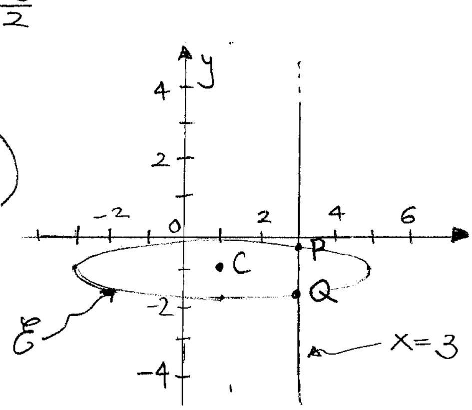
$$\frac{(x-1)^2}{16} + (y+1)^2 = 1$$

Intersezione con la retta di eq. $x=3$: $\begin{cases} \frac{(3-1)^2}{16} + (y+1)^2 = 1 \\ x=3 \end{cases} \iff$

$\iff \begin{cases} (y+1)^2 = \frac{3}{4} \\ x=3 \end{cases} \iff \begin{cases} y_{1/2} = -1 \pm \frac{\sqrt{3}}{2} \\ x=3 \end{cases}$

$P = \left(3, -1+\frac{\sqrt{3}}{2}\right)$ $Q = \left(3, -1-\frac{\sqrt{3}}{2}\right)$

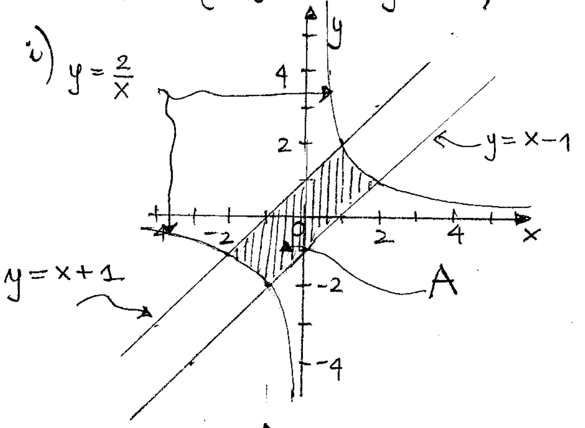
$d(P,Q) = \sqrt{3}$



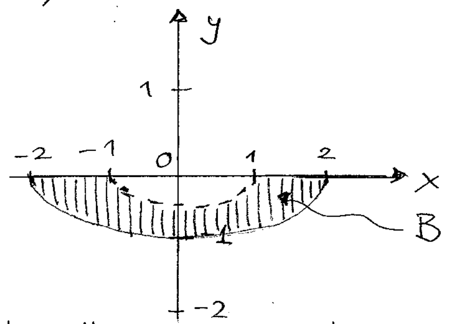
2) $A = \{(x,y) \in \mathbb{R}^2 : \; xy \leq 2, \; x-1 \leq y \leq x+1\}$

$B = \{(x,y) \in \mathbb{R}^2 : \; y \leq 0, \; 1 < x^2 + 4y^2 \leq 4\}$

i)



$$xy \leq 2 \iff \begin{cases} y \leq \frac{2}{x} & \text{se } x > 0 \\ y \geq \frac{2}{x} & \text{se } x < 0 \\ y \in \mathbb{R} & \text{se } x = 0 \end{cases}$$



$$1 < x^2 + 4y^2 \iff 1 < x^2 + \frac{y^2}{1/4}$$

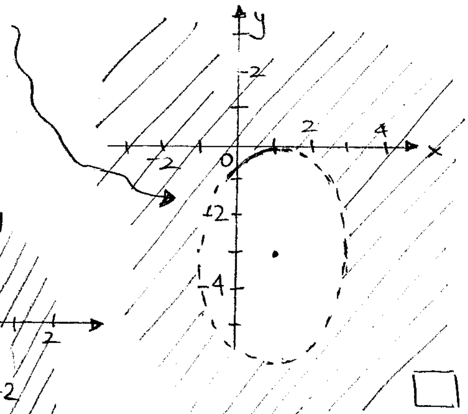
$$x^2 + 4y^2 \leq 4 \iff \frac{x^2}{4} + y^2 \leq 1$$

ii) Le rette orizzontali di eq $y = k$ con $k \in]-\infty, -2[\cup]2, +\infty[$ non intersecano mai l'insieme A.

