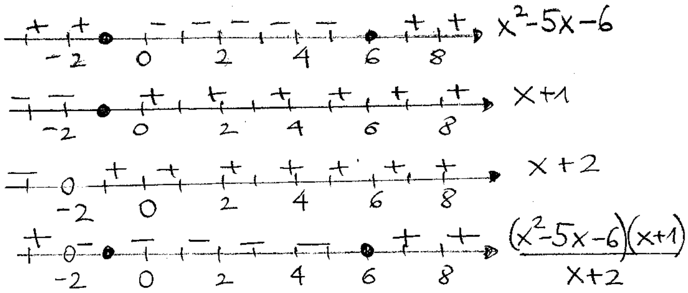
Università degli Studi di Trento – Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione – CdL in Filosofia
Verifica settimanale di ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2010-2011 — Rovereto, 3-7 ottobre 2011

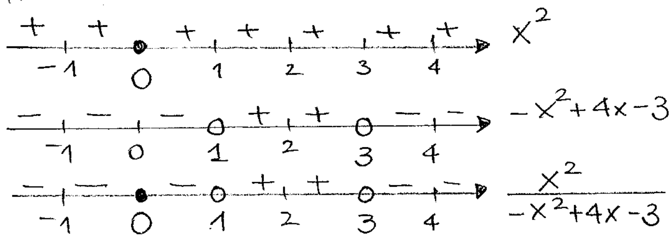
1) i) $A = \left\{ x \in \mathbb{R} : \dfrac{(x^2-5x-6)(x+1)}{x+2} < 0 \right\} = \underline{\underline{\left]-2,6\right[\setminus \{-1\}}}$.

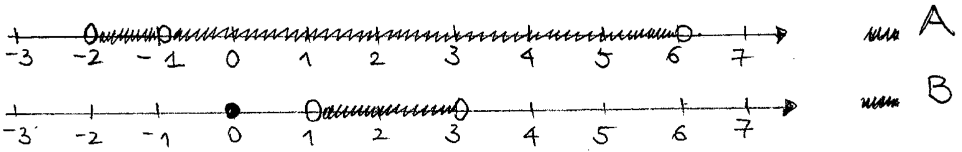
Infatti



$B = \left\{ x \in \mathbb{R} : \dfrac{x^2}{-x^2+4x-3} \geq 0 \right\} =$

$= \underline{\underline{\{0\} \cup \left]1,3\right[}}$. Infatti

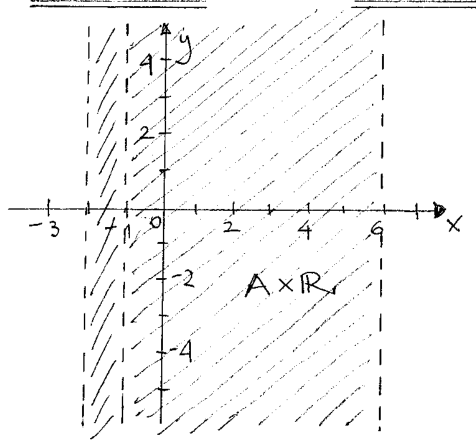


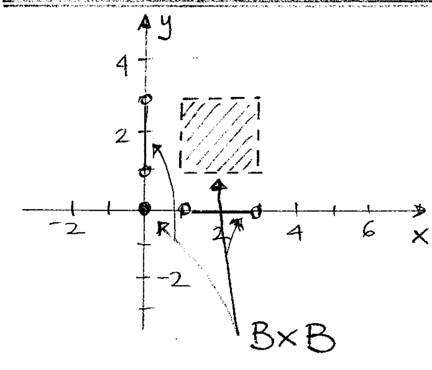


Oss. A non è un intervallo, perchè "ha un buco" (manca -1).

B non è un intervallo, perchè "mancano tutti i nr. reali tra 0 e 1".

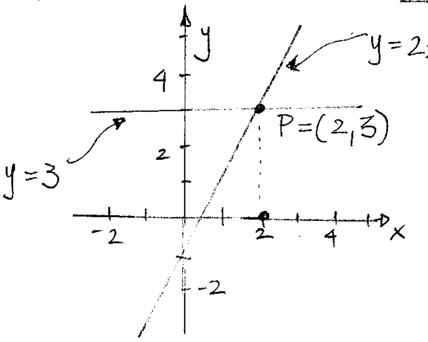
ii) $\underline{A \cup B = A}$ $\qquad$ $\underline{A \cap B = B}$ $\qquad$ $\underline{A \setminus B = \left]-2,-1\right[\cup \left]-1,0\right[\cup \left]0,1\right] \cup \left[3,6\right[}$.





