Università degli Studi di Trento – Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione – CdL in Filosofia
Verifica settimanale di ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2011-2012 – Rovereto, 12-16 dicembre 2011

1) $f(x)=(x^2-3)e^{-x}$ • $\operatorname{dom} f=\mathbb{R}=\,]-\infty,+\infty[$

• segno di f $\qquad + \quad + \quad -\sqrt{3} \quad - \quad - \quad - \quad - \quad \sqrt{3} \quad + \quad f$ $\quad (-2,\ -1,\ 0,\ 1,\ 2)$

• $\lim\limits_{x\to-\infty} f(x)=+\infty \qquad \lim\limits_{x\to+\infty} f(x)=0 \qquad y=0$ è asintoto orizzontale per f, per $x\to+\infty$.

• f continua su $\mathbb{R}$ (composizione e prodotto di funzioni continue)

• $\operatorname{dom} f'=\mathbb{R} \qquad f'(x)=2xe^{-x}-(x^2-3)e^{-x}=e^{-x}(-x^2+2x+3)$
$$=e^{-x}(-x+3)(x+1)$$

$- \quad - \quad (-1) \quad + \quad + \quad + \quad + \quad (3) \quad - \quad - \quad f'$ $\quad (-2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4)$

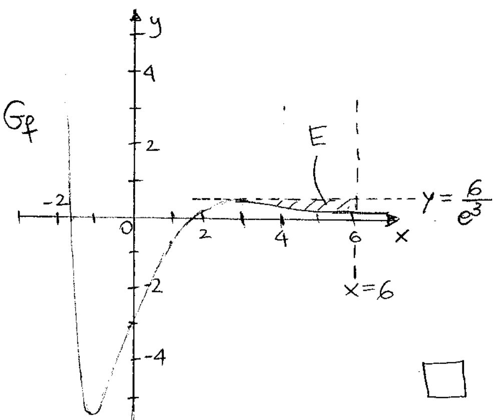
$\searrow \quad \nearrow \quad \searrow \quad f$

$x=-1$ pt. critico; risulta che è un pt. minimo loc. stretto per f

$f(-1)=-2e \sim -5.4$

$x=3$ pt. critico; risulta che è un pt. massimo loc. stretto per f

$f(3)=\dfrac{6}{e^3}\sim 0.3$



ii) $y=\dfrac{6}{e^3}$ □

iii) F è derivabile e $F'(x)=-e^{-x}(-x^2-2x+1)+e^{-x}(-2x-2)$
$$=e^{-x}(x^2+\cancel{2x}-1-\cancel{2x}-2)=e^{-x}(x^2-3)=f(x)$$
$\forall x\in\mathbb{R}$. □

iv) $\operatorname{area}(E)=\displaystyle\int_3^6\left(\frac{6}{e^3}-f(x)\right)dx=\left[\frac{6}{e^3}x\right]_3^6-\int_3^6 f(x)\,dx=$
$$=\frac{6}{e^3}\cdot 3-\Big[F(x)\Big]_3^6=\frac{18}{e^3}-\left[-47e^{-6}+14e^{-3}\right]=\underline{\underline{\frac{4}{e^3}+\frac{47}{e^6}}}.$$ ■

2) i) $\displaystyle\int_{-1}^0(-2x-1)^5dx=-\frac{1}{2}\int_{-1}^0-2(-2x-1)^5dx=-\frac{1}{2}\left[\frac{(-2x-1)^6}{6}\right]\Bigg|_{-1}^0=-\frac{1}{2}\left(\frac{1}{6}-\frac{1}{6}\right)=\underline{\underline{0}}.$

$\displaystyle\int_0^1\left[(2^x+1)^4\,2^x+\frac{1}{2x-3}\right]dx=\frac{1}{\log 2}\int_0^1(2^x+1)^4\,2^x\log 2\,dx+\frac{1}{2}\int_0^1\frac{2}{2x-3}dx=$
$$=\frac{1}{\log 2}\left[\frac{(2^x+1)^5}{5}\right]_0^1+\frac{1}{2}\Big[\log|2x-3|\Big]_0^1=\frac{1}{\log 2}\left(\frac{3^5}{5}-\frac{2^5}{5}\right)-\frac{1}{2}\log 3=$$
$$=\underline{\underline{\frac{1}{\log 2}\cdot\frac{211}{5}-\frac{1}{2}\log 3}}.$$

$$\int_{-1}^{0} \frac{3x}{x^2+4}\,dx = \frac{3}{2}\int_{-1}^{0}\frac{2x}{x^2+4}\,dx = \frac{3}{2}\Big[\log(x^2+4)\Big]_{-1}^{0} = \frac{3}{2}\left(\log 4 - \log 5\right) =$$

$$= \underline{\underline{\frac{3}{2}\log\frac{4}{5}}}\,.$$

ii) $$\int_{1}^{2} \frac{x + 2\sqrt[3]{x}}{x^2}\,dx = \int_{1}^{2}\left(\frac{1}{x} + 2x^{-5/3}\right)dx = \left[\log|x| + 2\,\frac{x^{-2/3}}{-2/3}\right]_{1}^{2} =$$