Le rette verticali di eq. $x = k$ con $k \in]-\infty, -2[\cup]2, +\infty[$ non intersecano mai l'insieme B. ■

3) i) $9(x-1)^2 + 4(y+3)^2 > 36 \iff$

$$\frac{(x-1)^2}{4} + \frac{(y+3)^2}{9} > 1$$



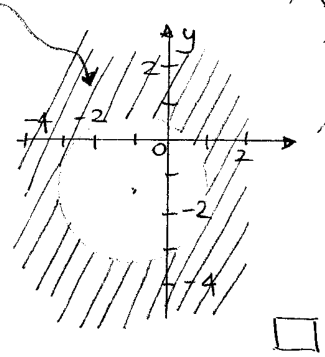
ii) $4x^2 + 4y^2 + 8x + 12y \geq 3$

$4(x^2 + 2x) + 4(y^2 + 3y) \geq 3$

$4(x+1)^2 - 4 + 4\left(y + \frac{3}{2}\right)^2 - 9 \geq 3$

$4(x+1)^2 + 4\left(y + \frac{3}{2}\right)^2 \geq 16$

$(x+1)^2 + \left(y + \frac{3}{2}\right)^2 \geq 4$



□

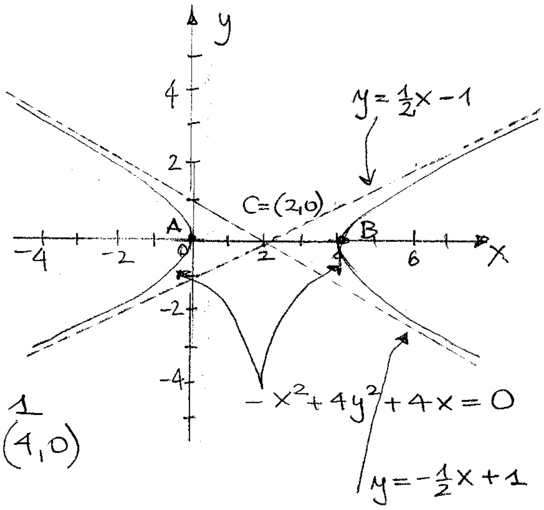
iii) $-x^2+4y^2+4x=0 \iff$

$-(x^2-4x)+4y^2=0 \iff$

$-(x-2)^2+4y^2=-4 \iff$

$\dfrac{(x-2)^2}{4}-y^2=1$

eq. iperbole: centro $C=(2,0)$
- semiassi $a=2$, $b=1$
- vertici $A=(0,0)$, $B=(4,0)$
- asintoti $y=\frac{1}{2}x-1$
  $y=-\frac{1}{2}x+1$