2) i) $2x-1=3 \iff 2x=4$

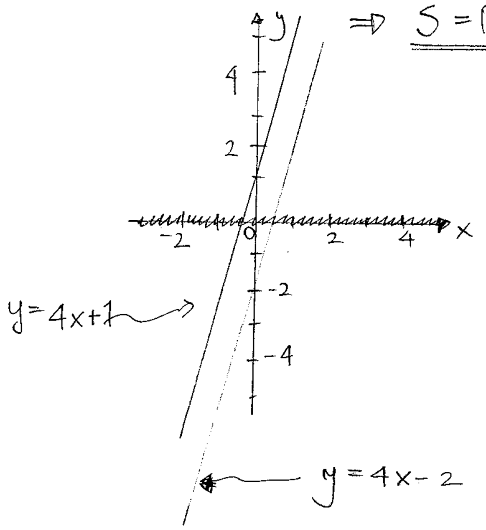
$\iff \underline{\underline{x=2}}$



<u>Interpret. geom.</u> : $x=2$ è la coordinata x del pt. di intersezione P della retta $y=2x-1$ e la retta $y=3$.

$4x-2 \leq 4x+1 \iff -2 \leq 1$

$\Rightarrow \underline{\underline{S=\mathbb{R}}}$



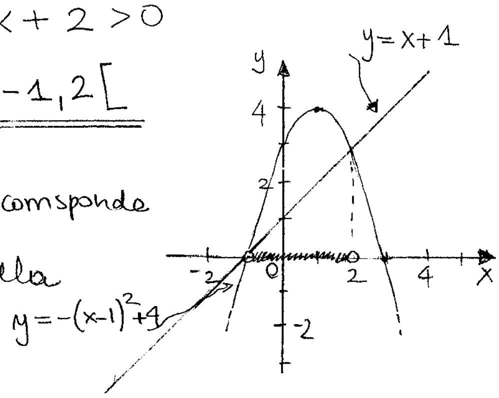
<u>Interpret. geom.</u> : La retta di eq. $y=4x-2$ è parallela alla retta di eq. $y=4x+1$. Inoltre, la retta $y=4x-2$ "sta sempre sotto" la retta $y=4x+1$.

$-(x-1)^2+4 > x+1 \iff$

$\iff -x^2+2x-1+4-x-1>0$

$\iff -x^2+x+2>0$

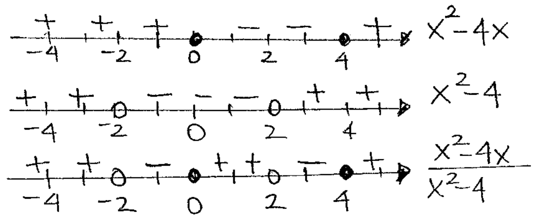
$\Rightarrow \underline{\underline{S=\,]-1,2[}}$



<u>Interpret. geom.</u>: la soluzione corrisponde alle coordinate x dei pt. (x,y) appartenenti alla parabola $y=-(x-1)^2+4$ per i quali $y=-(x-1)^2+4$ è maggiore di $y=x+1$, che intuitivamente significa le x per cui la parabola "sta sopra" la retta. $\square$

ii) $\dfrac{x(x^2-x)}{x^2-4} \leq x \iff \dfrac{x(x^2-x)-x(x^2-4)}{x^2-4} \leq 0 \quad \dfrac{-x^2+4x}{x^2-4} \leq 0$

$\iff \dfrac{x(x-4)}{x^2-4} \geq 0$

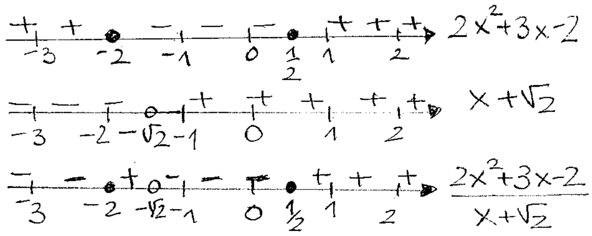


$\underline{\underline{S=\,]-\infty,-2[\,\cup\,[0,2[\,\cup\,[4,+\infty[}}$

$$\frac{x^2+3x}{x+\sqrt{2}} > \sqrt{2}-x \iff \frac{x^2+3x}{x+\sqrt{2}} + x - \sqrt{2} > 0$$

