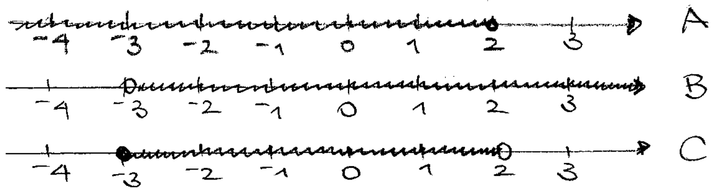
Università degli Studi di Trento – Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione – CdL in Filosofia
VERIFICA SETTIMANALE DI ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2012–2013 – Rovereto, 24–28 settembre 2012

1) i) $(a+1)x - 7a = 2a - 3x \iff (a+4)x = 9a$; quest'equazione ha soluzione solo se $a \in \mathbb{R}\setminus\{-4\}$; per tali a si ha $x = \dfrac{9a}{a+4}$ □

ii) $A =]-\infty, 2]$

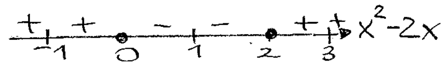
$B =]-3, +\infty[$

$C = [-3, 2[$



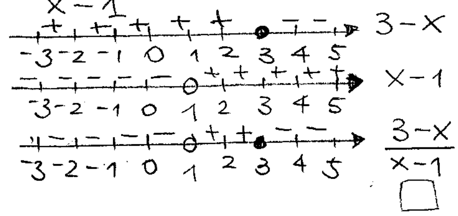
$A \cap B =]-3, 2]$ $\qquad (A\cap B) \cup C = \underline{\underline{[-3, 2]}}$.

2) $A = \{x \in \mathbb{R} : x^2 < 2x\} = \underline{\underline{]0, 2[}}$

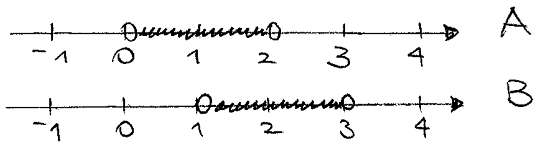


$B = \left\{x \in \mathbb{R} : \dfrac{2}{x-1} > 1\right\} = \underline{\underline{]1, 3[}}$

$\dfrac{2}{x-1} > 1 \iff \dfrac{2}{x-1} - 1 > 0 \iff \dfrac{3-x}{x-1} > 0$



a)



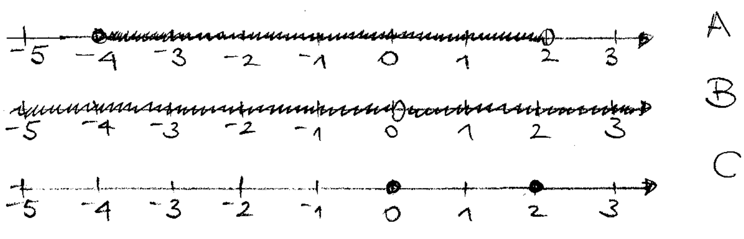
b) $A \cap B = \underline{\underline{]1, 2[}}$ $\qquad$ A e B non sono insiemi disgiunti.

$A \cup B = \underline{\underline{]0, 3[}}$

$B \setminus A = \underline{\underline{[2, 3[}}$ $\qquad \mathbb{R} \setminus B = \underline{\underline{]-\infty, 1] \cup [3, +\infty[}}$

3) $A = [-4, 2[$ $\qquad B = \left\{x \in \mathbb{R} : \dfrac{1}{x^2} > 0\right\} = \underline{\underline{\mathbb{R}\setminus\{0\}}}$.

