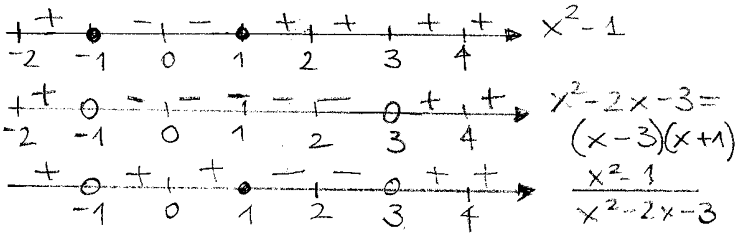
Università degli Studi di Trento – Facoltà di Scienze Cognitive
Corso di Laurea in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione – CdL in Filosofia
VERIFICA SETTIMANALE DI ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2012 – 2013 – Rovereto, 1–5 ottobre 2012

1) i) $A = \left\{x \in \mathbb{R} : \dfrac{x^2-1}{x^2-2x-3} < 0\right\}$

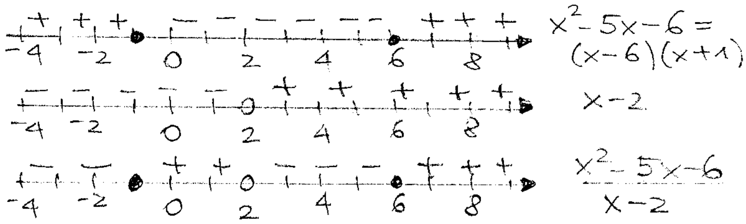
$=]1, 3[$



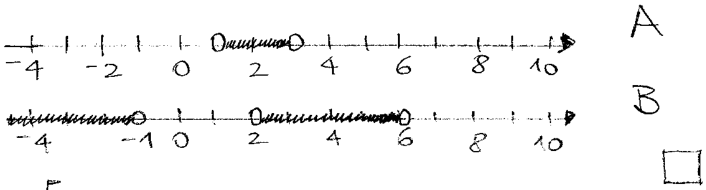
$B = \left\{x \in \mathbb{R} : \dfrac{-x^2+5x+6}{x-2} > 0\right\}$

$= \left\{x \in \mathbb{R} : \dfrac{x^2-5x-6}{x-2} < 0\right\}$

$=]-\infty, -1[\cup]2, 6[$

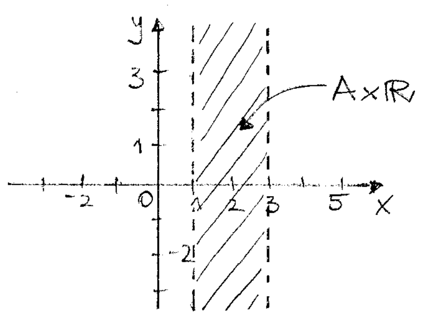


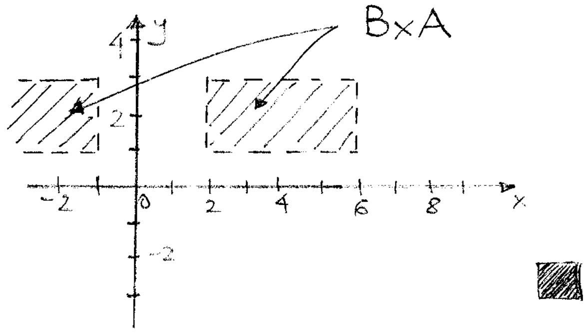
A è un intervallo, B non è un intervallo.



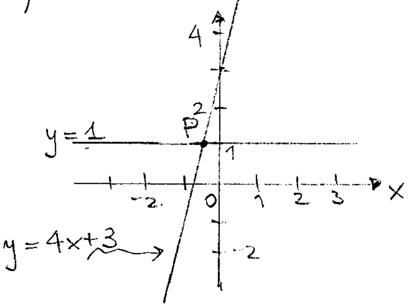
ii) $A \cup B =]-\infty, -1[\cup]1, 6[$;

$A \cap B =]2, 3[$; $A \setminus B =]1, 2]$;



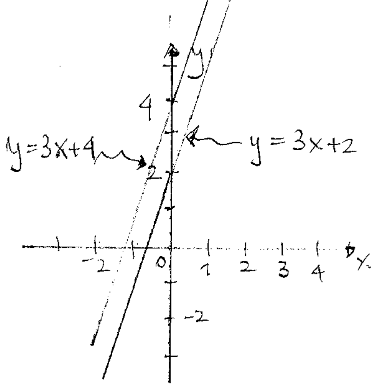


2) i) • $4x+3 = 1$ $\rightsquigarrow$ $4x = -2$ $x = -\frac{1}{2}$ $\Rightarrow$ $S = \left\{-\frac{1}{2}\right\}$.