$$\iff \frac{x^2+3x+x^2-2}{x+\sqrt{2}} > 0$$

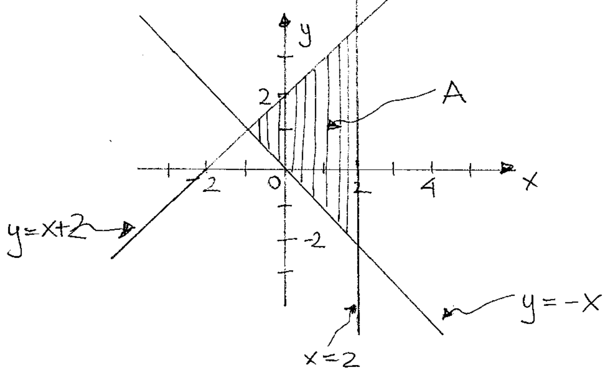
$$\iff \frac{2x^2+3x-2}{x+\sqrt{2}} > 0$$

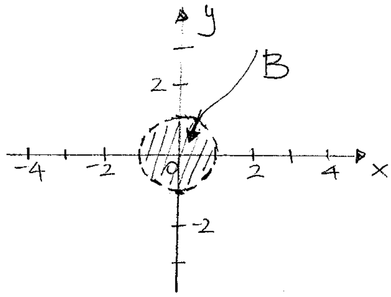


$$S = \left]-2, -\sqrt{2}\right[\cup \left]\tfrac{1}{2}, +\infty\right[$$

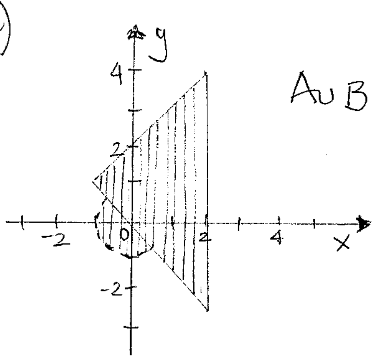
3) i) $A = \{(x,y) \in \mathbb{R}^2 : -1 \le x \le 2,\ -x \le y \le x+2\}$

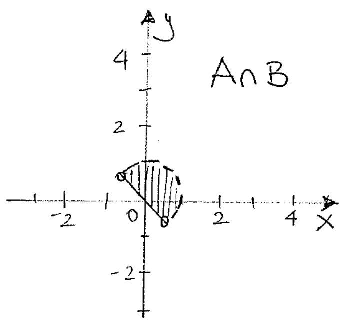
$B = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 < 1\}$

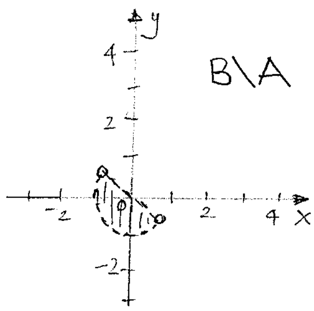




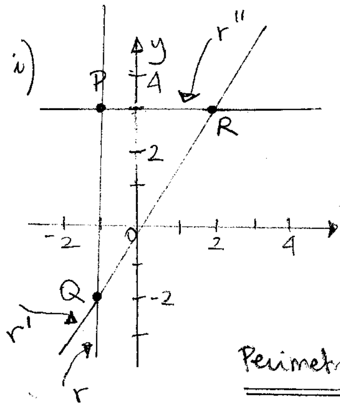
ii)







4) i)



