Università degli Studi di Trento – Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia cognitiva
CdL in Interfacce e Tecnologie della Comunicazione – CdL in Filosofia
PRIMA PROVA INTERMEDIA DI ANALISI MATEMATICA (CON ELEMENTI DI ALG.)
a.a. 2011-2012 – Rovereto, 3 novembre 2011

FILA A

a1) $A =]-\infty, 3[$ $\quad B =]2,4[$ $\quad C = \{3\}$ $\quad \underline{\underline{(A \cap B) \cup C =]2,3]}}$.

a2) $A =]4,7[$ $\quad$ i) $4 \in A$ F $\quad$ ii) $5 \subset A$ F

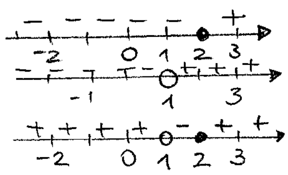
iii) $\{5,6\} \subset A$ V $\quad$ iv) $\phi \in \mathcal{P}(A)$ V.

a3) "C'è almeno uno studente presente all'esame di AM in data odierna che non svolge questo esercizio".

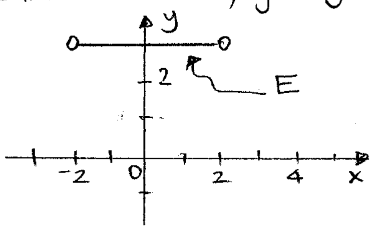
a4) $\dfrac{x}{x-1} < 2 \iff \dfrac{x - 2(x-1)}{x-1} < 0 \iff \dfrac{-x+2}{x-1} < 0 \iff$

$\iff \dfrac{x-2}{x-1} > 0$

$\underline{\underline{S =]-\infty, 1[\cup]2, +\infty[}}$.



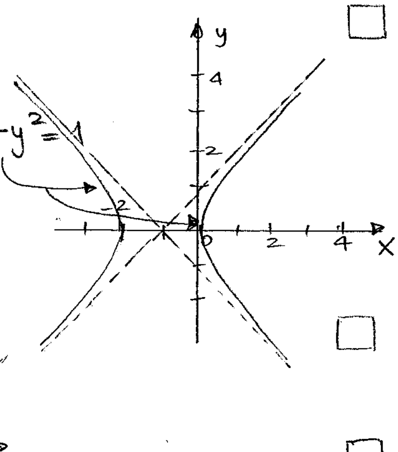
a5) $E = \{(x,y) \in \mathbb{R}^2 : x^2 < 4, y = 3\} = \{(x,y) \in \mathbb{R}^2 : -2 < x < 2, y = 3\}$.



a6) $\begin{cases} x \geq -1 \\ x^2 + 2x < 0 \end{cases}$ $\quad$ Poiché $x^2+2x = x(x+2)$ si ha $x^2+2x=0 \iff x=0$ o $x=-2$

$\iff \begin{cases} x \geq -1 \\ -2 < x < 0 \end{cases} \Rightarrow \underline{\underline{S = [-1, 0[}}$.

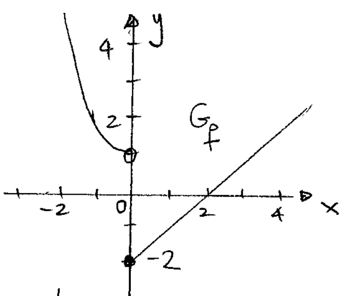
a7) $x^2 - y^2 + 2x = 0 \iff (x+1)^2 - 1 - y^2 = 0 \quad (x+1)^2 - y^2 = 1$



a8) $\underline{y = -2}$.

a9) $f : \mathbb{R} \to \mathbb{R} \quad f(x) = \begin{cases} x^2+1 & x < 0 \\ x-2 & x \geq 0 \end{cases}$

$\underline{\underline{f(\mathbb{R}) = [-2, +\infty[}}$.



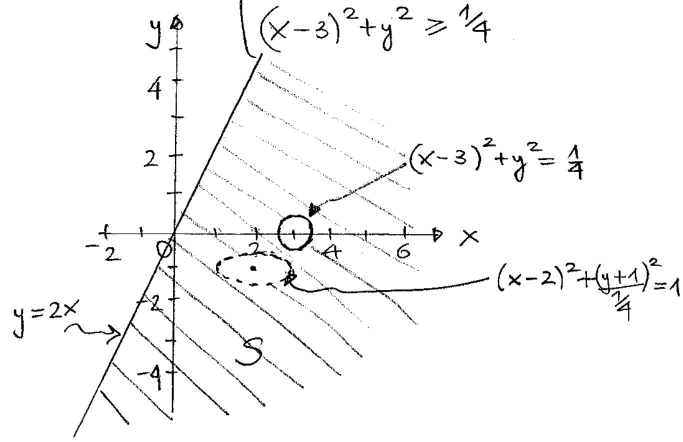
a10) $(fg)(3) = f(3)g(3) = \underline{\underline{2}}$; $\quad \left(\dfrac{g}{f}\right)(1) \nexists$ poiché $f(1) = 0$; $\quad (f-g)(-2) = f(-2) - g(-2) = \underline{\underline{-1}}$.

b1) i) Ricordo che $\text{non}[A \Leftrightarrow B] \Leftrightarrow [A \text{ e } (\text{non } B)] \text{ o } [B \text{ e } (\text{non } A)]$.

Quindi la negazione della frase data diventa: " Lo studente Mario Rossi supera l'esame di AM e non studia con impegno o lo studente Mario Rossi studia con impegno e non supera l'esame di AM."

ii) $\begin{cases} y \le 2x \\ (x-2)^2 - 4 + 4(y^2+2y) + 7 > 0 \\ (x-3)^2 + y^2 \ge 1/4 \end{cases} \Leftrightarrow \begin{cases} y \le 2x \\ (x-2)^2 + 4(y+1)^2 > 1 \\ (x-3)^2 + y^2 \ge 1/4 \end{cases}$

$\Leftrightarrow \begin{cases} y \le 2x \\ (x-2)^2 + \dfrac{(y+1)^2}{1/4} > 1 \\ (x-3)^2 + y^2 \ge 1/4 \end{cases}$



Le rette di equazione $y = 2x + q$ con $q > 0$ non intersecano mai l'insieme S.

b2) i) $P = (2,0)$ $m = -4$ $r: \; y = -4(x-2) \Rightarrow y = -4x + 8$.

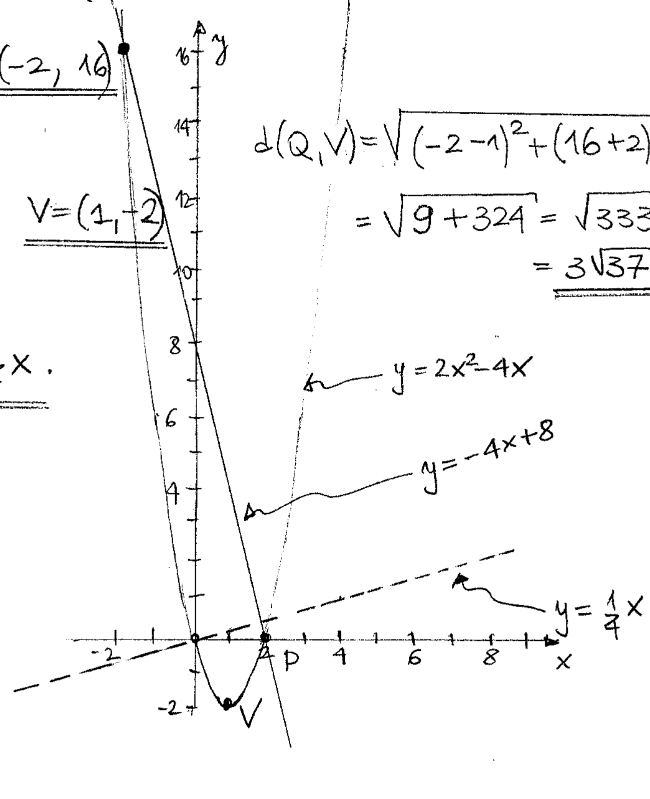
ii) $\begin{cases} y = 2x^2 - 4x \\ y = -4x + 8 \end{cases} \Leftrightarrow \begin{cases} 2x^2 - \cancel{4x} = -\cancel{4x} + 8 \\ y = -4x + 8 \end{cases} \Leftrightarrow \begin{cases} x^2 = 4 \\ y = -4x + 8 \end{cases} \begin{cases} x = \pm 2 \\ y = -4x + 8 \end{cases}$