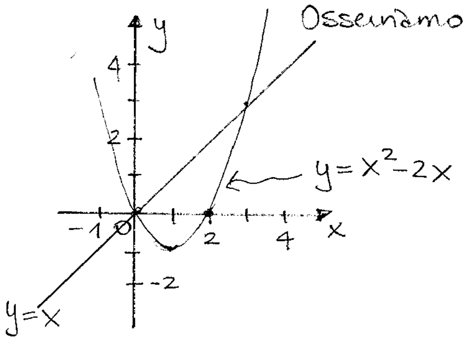
Interpret. geom.: Abbiamo trovato la coordinata x del punto di intersezione P delle due rette di eq. $y = 4x+3$ e $y = 1$.

• $3x+2 > 3x+4 \iff 2>4 \Rightarrow \underline{\underline{S=\emptyset}}$.



Interpret. geom.: La retta r di eq. $y=3x+2$ è parallela alla retta r' di eq. $y=3x+4$. Inoltre, la retta $y=3x+2$ "sta sempre sotto" la retta di eq. $y=3x+4$, e quindi la diseq. data non è verificata per nessun x.

• $x^2-2x \le x \iff x^2-3x\le 0 \Rightarrow \underline{\underline{S=[0,3]}}$.



Osserviamo che $y=x^2-2x \iff$

$y=(x-1)^2-1 \qquad V=(1,-1)$

Interpret. geom.: la soluzione corrisponde alle coordinate x dei pt. (x,y) appartenenti alla parabola di eq. $y=x^2-2x$, per i quali $y=x^2-2x$ è minore o uguale di $y=x$, che intuitivamente significa le x per cui la parabola "sta sotto" la retta.

□

ii) • $\dfrac{x(x^2-3x)}{x^2+1} > x \iff \dfrac{x(x^2-3x)-x(x^2+1)}{x^2+1}>0$

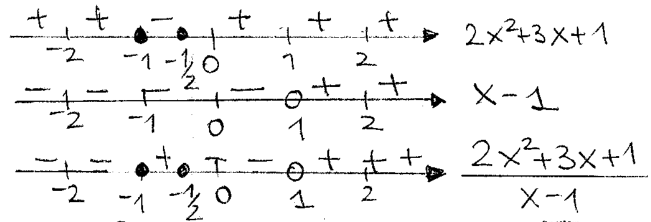
$\iff \dfrac{-3x^2-x}{x^2+1}>0 \iff \dfrac{3x^2+x}{x^2+1}<0$.

Osserviamo che $x^2+1>0 \ \forall x\in\mathbb{R}$, quindi basta determinare gli $x\in\mathbb{R}$ tali che $3x^2+x<0$. Si ha $\underline{\underline{S=\left]-\frac{1}{3},0\right[}}$.

• $\dfrac{x^2+5x}{x-1} > 1-x \iff \dfrac{x^2+5x}{x-1}+x-1>0$

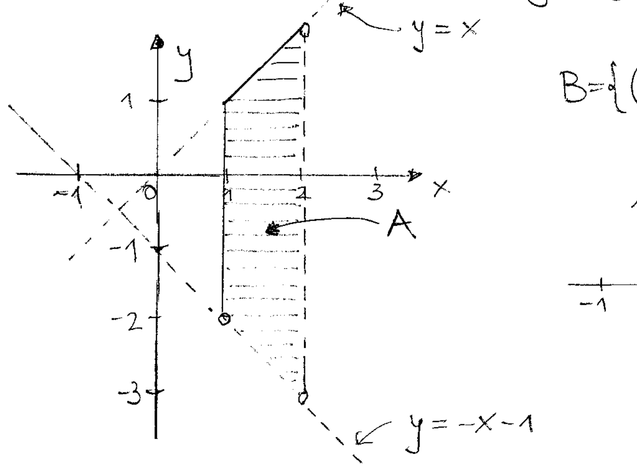
$\iff \dfrac{x^2+5x+x^2-2x+1}{x-1}>0 \iff \dfrac{2x^2+3x+1}{x-1}>0$

Ora $2x^2+3x+1=0 \iff x_{1/2}=\dfrac{-3\pm\sqrt{9-8}}{4}=\dfrac{-3\pm1}{4} \begin{cases} -1 \\ -\frac{1}{2}\end{cases}$