$r: \quad x = -1$

$r': \quad y = \frac{5}{3}x - \frac{1}{3}$

$r'': \quad y = 3$

ii) $d(P,Q) = 5$

$d(P,R) = 3$

$d(R,Q) = \sqrt{(3+2)^2+(2+1)^2}$

$= \sqrt{34}$

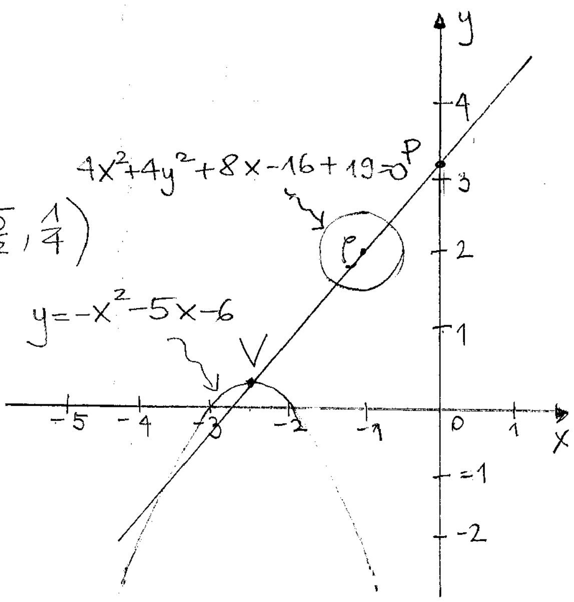
Perimetro triangolo di vertici $P, Q, R = 5+3+\sqrt{34} = 8+\sqrt{34}$

5) i) $y = -x^2 - 5x - 6$

$= -(x^2+5x) - 6$

$= -\left(x+\frac{5}{2}\right)^2 + \frac{25}{4} - 6$

$y = -\left(x+\frac{5}{2}\right)^2 + \frac{1}{4}$ $\qquad V = \left(-\frac{5}{2}, \frac{1}{4}\right)$



$4x^2+4y^2+8x-16y+19=0$

$4(x^2+2x) + 4(y^2-4y) + 19 = 0$

$\underbrace{4(x+1)^2-4} + \underbrace{4(y-2)^2-16} + 19 = 0$

$4(x+1)^2 + 4(y-2)^2 = 1$

$(x+1)^2 + (y-2)^2 = \frac{1}{4}$ $\qquad C = (-1, 2) \quad r = \frac{1}{2}$

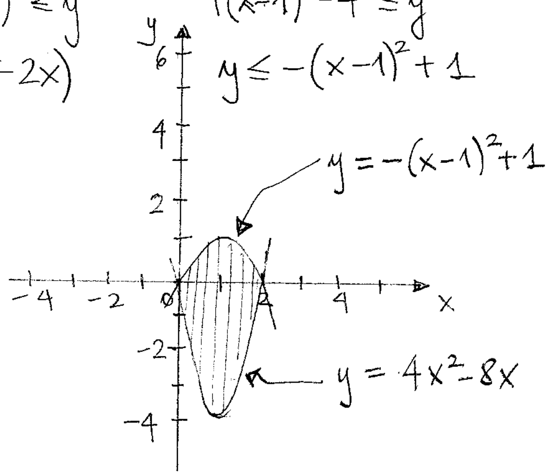
ii) Eq. della retta passante per V e C : $\frac{y-2}{\frac{1}{4}-2} = \frac{x+1}{-\frac{5}{2}+1}$

$\iff 4\frac{(y-2)}{1-8} = \frac{2(x+1)}{-5+2}$

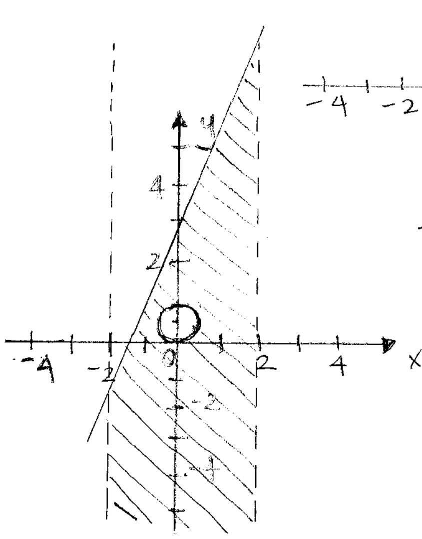
$\iff 4\frac{(y-2)}{-7} = \frac{2(x+1)}{-3}$ $\qquad y = \frac{7}{6}x + \frac{19}{6}$

iii) $P = \left(0, \frac{19}{6}\right)$ è il pt. di intersezione della retta r con l'asse y.

6) $\begin{cases} 4x^2-8x \le y \\ y \le -x^2+2x \end{cases}$ $\qquad \begin{matrix} 4(x^2-2x) \le y \\ y \le -(x^2-2x) \end{matrix}$ $\qquad \begin{matrix} 4(x-1)^2-4 \le y \\ y \le -(x-1)^2+1 \end{matrix}$



$\begin{cases} x^2 < 4 \\ x^2+y^2-y \geq 0 \\ y \leq 2x+3 \end{cases}$



$x^2+y^2-y=0$

$x^2+\left(y-\frac{1}{2}\right)^2 = \frac{1}{4}$