$P = (2,0)$ $Q = (-2, 16)$

iii) $y = 2(x^2 - 2x)$
$= 2(x-1)^2 - 2$ $V = (1, -2)$

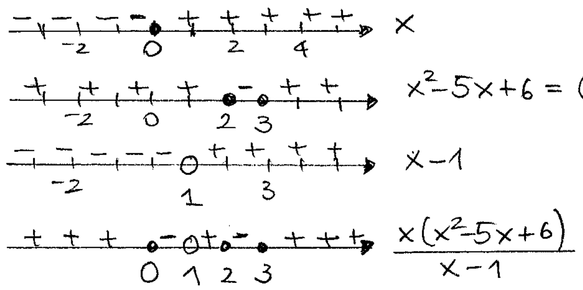
$d(Q,V) = \sqrt{(-2-1)^2 + (16+2)^2} =$
$= \sqrt{9 + 324} = \sqrt{333} =$
$= 3\sqrt{37}$

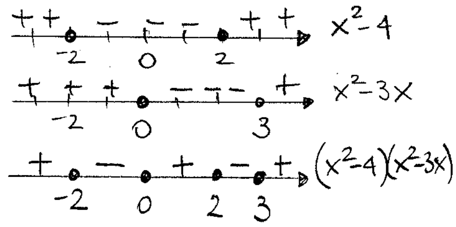
iv) r' ha eq. $y = \frac{1}{4}x$.



b3) $A = \left\{ x \in \mathbb{R} : \dfrac{x(x^2-5x+6)}{x-1} \leq 0 \right\}$ $\qquad B = \left\{ x \in \mathbb{R} : (x^2-4)(x^2-3x) \leq 0 \right\}$.

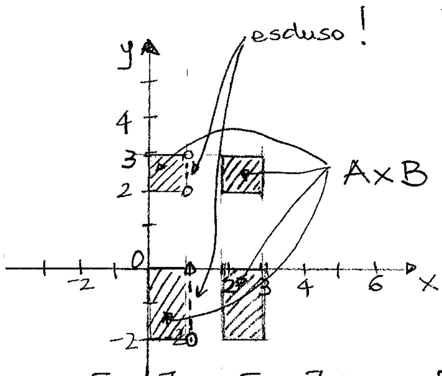
i)

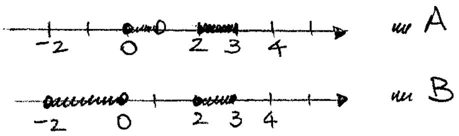




$A = [0,1[\cup [2,3]$ $\qquad B = [-2,0] \cup [2,3]$.

ii)



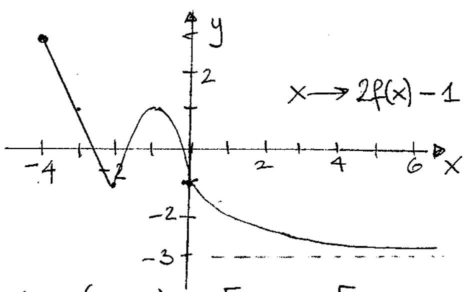


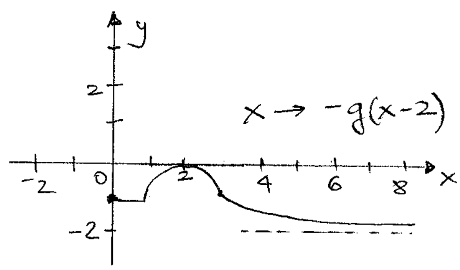
iii) $\mathbb{R} \setminus B =]-\infty,-2[\cup]0,2[\cup]3,+\infty[$.

iv) ⓐ $1 \in B$ (F) $\qquad$ ⓑ B è un intervallo (F) $\qquad$ ⓒ $[-2,0[\subseteq B$ (V)

b4) i) $A = \{ x \in \mathbb{R} : 0 \leq f(x) \leq 1 \} = [-3,0]$: A è limitato ; $\min A = -3$, $\max A = 0$

ii)





$\mathrm{dom}\,(2f-1) = [-4,+\infty[$ $\qquad$ $\mathrm{dom}\,[-g(x-2)] = [0,+\infty[$

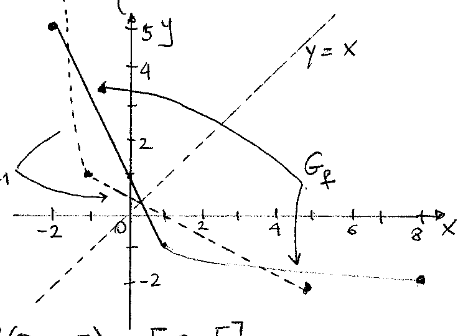
iii) $\min\limits_{[-2,1]} g = 0$ $\qquad$ $x=0$ pt. di minimo.

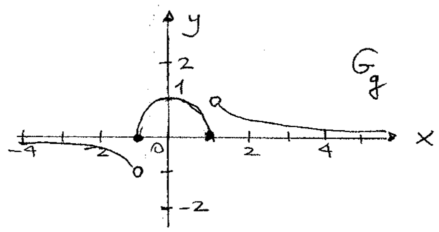
$\max\limits_{[-2,1]} g = 1$ $\qquad$ $x \in [-2,-1] \cup \{1\}$ pt. di massimo.

b5) $f : [-2,8] \to \mathbb{R}$ $\qquad$ $g : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} -2x+1 & \text{se } -2 \le x \le 1 \\ -\sqrt[3]{x} & \text{se } 1 < x \le 8 \end{cases} \qquad g(x) = \begin{cases} -x^4+1 & \text{se } -1 \le x \le 1 \\ \frac{1}{x} & \text{altrimenti.} \end{cases}$$

i)





$f([-2,8]) = [-2,5]$.

ii) $f^{-1} : [-2,5] \to [-2,8]$

iii) Oss. che $g(\mathbb{R}) =]-1,1] \subset \mathrm{dom} f = [-2,8]$, quindi $f \circ g : \mathbb{R} \to \mathbb{R}$.

Inoltre, poiché $g(x) \in]-1,1]$ $\forall x \in \mathbb{R}$ si ha

$$(f \circ g)(x) = \begin{cases} f(-x^2+1) & \text{se } -1 \le x \le 1 \\ f\left(\frac{1}{x}\right) & \text{altrimenti} \end{cases}$$

$$= \begin{cases} -2(-x^2+1)+1 & \text{se } -1 \le x \le 1 \\ -2 \cdot \frac{1}{x} + 1 & \text{altrimenti.} \end{cases}$$

$f(g(x)) = -2g(x)+1$ poiché $g(x) \in]-1,1] \subset [-2,1]$ $\forall x \in \mathbb{R}$.