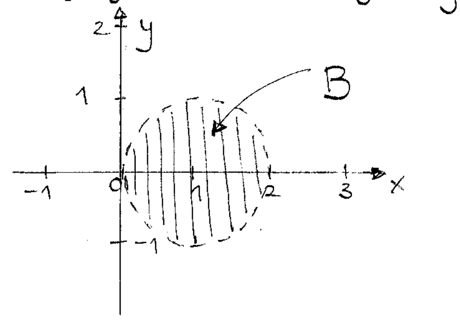


$$S = \left]-1, -\tfrac{1}{2}\right[\cup \left]1, +\infty\right[.$$

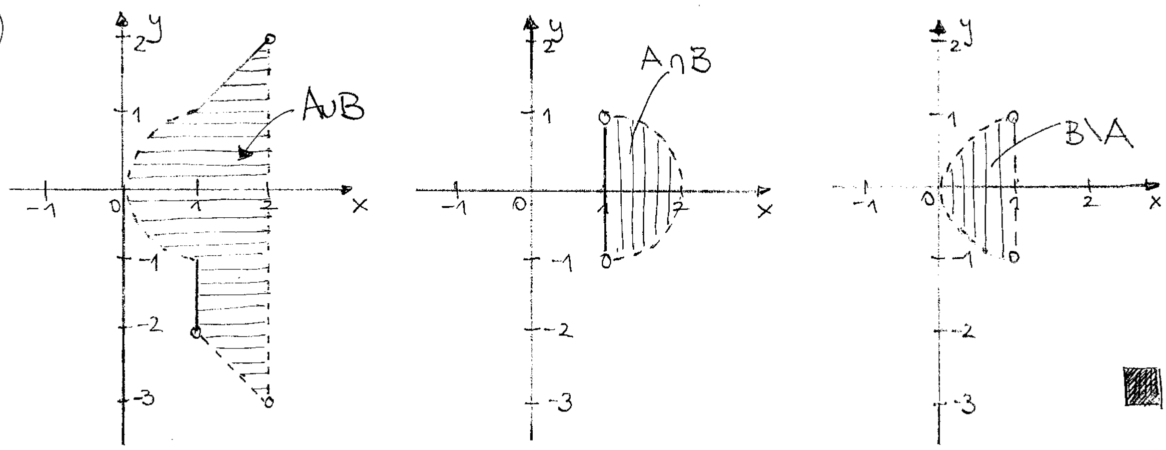
3) i) $A = \{(x,y) \in \mathbb{R}^2 : 1 \leq x < 2 ,\ -x-1 < y \leq x\}$



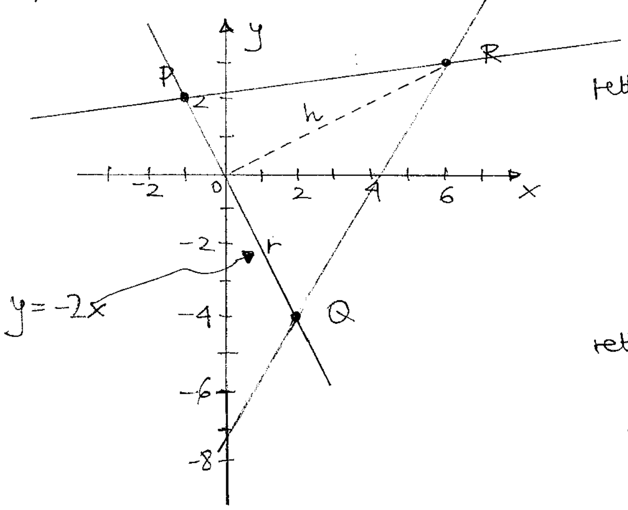
$B = \{(x,y) \in \mathbb{R}^2 : (x-1)^2 + y^2 < 1\}$



ii)



4) $P = (-1, 2) \quad Q = (2, -4) \quad R = (6, 3)$



retta pass. per P e Q :

$$\frac{y-2}{-4-2} = \frac{x+1}{2+1} \iff$$

$$y - 2 = -2(x+1) \Rightarrow \underline{y = -2x}$$

retta passante per P e R :

$$\frac{y-2}{3-2} = \frac{x+1}{6+1} \iff$$

$$y - 2 = \frac{1}{7}(x+1) \Rightarrow \underline{y = \frac{1}{7}x + \frac{15}{7}}$$

retta passante per Q e R :

$$\frac{y+4}{3+4} = \frac{x-2}{6-2} \iff y + 4 = \frac{7}{4}(x-2)$$

$$\Rightarrow \underline{y = \frac{7}{4}x - \frac{15}{2}}$$

ii) r' : ha equazione $\underline{\underline{y = \frac{1}{2}x}}$.

perimetro del triangolo di vertici P, Q ed R $= \overline{PQ} + \overline{QR} + \overline{PR}$

$$= \sqrt{(-1-2)^2 + (2+4)^2} + \sqrt{(2-6)^2 + (-4-3)^2} + \sqrt{(-1-6)^2 + (2-3)^2}$$

$$= \sqrt{9+36} + \sqrt{16+49} + \sqrt{49+1} = \underline{\underline{3\sqrt{5} + \sqrt{65} + 5\sqrt{2}}}.$$

area del triangolo di vertici P, Q ed R è $A = \frac{\overline{PQ} \cdot \overline{RO}}{2}$

Poichè $\overline{RO} = \sqrt{6^2 + 3^2} = \sqrt{36+9} = \sqrt{45}$, si ha $A = \frac{\sqrt{45} \cdot \sqrt{45}}{2}$,

e quindi $\underline{\underline{A = \frac{45}{2}}}$.