$C = \{x \in \mathbb{R} : x^2(2-x)(-x^2-1) = 0\} = \underline{\underline{\{0, 2\}}}$



$A \cap B = [-4, 0[\cup]0, 2[$ (V); $\qquad A \cup B = \mathbb{R}\setminus\{0\}$ (F) (si ha $A \cup B = \mathbb{R}$);

$]0, 1[\subset B$ (V); $\qquad \{-1, 2\} \subseteq C$ (F) (poiché $-1 \notin C$);

$\{2\} \in \mathcal{P}(C)$ (V) (poiché $\{2\}$ è un sottoinsieme di C, ossia $\{2\} \subset C$);

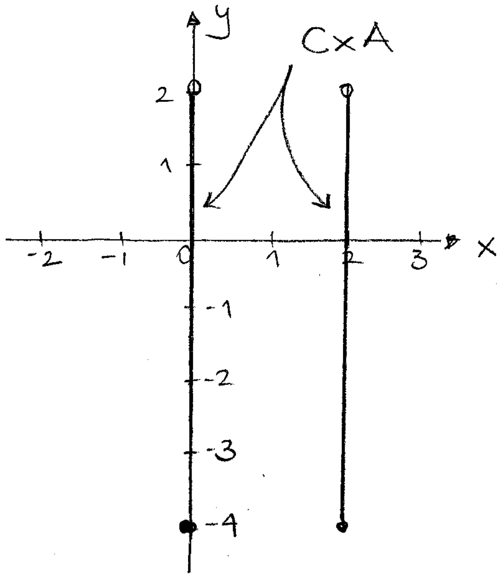
$[-1,2[\ \subset \mathcal{P}(A)$ (F) (non è corretta la scrittura; è corretto $[-1,2[\ \in \mathcal{P}(A)$ oppure $[-1,2[\ \subset A$);

$(-4,2) \in A \times B$ (V) (poiché $-4 \in A$, $2 \in B$);

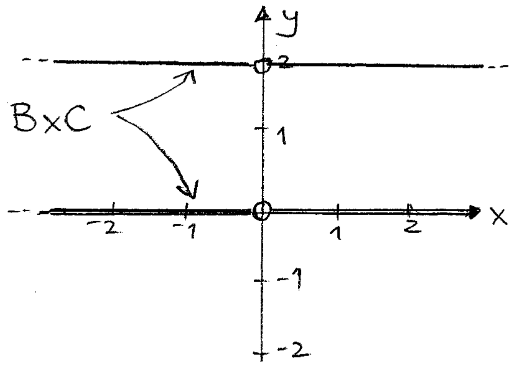
$]-4,-2[\ \in A \times C$ (F) (non è corretta la scrittura; inoltre $-2 \notin C$ una scrittura corretta sarebbe $(-4,0) \in A \times C$, oppure $(-4,2) \in A \times C$);

$\mathbb{R} \setminus B = \{0\}$ (V). □

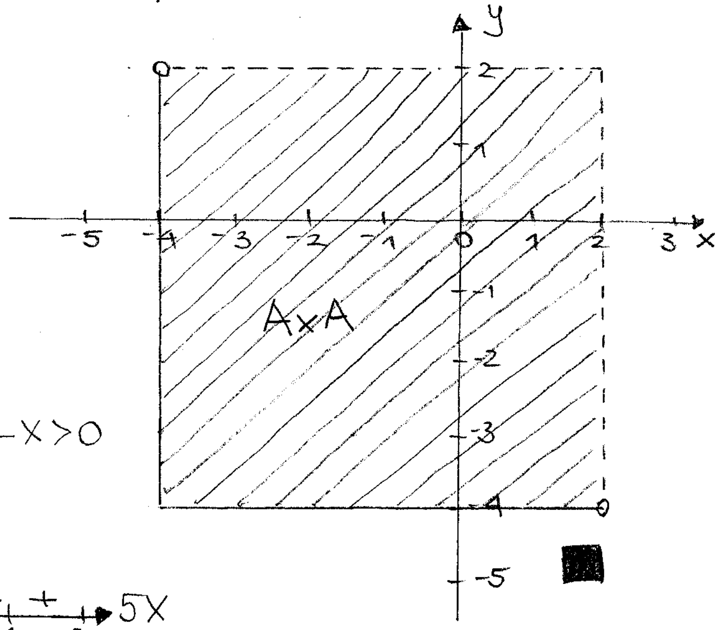
ii) $C \times A = \{0,2\} \times [-4,2[$