□

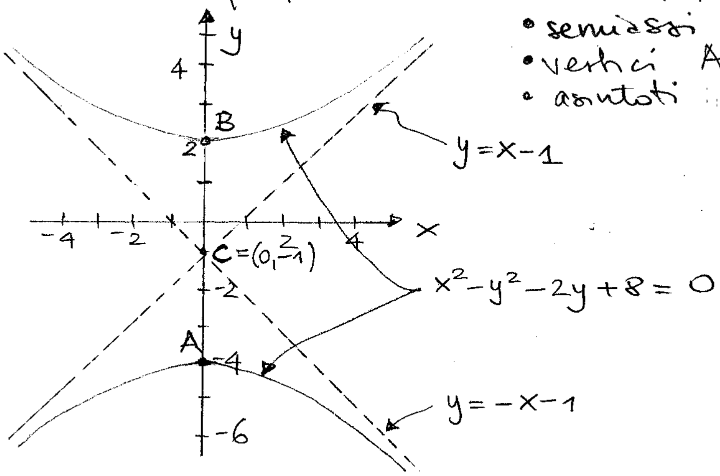
iv) $x^2-y^2-2y+8=0 \iff$

$x^2-(y^2+2y)+8=0 \iff$

$x^2-(y+1)^2+1+8=0 \iff x^2-(y+1)^2=-9$

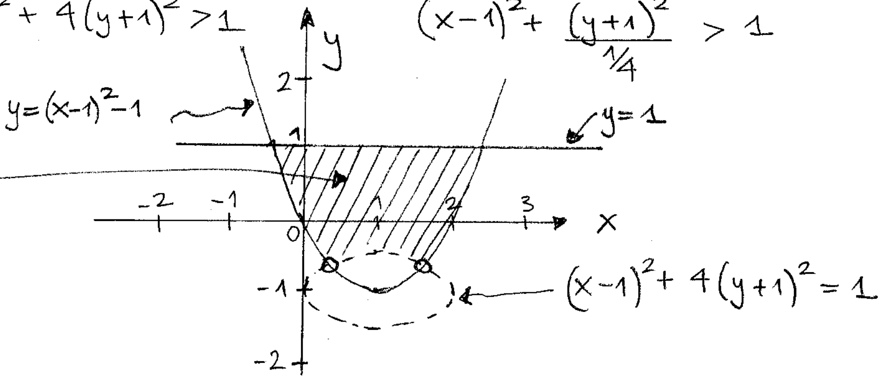
$\iff -\dfrac{x^2}{9}+\dfrac{(y+1)^2}{9}=1$

eq. iperbole: centro $C=(0,-1)$
- semiassi $a=b=3$
- vertici $A=(0,-4)$ $B=(0,2)$
- asintoti $y=x-1$
  $y=-x-1$



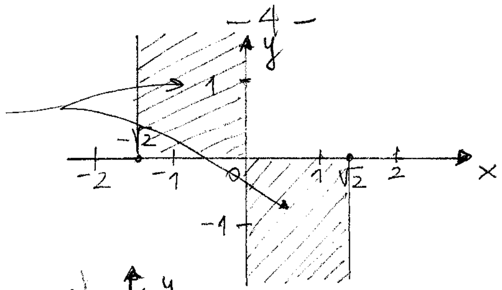
■

4) $\begin{cases} x^2-2x\le y\le 1 \\ (x-1)^2+4(y+1)^2>1 \end{cases} \iff \begin{cases} (x-1)^2-1\le y\le 1 \\ (x-1)^2+\dfrac{(y+1)^2}{1/4}>1 \end{cases}$