FILA B

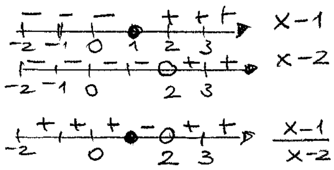
a1) $A =]-\infty, 3[$ $\quad B =]2,4[$ $\quad C = \{3\}$ $\quad \underline{(A \setminus B) \cup C =]-\infty, 2] \cup \{3\}}.$ ☐

a2) $A = [4,7[$ $\quad$ i) $4 \in A$ (V) $\quad$ ii) $5 \subset A$ (F)

iii) $\{5,6\} \subset A$ (V) $\quad$ iv) $\{7\} \in \mathcal{P}(A)$ (F). ☐

a3) "Tutti gli studenti presenti all'esame di AM in data odierna non svolgono questo esercizio." ☐

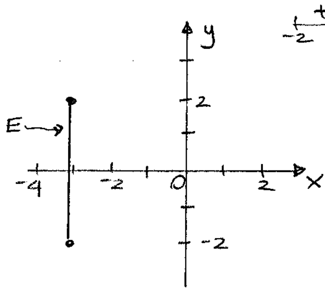
a4) $\frac{x}{x-2} < -1 \iff \frac{x+(x-2)}{x-2} < 0 \iff \frac{2x-2}{x-2} < 0 \iff \frac{x-1}{x-2} < 0$



$\underline{S =]1, 2[}.$ ☐

a5) $E = \{(x,y) \in \mathbb{R}^2 : \ x = -3, \ y^2 \le 4\}$

$= \{(x,y) \in \mathbb{R}^2 : x = -3, \ -2 \le y \le 2\}$



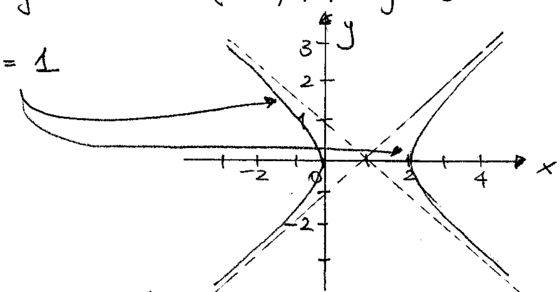
☐

a6) $\begin{cases} x \ge 1 \\ -x^2+2x \ge 0 \end{cases}$ $\quad$ Poiché $-x^2+2x = x(-x+2)$ si ha $-x^2+2x=0 \iff x=0$ o $x=2$

$\iff \begin{cases} x \ge 1 \\ 0 \le x \le 2 \end{cases} \Rightarrow \underline{S = [1,2]}.$ ☐

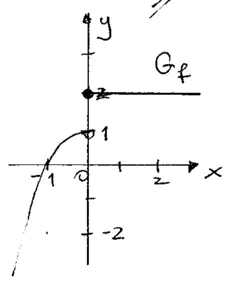
a7) $-x^2+y^2+2x=0 \iff -(x^2-2x)+y^2=0 \quad -(x-1)^2+1+y^2=0$

$\iff (x-1)^2 - y^2 = 1$



a8) $\underline{x = -1}.$ ☐ ☐

a9) $f: \mathbb{R} \to \mathbb{R}$, $\quad f(x) = \begin{cases} -x^2+1 & \text{se } x<0 \\ 2 & \text{se } x \ge 0 \end{cases}$



$\underline{f(\mathbb{R}) =]-\infty, 1[\cup \{2\}}.$ ☐

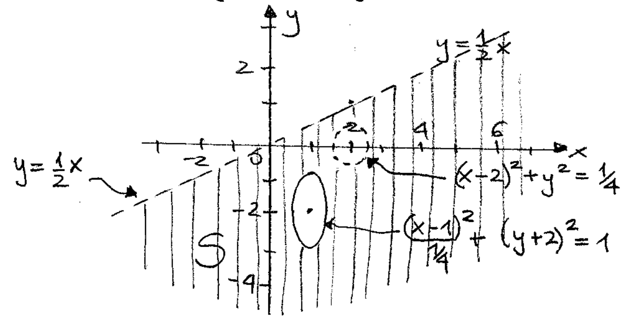
a10) $(fg)(3) = f(3) \cdot g(3) = 2 \cdot 0 = \underline{0}$; $\quad \left(\frac{g}{f}\right)(1) \nexists$ poiché $f(1)=0$; $\quad (f+g)(-2) = f(-2)+g(-2) =$

$\underline{= 1}.$ ■

b1) i) "Lo Stato Italiano supera la crisi e l'Europa non lo sostiene o l'Europa lo sostiene e lo Stato Italiano non supera la crisi".

ii) $\begin{cases} y < \frac{1}{2}x \\ 4(x^2-2x) + (y+2)^2 - 4 + 7 \geq 0 \\ (x-2)^2 + y^2 > \frac{1}{4} \end{cases} \iff \begin{cases} y < \frac{1}{2}x \\ 4(x-1)^2 + (y+2)^2 \geq 1 \\ (x-2)^2 + y^2 > \frac{1}{4} \end{cases}$

$\iff \begin{cases} y < \frac{1}{2}x \\ \frac{(x-1)^2}{1/4} + (y+2)^2 \geq 1 \\ (x-2)^2 + y^2 > \frac{1}{4} \end{cases}$



le rette di eq. $\underline{\underline{y = \frac{1}{2}x + q}}$ con $q \geq 0$ non intersecano mai l'insieme S.

b2) i) $P = (-2, 0)$ $m = 4$ $r: \quad y = 4(x+2) \Rightarrow \underline{\underline{y = 4x + 8}}$

ii) $\begin{cases} y = 2x^2 + 4x \\ y = 4x + 8 \end{cases} \iff \begin{cases} 2x^2 + \cancel{4x} = \cancel{4x} + 8 \\ y = 4x + 8 \end{cases} \iff \begin{cases} x^2 = 4 \\ y = 4x + 8 \end{cases} \iff \begin{cases} x = \pm 2 \\ y = 4x + 8 \end{cases}$

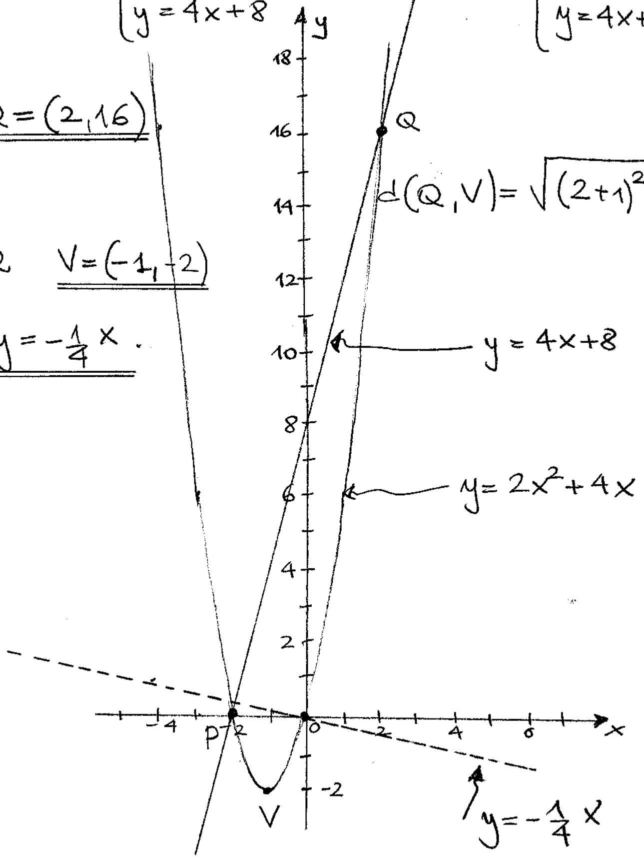
$P = (-2, 0)$ $\underline{\underline{Q = (2, 16)}}$

iii) $y = 2(x^2 + 2x)$

$= 2(x+1)^2 - 2$ $\underline{\underline{V = (-1, -2)}}$

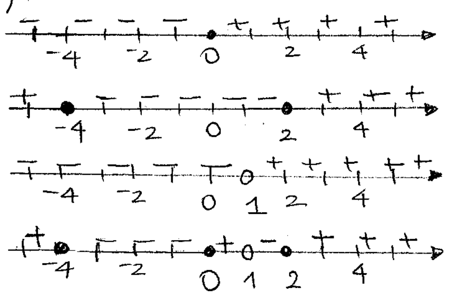
$d(Q, V) = \sqrt{(2+1)^2 + (16+2)^2} = \sqrt{333}$
$= \underline{\underline{3\sqrt{37}}}$