$B \times C = \mathbb{R} \setminus \{0\} \times \{0,2\}$



$A \times A = [-4,2[\ \times [-4,2[$

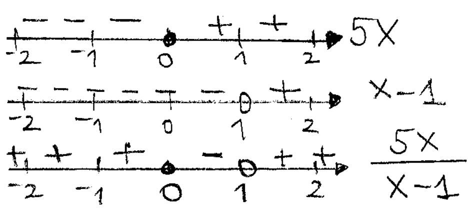


■

4) i) $A = \left\{ x \in \mathbb{R} : \dfrac{4x+x^2}{x-1} > x \right\}$

$$\frac{4x+x^2}{x-1} > x \iff \frac{4x+x^2}{x-1} - x > 0$$

$$\iff \frac{5x}{x-1} > 0$$

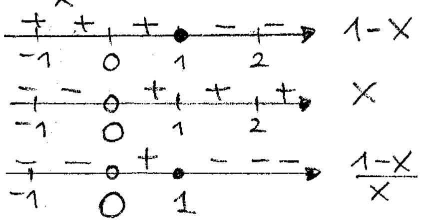


Si ha $A = \underline{\underline{]-\infty,0[\ \cup\ ]1,+\infty[}}$ A <u>non</u> è un intervallo.

$B = \{x \in \mathbb{R} : 1 \leq \frac{1}{x} \leq 2\}$

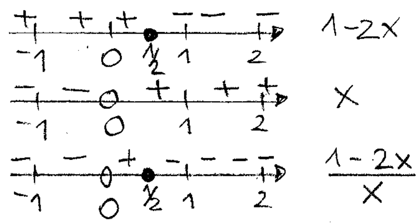
$\frac{1}{x} \geq 1 \iff \frac{1}{x} - 1 \geq 0$

$\iff \frac{1-x}{x} \geq 0$



$\frac{1}{x} \leq 2 \iff \frac{1}{x} - 2 \leq 0$

$\frac{1-2x}{x} \leq 0$

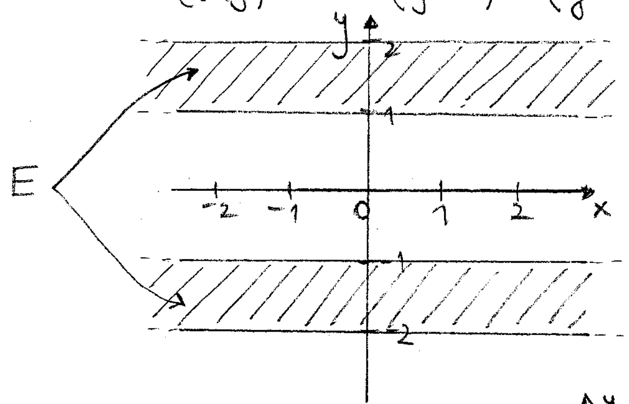


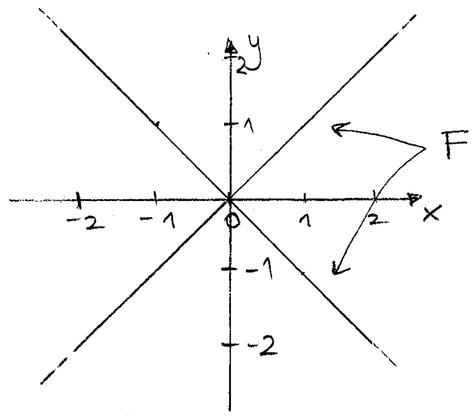
$$B = \left]0,1\right] \cap \left(\left]-\infty,0\right[\cup \left[\tfrac{1}{2},+\infty\right[\right) = \underline{\underline{\left[\tfrac{1}{2},1\right]}}.$$