5) i) $y + x^2 + 4x + 1 = 0 \iff y = -x^2 - 4x - 1$

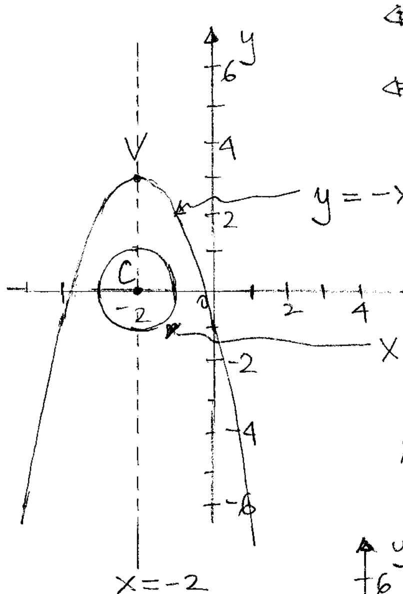
$$= -(x^2 + 4x) - 1$$

$$y = -(x+2)^2 + 3 \qquad V = (-2, 3)$$

$$x^2 + y^2 + 4x + 3 = 0 \iff \underbrace{x^2 + 4x}_{} + y^2 + 3 = 0$$

$$\iff (x+2)^2 - 4 + y^2 + 3 = 0$$

$$\iff (x+2)^2 + y^2 = 1 \qquad C = (-2, 0),\ r = 1$$

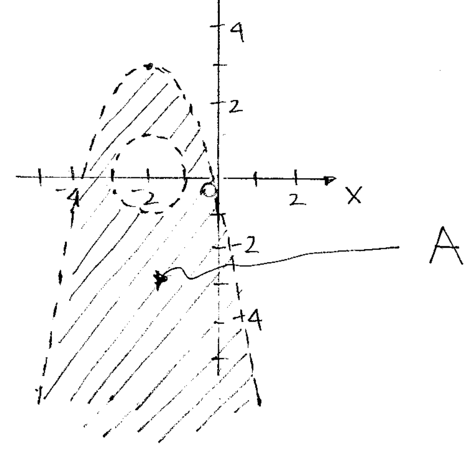


$y = -x^2 - 4x - 1$

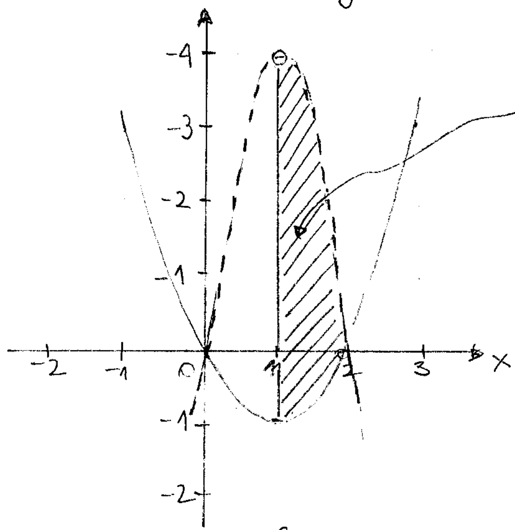
$x^2 + y^2 + 4x + 3 = 0$

ii) L'eq. della retta passante per V e C è $x = -2$.

iii)



6) $\begin{cases} x \geq 1 \\ y - x^2 + 2x \geq 0 \\ y < -4x^2 + 8x \end{cases} \iff \begin{cases} x \geq 1 \\ y \geq x^2 - 2x \\ y < -4(x^2 - 2x) \end{cases} \iff \begin{cases} x \geq 1 \\ y \geq (x-1)^2 - 1 \\ y < -4(x-1)^2 + 4 \end{cases}$



$\begin{cases} 1 < x^2 + y^2 \leq 4 \\ x^2 \geq 1 \\ y - 4x \leq 0 \end{cases} \iff \begin{cases} 1 < x^2 + y^2 \leq 4 \\ x \leq -1 \text{ o } x \geq 1 \\ y \leq 4x \end{cases}$