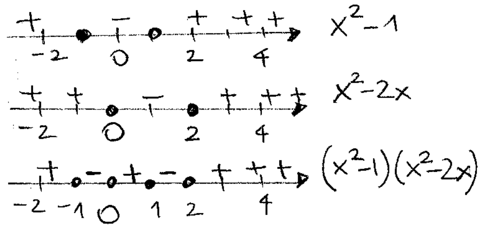
iv) r' ha eq $\underline{\underline{y = -\frac{1}{4}x}}$.



b3) $A = \left\{ x \in \mathbb{R} : \dfrac{x(x^2+2x-8)}{x-1} \leq 0 \right\}$ $B = \left\{ x \in \mathbb{R} : (x^2-1)(x^2-2x) \leq 0 \right\}$.

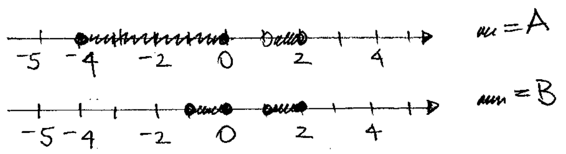
i)



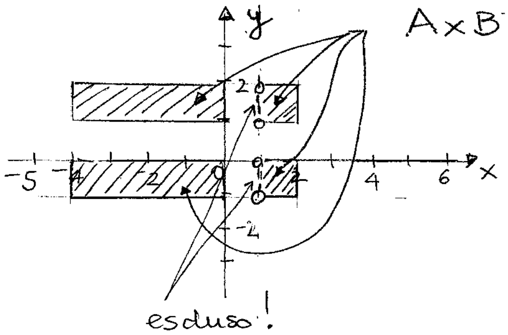


$A = [-4,0] \cup]1,2]$

$B = [-1,0] \cup [1,2]$.



ii)



iii) $\mathbb{R} \setminus B =]-\infty,-1[\cup]0,1[\cup]2,+\infty[$.

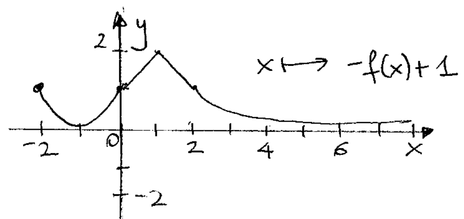
iv) ⓐ $1 \in A$ (F) ⓑ A è un intervallo (F) ⓒ $[-2,0] \subseteq A$ (V)

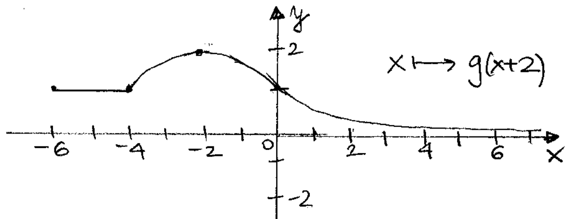
b4) i) $A = \{ x \in \mathbb{R} : 0 \leq f(x) \leq 1 \} = [-2,0] \cup [2,+\infty[$.

A è limitato inferiormente, ma non è limitato superiormente.

$\min A = -2$, $\nexists \max A$.

ii)





$\operatorname{dom}(-f+1) = [-2,+\infty[$ $\operatorname{dom}(g(x+2)) = [-6,+\infty[$

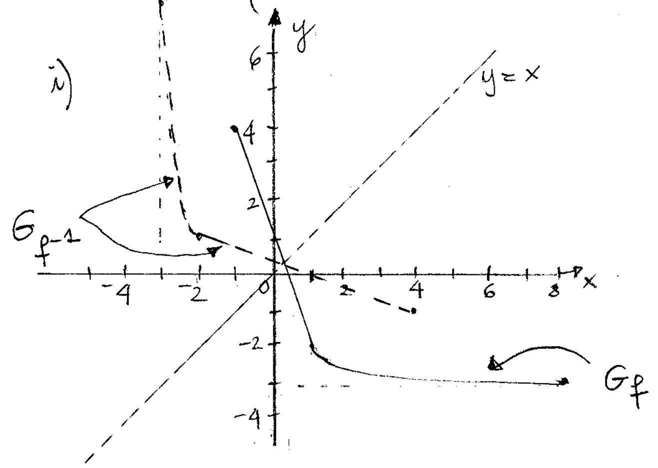
iii) $\min\limits_{[-4,1]} g = 1$ $x \in [-4,-2]$ pt. di minimo.

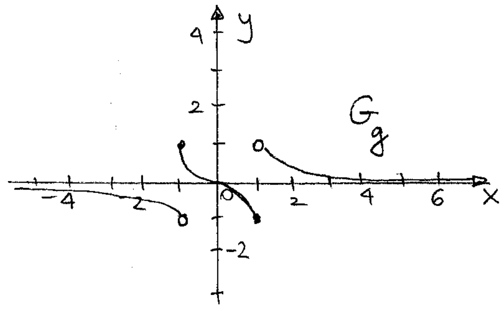
$\max\limits_{[-4,1]} g = 2$ $x = 0$ pt. di massimo.

b5) $f : [-1,8] \to \mathbb{R}$ $\qquad$ $g : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} -3x+1 & \text{se } -1 \le x \le 1 \\ -\sqrt[3]{x}-1 & \text{se } 1 < x \le 8 \end{cases} \qquad g(x) = \begin{cases} -x^3 & \text{se } -1 \le x \le 1 \\ \frac{1}{x} & \text{altrimenti} \end{cases}$$

i)





$$\underline{\underline{f([-1,8]) = [-3,4].}}$$

ii) $f^{-1} : [-3,4] \to [-1,8]$

iii) Oss. che $g(\mathbb{R}) = [-1,1] \subset \operatorname{dom} f = [-1,8]$, quindi $f \circ g : \mathbb{R} \to \mathbb{R}$.

Inoltre, poiché $g(x) \in [-1,1]$ $\forall x \in \mathbb{R}$, si ha

$$(f \circ g)(x) = \begin{cases} f(-x^3) & \text{se } -1 \le x \le 1 \\ f\left(\frac{1}{x}\right) & \text{altrimenti} \end{cases}$$

$$= \begin{cases} -3(-x^3)+1 & \text{se } -1 \le x \le 1 \\ -3 \cdot \frac{1}{x} + 1 & \text{altrimenti} \end{cases}$$

$f(g(x)) = -3\,g(x)+1$ poiché $g(x) \in [-1,1]$ $\forall x \in \mathbb{R}$. ■

FILA C

a1) $A =]-\infty, 4[\qquad B =]2,5[\qquad C = \{3\} \qquad (B \setminus A) \cup C = \{3\} \cup [4,5[.$

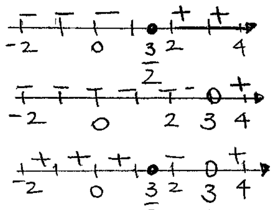
a2) $A = [4,6[\qquad$ i) $4 \in A$ V $\qquad$ ii) $5 \subset A$ F

iii) $\{5,6\} \subset A$ F $\qquad$ iv) $[4,5[\in \mathcal{P}(A)$ V.

a3) "Tutti gli studenti presenti all'esame di AM in data odierna svolgono questo esercizio."