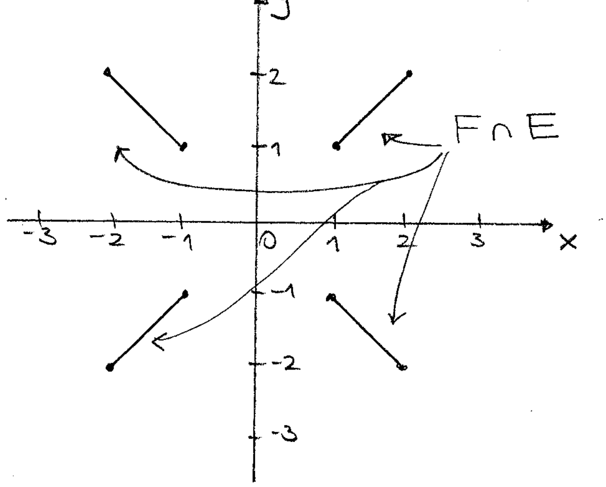
ii) $A \cup B = \underline{\underline{\left]-\infty,0\right[\cup \left[\tfrac{1}{2},+\infty\right[}}$; $A \cap B = \underline{\underline{\emptyset}}$; $B \setminus A = \underline{\underline{B}}$

5) $E = \{(x,y) \in \mathbb{R}^2 : 1 \leq y^2 \leq 4\} = \{(x,y) \in \mathbb{R}^2 : y \in [-2,-1] \cup [1,2]\}$

$F = \{(x,y) \in \mathbb{R}^2 : (y = x) \text{ o } (y = -x)\}$





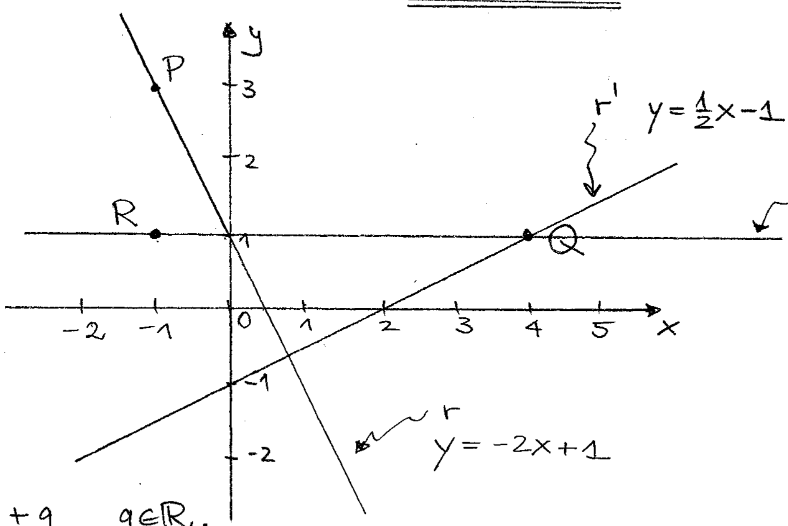


6) i) $m = -2 \qquad P = (-1, 3)$

$$y = 3 - 2(x+1) \quad \Rightarrow \quad \underline{\underline{y = -2x + 1}}\,.$$

ii) r' ha pendenza $m' = \frac{1}{2}$; $\qquad Q = (4, 1)$

$$y = 1 + \tfrac{1}{2}(x-4) \quad \Rightarrow \quad \underline{\underline{y = \tfrac{1}{2}x - 1}}\,.$$



iii) $y = \frac{1}{2}x + q \qquad q \in \mathbb{R}$.

iv) $y = 1$ (retta orizzontale passante per R e Q)