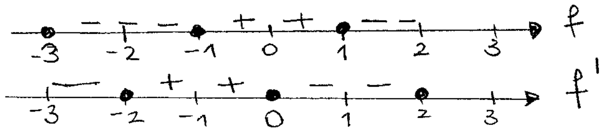
$$= \underline{\underline{\log 2 - 3\cdot\frac{1}{2^{2/3}} + 3}}\,.$$

$$\int_{0}^{1} 3x^2 e^{-x^3}\,dx = -\int_{0}^{1}\left[-3x^2e^{-x^3}\right]dx = \left[-e^{-x^3}\right]_{0}^{1} = -e^{-1} + 1 = \underline{\underline{1 - \frac{1}{e}}}\,.$$

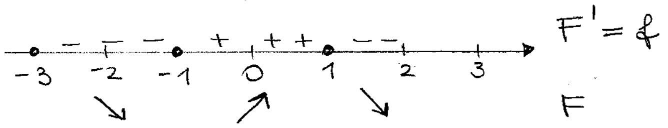
$$\int_{-2}^{3} e^{-|x|}\,dx = \int_{-2}^{0} e^{x}\,dx + \int_{0}^{3} e^{-x}\,dx = \Big[e^{x}\Big]_{-2}^{0} - \Big[e^{-x}\Big]_{0}^{3} = \left(1 - \frac{1}{e^2}\right) - \left(\frac{1}{e^3} - 1\right)$$

$$= \underline{\underline{2 - \frac{1}{e^2} - \frac{1}{e^3}}}\,.$$

3) i)



ii)

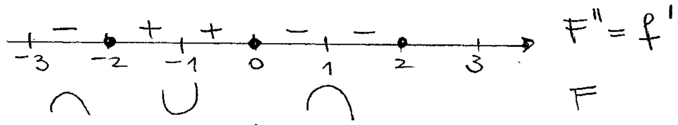


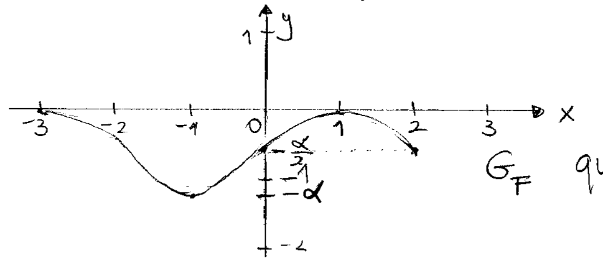
F decresce su $[-3,-1]$

F cresce su $[-1, 1]$

F decresce su $[1, 2]$.

iii)



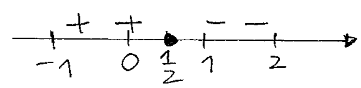


G_F qualitativo

4) $F'(x) = f(x) \quad \forall x \in [-1,2]$.

Quindi $F'(x)=0 \iff f(x)=0 \implies x=\frac{1}{2}$ è pt. critico per f.

Osserviamo inoltre



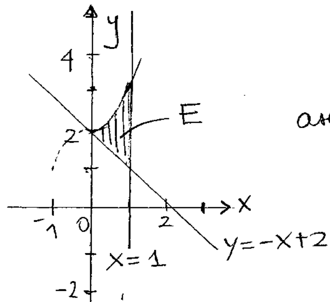
$F'(x) = f(x)$

$F(x)$

Quindi $x=\frac{1}{2}$ è pt. di massimo per F e $F(\frac{1}{2}) = \int_{-1}^{1/2} f(t)\,dt = \max_{[-1,2]} F$

$x=-1$ e $x=2$ sono pt. di minimo per F e $F(-1)=F(2)=0=\min_{[-1,2]} F$.

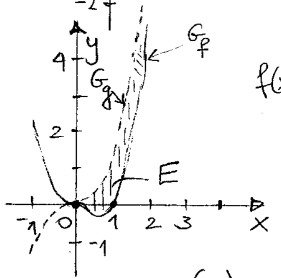
5) i)



$$\text{area}(E) = \int_0^1 \left(x^3+2-(-x+2)\right)dx = \int_0^1 \left(x^3+\cancel{2}+x-\cancel{2}\right)dx$$

$$= \int_0^1 (x^3+x)\,dx = \left[\frac{x^4}{4}+\frac{x^2}{2}\right]_0^1 = \frac{1}{4}+\frac{1}{2} = \underline{\underline{\frac{3}{4}}};$$

ii)



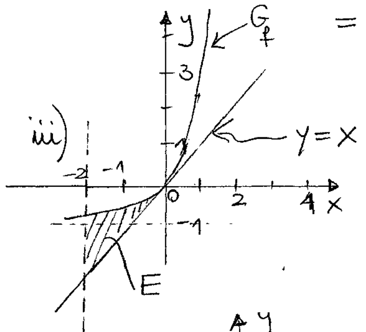
$$f(x) = x^4-x^3 = x^3(x-1)$$

$$x^4-x^3 = x^3 \iff x^3(x-2)=0 \iff x=0 \text{ o } x=2$$

$$\text{area}(E) = \int_0^2 \left(x^3-(x^4-x^3)\right)dx = \int_0^2 (2x^3-x^4)\,dx = \left[2\frac{x^4}{4}-\frac{x^5}{5}\right]_0^2$$

$$= \frac{1}{\cancel{2}}\cdot\cancel{16}^{\,8} - \frac{32}{5} = \underline{\underline{\frac{8}{5}}};$$