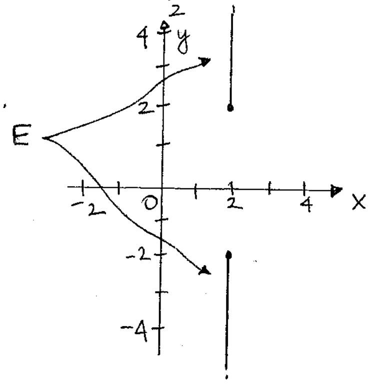
a4) $\dfrac{x}{x-3} < -1 \iff \dfrac{x + (x-3)}{x-3} < 0 \iff \dfrac{2x-3}{x-3} < 0$

$S =]3/2, 3[\,.$



a5) $E = \{(x,y) \in \mathbb{R}^2 : \ x=2,\ y^2 \geq 4\} =$

$= \{(x,y) \in \mathbb{R}^2 : \ x = 2,\ y \leq -2 \text{ o } y \geq 2\}.$



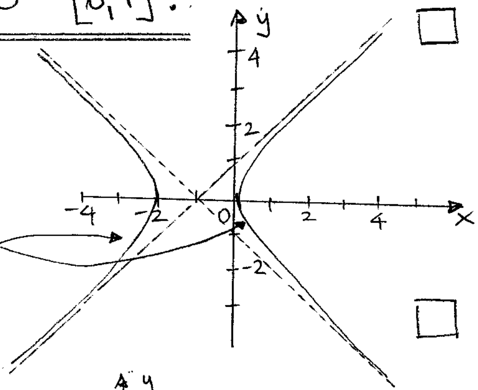
a6) $\begin{cases} x \leq 1 \\ -x^2+2x \geq 0 \end{cases} \iff \begin{cases} x \leq 1 \\ x^2-2x \leq 0 \end{cases} \iff$

$\iff \begin{cases} x \leq 1 \\ 0 \leq x \leq 2 \end{cases} \Rightarrow S = [0,1].$

a7) $-x^2+y^2-2x=0 \iff -(x^2+2x) + y^2 = 0$

$\iff -(x+1)^2 + 1 + y^2 = 0$

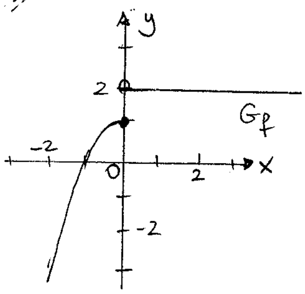
$\iff (x+1)^2 - y^2 = 1$



a8) $x = -2$

a9) $f: \mathbb{R} \to \mathbb{R} \qquad f(x) = \begin{cases} -x^2+1 & x \leq 0 \\ 2 & x > 0 \end{cases}$

$f(\mathbb{R}) =]-\infty, 1] \cup \{2\}.$

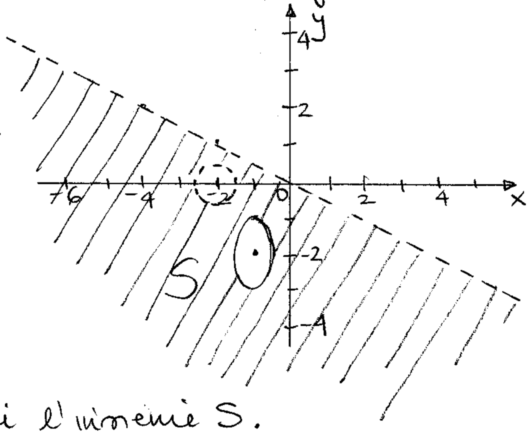


a10) $(fg)(1) = f(1)g(1) = 1\,; \qquad \left(\dfrac{g}{f}\right)(0) = \dfrac{g(0)}{f(0)} = 1 \qquad (f+g)(-2) = f(-2) + g(-2) = 2.$

b1) i) "Lo Stato Italiano supera la crisi e l'Europa non lo sostiene o l'Europa lo sostiene e lo Stato Italiano non supera la crisi."

ii) $\begin{cases} y < -\frac{1}{2}x \\ 4(x^2+2x) + (y^2+4y) + 7 \geq 0 \\ (x+2)^2 + y^2 > 1/4 \end{cases} \iff \begin{cases} y < -\frac{1}{2}x \\ 4(x+1)^2 - 4 + (y+2)^2 - 4 + 7 \geq 0 \\ (x+2)^2 + y^2 > 1/4 \end{cases}$

$\iff \begin{cases} y < -\frac{1}{2}x \\ \frac{(x+1)^2}{1/4} + (y+2)^2 \geq 1 \\ (x+2)^2 + y^2 > 1/4 \end{cases}$



Le rette di eq. $y = -\frac{1}{2}x + q$ con $q \geq 0$ non intersecano mai l'insieme S.

b2) i) $P = (-2,0)$ $\quad m = 1$ $\quad r: \; y = x+2$

ii) $\begin{cases} y = 2x^2+4x \\ y = x+2 \end{cases} \iff \begin{cases} 2x^2+4x = x+2 \\ y = x+2 \end{cases} \iff \begin{cases} 2x^2+3x-2 = 0 \\ y = x+2 \end{cases}$

$\iff \begin{cases} x_{1/2} = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4} \begin{matrix} -2 \\ 1/2 \end{matrix} \\ y = x+2 \end{cases}$

$P = (-2,0)$ $\quad Q = \left(\frac{1}{2}, \frac{5}{2}\right)$.

iii) $y = 2x^2+4x$

$= 2(x^2+2x) = 2(x+1)^2 - 2$

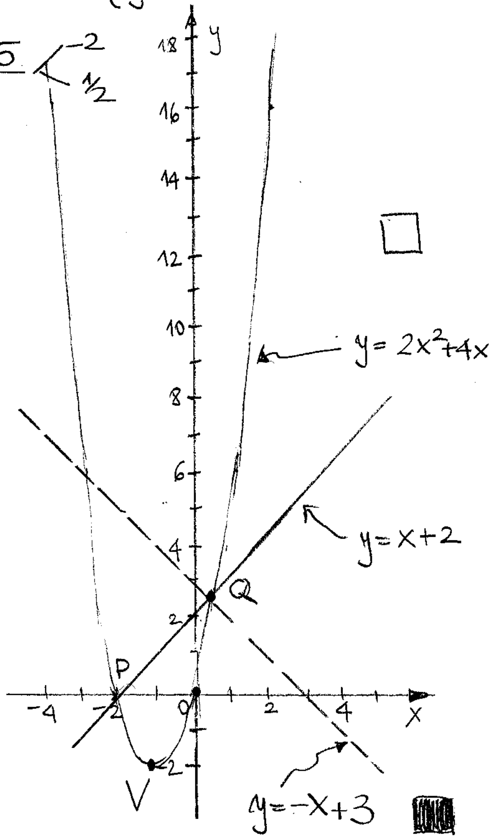
$V = (-1,-2)$

$d(V,Q) = \sqrt{\left(\frac{1}{2}+1\right)^2 + \left(\frac{5}{2}+2\right)^2}$

$= \sqrt{\frac{9}{4} + \frac{81}{4}} = \sqrt{\frac{90}{4}} = \frac{3\sqrt{10}}{2}$