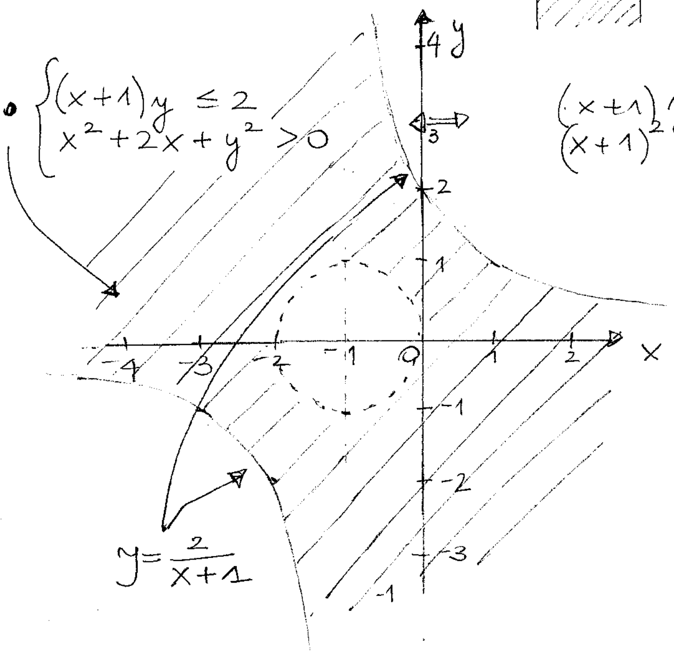


□

- $\begin{cases} x^2 \leq 2 \\ xy \leq 0 \end{cases}$



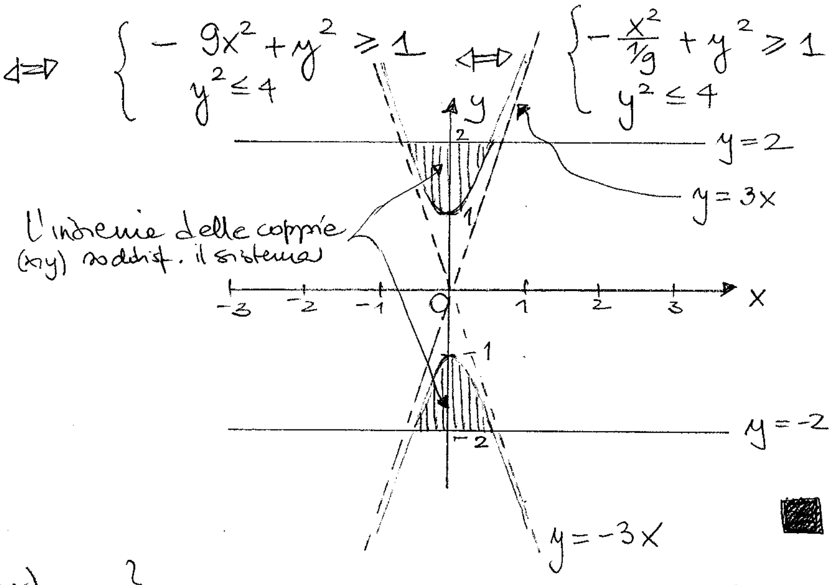
- $\begin{cases} (x+1)y \leq 2 \\ x^2+2x+y^2 > 0 \end{cases}$ $\iff$ $\begin{cases} (x+1)y \leq 2 \\ (x+1)^2 + y^2 > 1 \end{cases}$



$(x+1)y \leq 2 \iff$

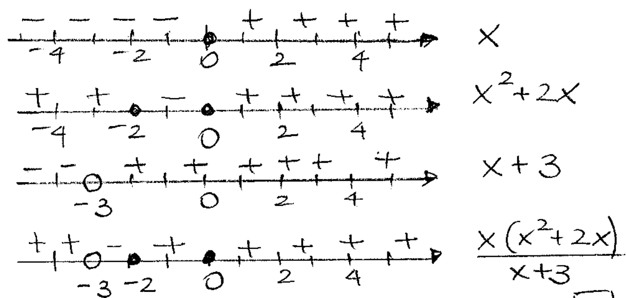
$$\begin{cases} (-1, y) & \text{se } x=-1 \\ y \leq \dfrac{2}{x+1} & \text{se } x>-1 \\ y \geq \dfrac{2}{x+1} & \text{se } x<-1 \end{cases}$$

- $\begin{cases} 9x^2 - y^2 \leq -1 \\ y^2 \leq 4 \end{cases}$ $\iff$ $\begin{cases} -9x^2 + y^2 \geq 1 \\ y^2 \leq 4 \end{cases}$ $\iff$ $\begin{cases} -\dfrac{x^2}{1/9} + y^2 \geq 1 \\ y^2 \leq 4 \end{cases}$



5) $A = \left\{ x \in \mathbb{R} : \dfrac{x(x^2+2x)}{x+3} \leq 0 \right\}$

$= \underline{]-3, -2] \cup \{0\}}$



<u>A è limitato inferiormente</u>

<u>e superiormente.</u>

<u>$\nexists$ min A</u> ; <u>max A = 0</u> .

• $B = \left\{ x \in \mathbb{R} : \dfrac{3x}{x^2+2} \geq -1 \right\} = \left]-\infty, -2\right] \cup \left[-1, +\infty\right[$

Notiamo $\dfrac{3x}{x^2+2} \geq -1 \iff 3x \geq -(x^2+2)$ (NOTA: $x^2+2 > 0 \ \forall x \in \mathbb{R}$)

$\iff x^2+3x+2 \geq 0$

$\iff (x+2)(x+1) \geq 0$

B non è limitato inferiormente e non è limitato superiormente;

$\nexists \min B$, $\nexists \max B$. $\square$

• $C = \left]0, 4\right] \cup \{-2\}$: C è limitato inferiormente e superiormente.

$\min C = -2$, $\max C = 4$. $\blacksquare$