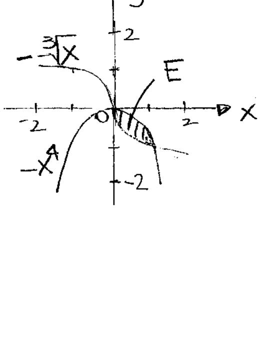
iii)



$$\text{area}(E) = \int_{-2}^0 (e^x-1-x)\,dx = \left[e^x-x-\frac{x^2}{2}\right]_{-2}^0$$

$$= 1-\left(e^{-2}+\cancel{2}-\cancel{2}\right) = \underline{\underline{1-\frac{1}{e^2}}};$$

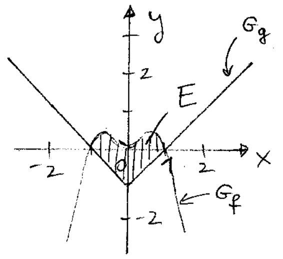
iv)



$$\text{area}(E) = \int_0^1 \left(-x^4+\sqrt[3]{x}\right)dx = \left[-\frac{x^5}{5}+\frac{x^{4/3}}{4/3}\right]_0^1$$

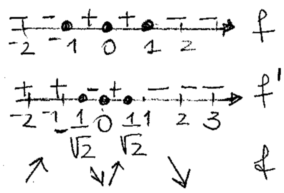
$$= -\frac{1}{5}+\frac{3}{4} = \frac{-4+15}{20} = \underline{\underline{\frac{11}{20}}};$$

v)



$$f(x) = x^2 - x^4 = x^2(1-x^2)$$

$$f'(x) = 2x - 4x^3 = 2x(1-2x^2)$$

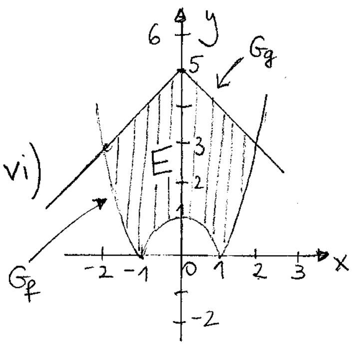


$$\text{area}(E) = 2\int_0^1 \left[(x^2 - x^4) - (x-1)\right] dx$$

$$= 2\left[\frac{x^3}{3} - \frac{x^5}{5} - \frac{x^2}{2} + x\right]_0^1 = 2\left(\frac{1}{3} - \frac{1}{5} - \frac{1}{2} + 1\right)$$

$$= 2\left(\frac{10-6-15+30}{30}\right) = \cancel{2}\cdot\frac{19}{\cancel{30}_{15}} = \underline{\underline{\frac{19}{15}}};$$

vi)



$$\begin{cases} x > 1 \\ x^2 - 1 = -x + 5 \end{cases} \Leftrightarrow x^2 + x - 6 = 0$$

$$\Leftrightarrow (x+3)(x-2) = 0$$

$$\text{area}(E) = 2\int_0^2 \left[(-x+5) - |x^2 - 1|\right] dx$$

$$= 2\int_0^1 \left[(-x+5) - (1-x^2)\right] dx + 2\int_1^2 \left[(-x+5) - (x^2-1)\right] dx$$

$$= 2\int_0^1 (x^2 - x + 4)\,dx + 2\int_1^2 (-x^2 - x + 6)\,dx$$

$$= 2\left[\frac{x^3}{3} - \frac{x^2}{2} + 4x\right]_0^1 + 2\left[-\frac{x^3}{3} - \frac{x^2}{2} + 6x\right]_1^2$$

$$= 2\left[\frac{1}{3} - \frac{1}{2} + 4\right] + 2\left[\left(-\frac{8}{3} - 2 + 12\right) - \left(-\frac{1}{3} - \frac{1}{2} + 6\right)\right]$$

$$= 2\left[-\frac{\cancel{6}^{2}}{\cancel{3}} + 8\right] = \underline{\underline{12}}.$$

6) i) $P_9^{4,5} = \frac{9!}{4!\,5!} = \frac{9\cdot \cancel{8}^{2}\cdot 7\cdot \cancel{6}\cdot \cancel{5!}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{5!}} = 18\cdot 7 = \underline{\underline{126}}.$

ii) $C_{15,6} = \frac{15!}{6!\,(15-6)!} = \frac{15!}{6!\;9!} = \frac{\cancel{15}^{5}\cdot\cancel{14}^{7}\cdot 13\cdot\cancel{12}\cdot\cancel{10}\cdot\cancel{9!}}{\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{9!}} = \underline{\underline{455}}.$

iii) $D_{9,3} = \frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{9\cdot 8\cdot 7\cdot\cancel{6!}}{\cancel{6!}} = 72\cdot 7 =$

$= \underline{\underline{504}}.$