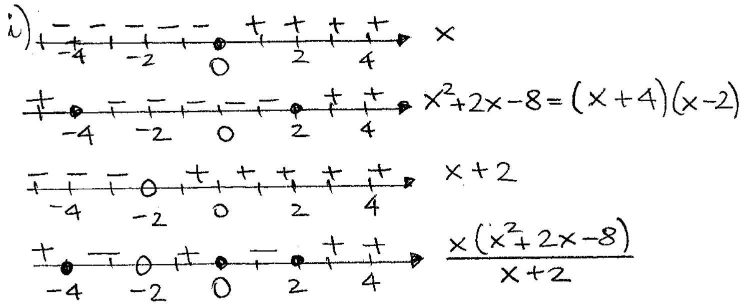
iv) r' ha eq. $y = \frac{5}{2} - \left(x - \frac{1}{2}\right)$

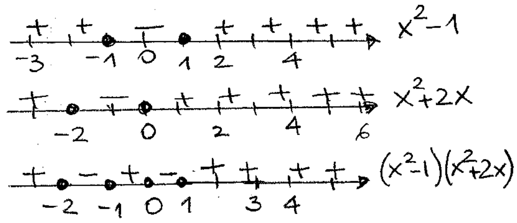
$y = -x + 3$



b3) $A=\left\{x\in\mathbb{R}:\ \dfrac{x(x^2+2x-8)}{x+2}\leq 0\right\}$ $\qquad$ $B=\{x\in\mathbb{R}:\ (x^2-1)(x^2+2x)\leq 0\}$.

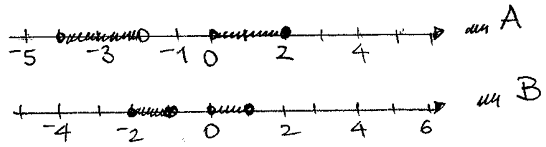
i)





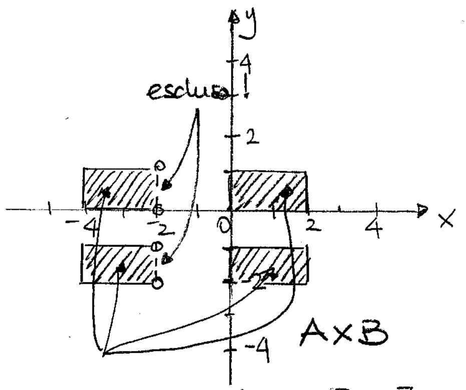
$A=[-4,-2[\ \cup\ [0,2]$

$B=[-2,-1]\cup[0,1]$.



☐

ii)



☐

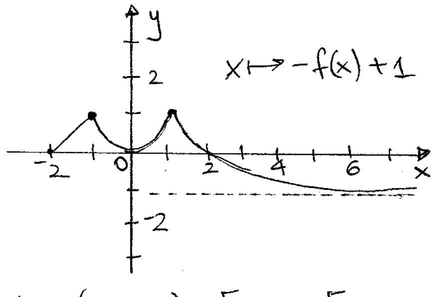
iii) $\mathbb{R}\setminus B=\ ]-\infty,-2[\ \cup\ ]-1,0[\ \cup\ ]1,+\infty[$ ☐

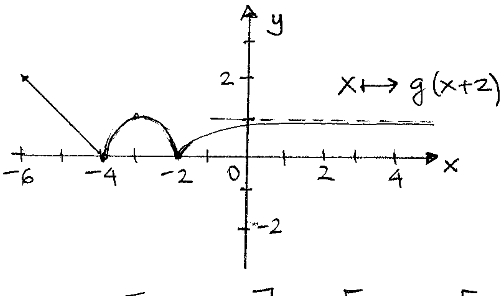
iv) ⓐ $-2\in A$ (F) $\qquad$ ⓑ A non è un intervallo (V) $\qquad$ ⓒ $[-2,0[\ \subseteq \mathbb{R}\setminus A$ (V)

b4) i) $A=\{x\in\mathbb{R}:\ 0\leq f(x)\leq 1\}=[-2,2]$: A è limitato; $\min A=-2$; $\max A=2$.

☐

ii)





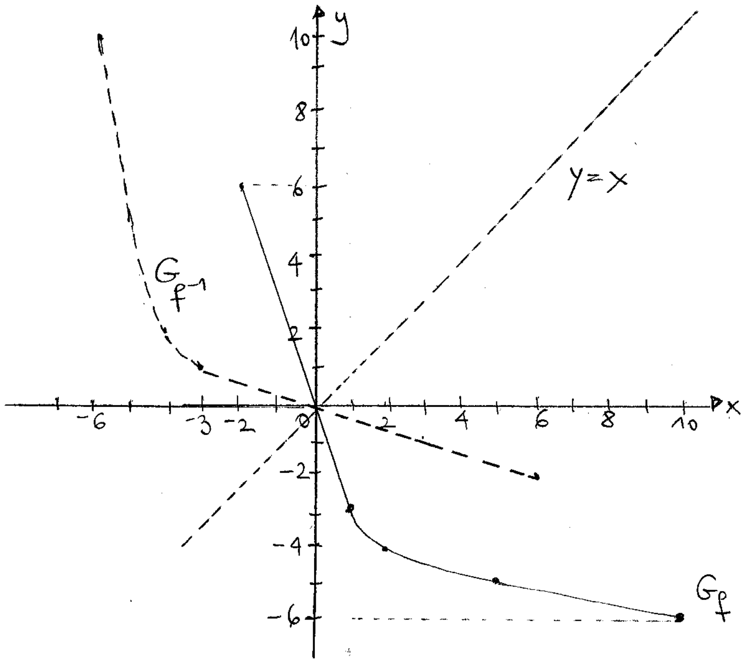
$\mathrm{dom}(-f+1)=[-2,+\infty[$ $\qquad$ $\mathrm{dom}\,[g(x+2)]=[-6,+\infty[$. ☐

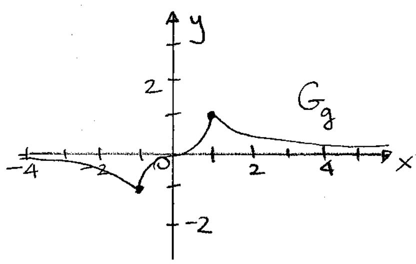
iii) $\min\limits_{[-4,1]} g=0$ $\qquad$ $x\in\{-2,0\}$ sono pt. di minimo.

$\max\limits_{[-4,1]} g=2$ $\qquad$ $x=-4$ pt. di massimo.

b5) $f : [-2,10] \to \mathbb{R}$ $\qquad\qquad$ $g : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} -3x & \text{se } -2 \le x \le 1 \\ -\sqrt{x-1} - 3 & \text{se } 1 < x \le 10 \end{cases} \qquad g(x) = \begin{cases} x^3 & \text{se } -1 \le x \le 1 \\ \frac{1}{x} & \text{altrimenti.} \end{cases}$$





$$\underline{\underline{f([-2,10]) = [-6,6]}}$$

ii) $f^{-1} : [-6,6] \to [-2,10]$

iii) Oss. che $g(\mathbb{R}) = [-1,1] \subset \operatorname{dom} f = [-2,10]$, quindi $f \circ g : \mathbb{R} \to \mathbb{R}$

Inoltre, poiché $g(x) \in [-1,1]$ $\forall x \in \mathbb{R}$ si ha

$$(f \circ g)(x) = -3\, g(x) = \begin{cases} -3x^3 & \text{se } -1 \le x \le 1 \\ -\frac{3}{x} & \text{altrimenti} \end{cases}$$

FILA (D)

a1) $A =]-2,3[\qquad B = [2,+\infty[\qquad C = \{3\} \qquad (A \cap B) \cup C = [2,3].$

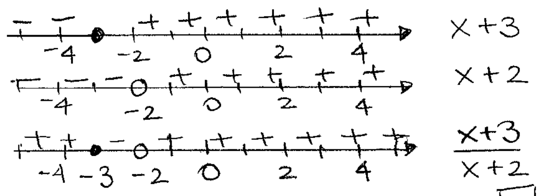
a2) $A = [4,9[\qquad$ i) $4 \in A$ (V) $\qquad$ ii) $\{5\} \subset \mathcal{P}(A)$ (F)

iii) $\{5\} \subset A$ (V) $\qquad$ iv) $\phi \in A$ (F).

a3) "C'è almeno uno studente presente all'esame di AM in data odierna che non svolge la prima parte del compito".

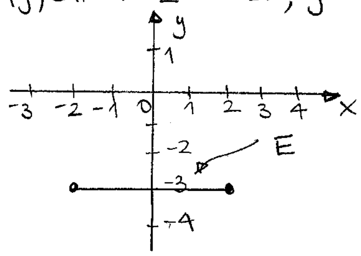
a4) $\dfrac{x}{x+2} > 3 \iff \dfrac{x-3(x+2)}{x+2} > 0 \iff \dfrac{-2x-6}{x+2} > 0 \iff$

$\iff \dfrac{x+3}{x+2} < 0$



$S =]-3,-2[$.

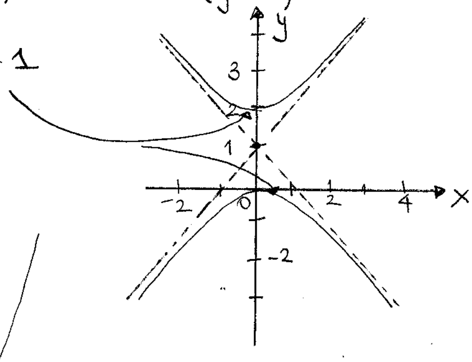
a5) $E = \{(x,y) \in \mathbb{R}^2 : \ x^2 \leq 4, \ y = -3\} = \{(x,y) \in \mathbb{R}^2 : \ -2 \leq x \leq 2, \ y = -3\}.$



a6) $\begin{cases} x \leq -1 \\ x^2 + 2x < 0 \end{cases} \iff \begin{cases} x \leq -1 \\ -2 < x < 0 \end{cases} \implies S =]-2,-1].$

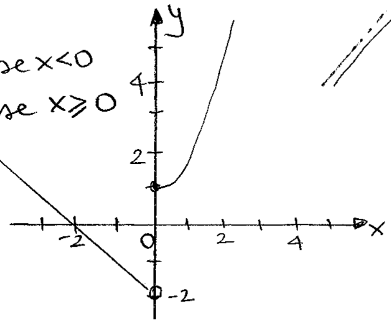
a7) $x^2 - y^2 + 2y = 0 \iff x^2 - (y^2 - 2y) = 0 \qquad x^2 - (y-1)^2 + 1 = 0$

$\iff -x^2 + (y-1)^2 = 1$



a8) $y = 3$.

a9) $f : \mathbb{R} \to \mathbb{R} \qquad f(x) = \begin{cases} -x-2 & \text{se } x < 0 \\ x^2 + 1 & \text{se } x \geq 0 \end{cases}$

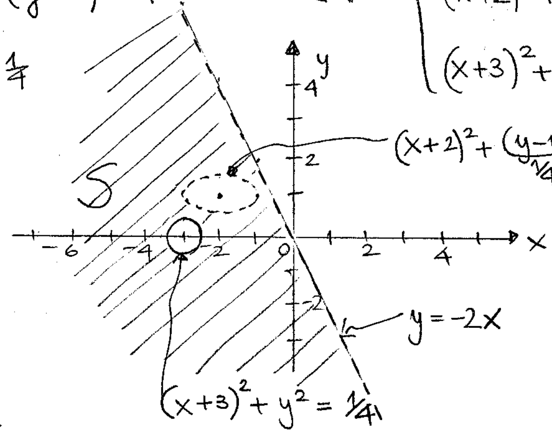


$f(\mathbb{R}) =]-2,+\infty[$

a10) $(fg)(1) = f(1)g(1) = \underline{0} \quad ; \quad \left(\dfrac{f}{g}\right)(0) = \dfrac{f(0)}{g(0)} = \underline{-1} \quad ; \quad (f-g)(3) = f(3) - g(3) = \underline{-1}.$

b1) i) "Lo studente Tizio Caio supera l'esame di AM e non frequenta con serietà le lezioni o lo studente Tizio Caio frequenta con serietà le lezioni e non supera l'esame di AM."

ii) $\begin{cases} y < -2x \\ (x+2)^2 - 4 + 4(y-1)^2 - 4 + 7 > 0 \\ (x+3)^2 + y^2 \geq \frac{1}{4} \end{cases} \iff \begin{cases} y < -2x \\ (x+2)^2 + \frac{(y-1)^2}{1/4} > 1 \\ (x+3)^2 + y^2 \geq \frac{1}{4} \end{cases}$



Le rette di equazione

$\underline{\underline{y = -2x + q \text{ con } q \geq 0}}$ non intersecano mai l'insieme S.

b2) i) $P = (2,0)$ $\quad m = -1$ $\quad r: y = -(x-2) \Rightarrow \underline{\underline{y = -x + 2}}.$

ii) $\begin{cases} y = 2x^2 - 4x \\ y = -x + 2 \end{cases} \iff \begin{cases} 2x^2 - 4x = -3x + 2 \\ y = -x + 2 \end{cases} \iff \begin{cases} 2x^2 - 3x - 2 = 0 \\ y = -x + 2 \end{cases}$

$\iff \begin{cases} x_{1/2} = \frac{3 \pm \sqrt{9+16}}{4} \\ y = -x + 2 \end{cases} \iff \begin{cases} x_{1/2} = \frac{3 \pm 5}{4} \begin{smallmatrix} \nearrow -\frac{1}{2} \\ \searrow 2 \end{smallmatrix} \\ y = -x + 2 \end{cases}$

$P = (2,0) \quad \underline{\underline{Q = \left(-\frac{1}{2}, \frac{5}{2}\right)}}$

iii) $y = 2(x^2 - 2x)$

$= 2(x-1)^2 - 2$

$V = (1, -2)$

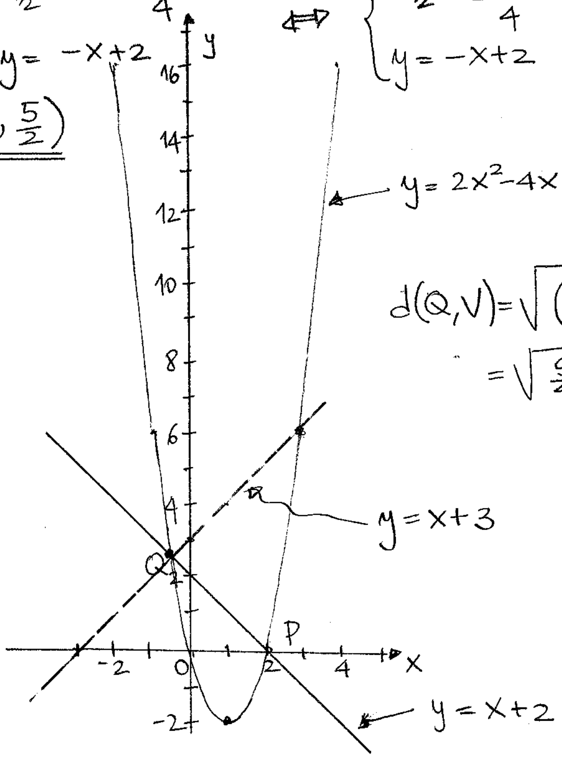
$d(Q,V) = \sqrt{\left(-\frac{1}{2} - 1\right)^2 + \left(\frac{5}{2} + 2\right)^2}$

$= \sqrt{\frac{9}{4} + \frac{81}{4}} = \sqrt{\frac{90}{4}} = \underline{\underline{\frac{3\sqrt{10}}{2}}}$

iv) r' ha equazione

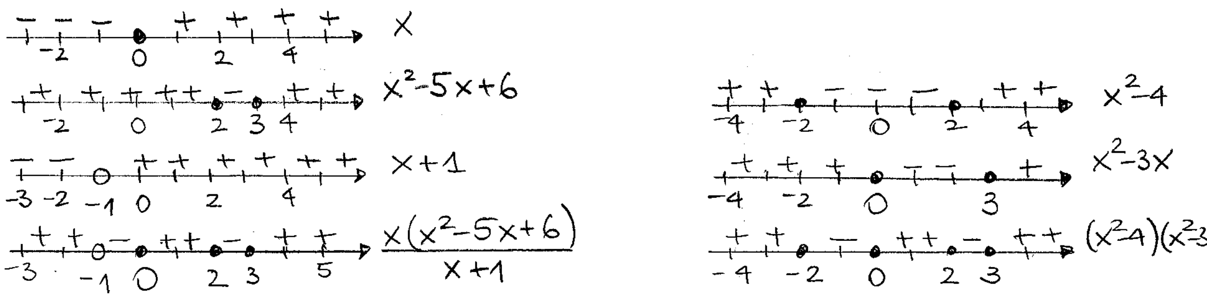
$y = \frac{5}{2} + \left(x + \frac{1}{2}\right)$

$\underline{\underline{y = x + 3}}.$

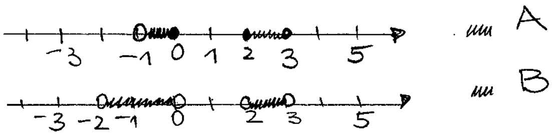


b3) $A = \left\{ x \in \mathbb{R} : \dfrac{x(x^2-5x+6)}{x+1} \le 0 \right\}$ $\qquad$ $B = \left\{ x \in \mathbb{R} : (x^2-4)(x^2-3x) < 0 \right\}$

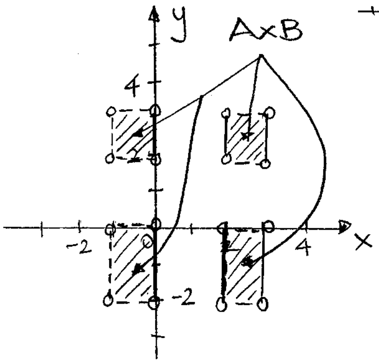
i)



$\underline{\underline{A = \left]-1, 0\right] \cup [2, 3]}}$ $\qquad$ $\underline{\underline{B = \left]-2, 0\right[\cup \left]2, 3\right[.}}$



ii)



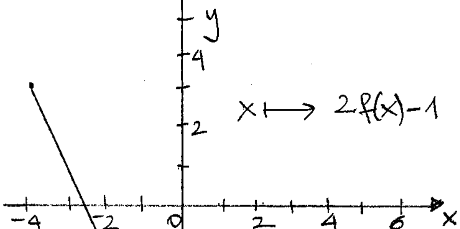
iii) $\underline{\underline{\mathbb{R} \setminus A = \left]-\infty, -1\right] \cup \left]0, 2\right[\cup \left]3, +\infty\right[.}}$

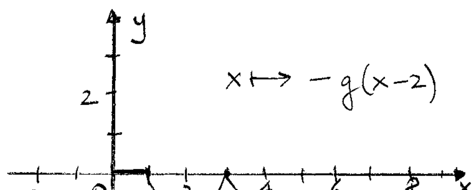
iv) ⓐ $3 \in B$ Ⓕ $\qquad$ ⓑ B non è un intervallo Ⓥ $\qquad$ ⓒ $\left]-2, 0\right[\subseteq B$ Ⓥ

b4) i) $A = \{ x \in \mathbb{R} : 0 \le f(x) \le 1 \} = \underline{\underline{[-3, -2] \cup [0, 1]}}$: A è limitato;

$\underline{\underline{\min A = -3}}$, $\underline{\underline{\max A = 1}}$.

ii)





$\mathrm{dom}\,(2f-1) = \left[-4, +\infty\right[$ $\qquad$ $\mathrm{dom}\left[-g(x-2)\right] = \left[0, +\infty\right[.$

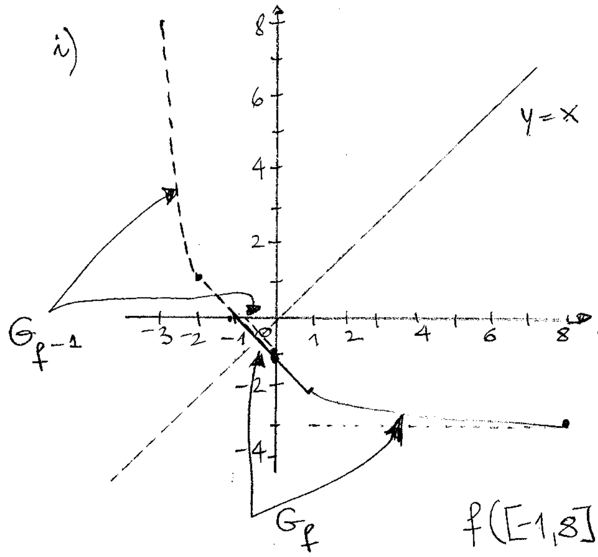
iii) $\min\limits_{[-2,1]} g = \underline{\underline{0}}$ $\qquad$ $\underline{\underline{x \in [-2, -1] \cup \{1\} \text{ pt. di minimo.}}}$

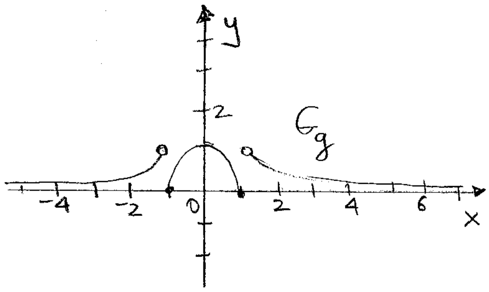
$\max\limits_{[-2,1]} g = \underline{\underline{1}}$ $\qquad$ $\underline{\underline{x = 0 \text{ pt. di massimo.}}}$

b5) $f: [-1,8] \to \mathbb{R}$, $g: \mathbb{R} \to \mathbb{R}$,

$$f(x) = \begin{cases} -x-1 & \text{se } -1 \le x \le 1 \\ -\sqrt[3]{x}-1 & \text{se } 1 < x \le 8 \end{cases} \qquad g(x) = \begin{cases} -x^4+1 & \text{se } -1 \le x \le 1 \\ \frac{1}{x^2} & \text{altrimenti} \end{cases}$$

i)





$$f([-1,8]) = [-3,0]$$ $\square$

ii) $f^{-1}: [-3,0] \to [-1,8]$ $\square$

iii) Oss. che $g(\mathbb{R}) = [0,1] \subset \operatorname{dom} f = [-1,8]$, quindi $f \circ g: \mathbb{R} \to \mathbb{R}$.
Inoltre, poiché $g(x) \in [0,1]$ $\forall x \in \mathbb{R}$ si ha

$$(f \circ g)(x) = -g(x) - 1$$

$$= \begin{cases} -(-x^4+1)-1 & \text{se } -1 \le x \le 1 \\ -\frac{1}{x^2}-1 & \text{altrimenti} \end{cases}$$

$$= \begin{cases} x^4-2 & \text{se } -1 \le x \le 1 \\ -\frac{1}{x^2}-1 & \text{altrimenti.} \end{cases}$$ $\blacksquare$