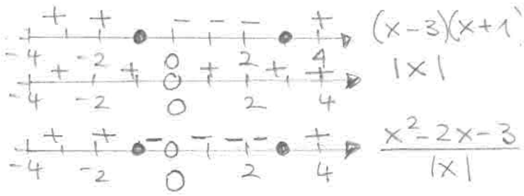
Università degli Studi di Trento – Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione – CdL in Filosofia
ESAME SCRITTO DI ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2012 – 2013 – Rovereto, 3 giugno 2013

FILA A

1) i) $A = \left\{ x \in \mathbb{R} : \dfrac{x^2 - 2x - 3}{|x|} \leq 0 \right\}$

$= \underline{[-1, 3] \setminus \{0\}}$



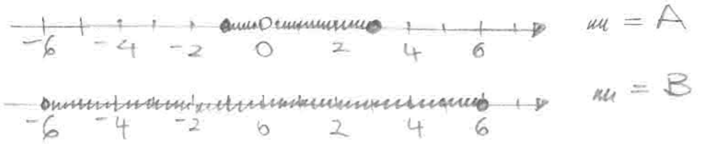
$B = \left\{ x \in \mathbb{R} : \log_2 (2 + |x|) \leq 3 \right\}$

$= \underline{[-6, 6]}$

$\log_2 (2+|x|) \leq \log_2 8$

$\Longleftrightarrow \quad 0 < 2 + |x| \leq 8$

$\forall x \in \mathbb{R} \qquad |x| \leq 6$

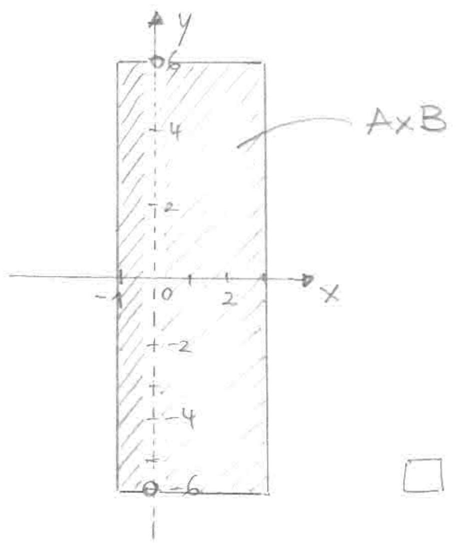


ii) $A \cup B = B = \underline{[-6, 6]}$ $\qquad A \setminus B = \underline{\emptyset}$

iii) A e B non sono disgiunti, poiché $A \cap B = A \neq \emptyset$.

A e B sono due insiemi limitati.

iv)



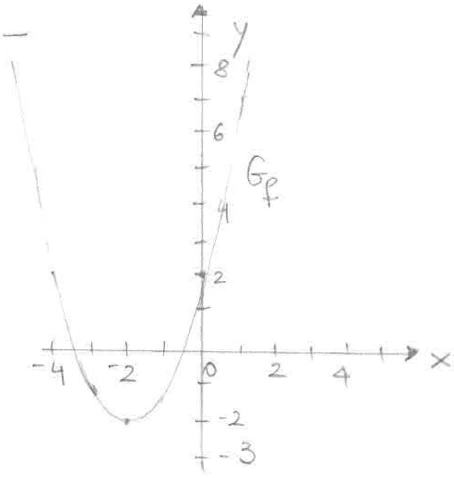
v) $x = k$ con

$\underline{k \in \left]-\infty, -1\right[\cup \{0\} \cup \left]3, +\infty\right[}$.

2) $f(x) = ax^2 + bx + c$ $\qquad f(0) = 2 \Longleftrightarrow c = 2$

$f'(x) = 2ax + b$ $\qquad f'(0) = 4 \Longleftrightarrow b = 4$ $\qquad \underline{f(x) = x^2 + 4x + 2}$, $x \in \mathbb{R}$

$f''(x) = 2a$ $\qquad f''(1) = 2 \Longleftrightarrow a = 1$



ii) $f(x) = x^2 + 4x + 2$

$= (x+2)^2 - 4 + 2$

$= (x+2)^2 - 2$

iii) $V = (x_0, f(x_0)) = \underline{\underline{(-2,-2)}}$

$\mathcal{E}: \quad x^2 + 4y^2 + 2x + 32y + 62 = 0$

$(x+1)^2 - 1 + 4(y^2 + 8y) + 62 = 0$

$(x+1)^2 - 1 + 4(y+4)^2 - 64 + 62 = 0$

$(x+1)^2 + 4(y+4)^2 = 3$

$\frac{(x+1)^2}{3} + \frac{(y+4)^2}{3/4} = 1 \qquad \underline{\underline{C = (-1,-4)}} \qquad a = \sqrt{3}, \; b = \frac{\sqrt{3}}{2}$

$d(V, C) = \sqrt{(-2+1)^2 + (-2+4)^2} = \sqrt{1+4} = \underline{\underline{\sqrt{5}}}$.

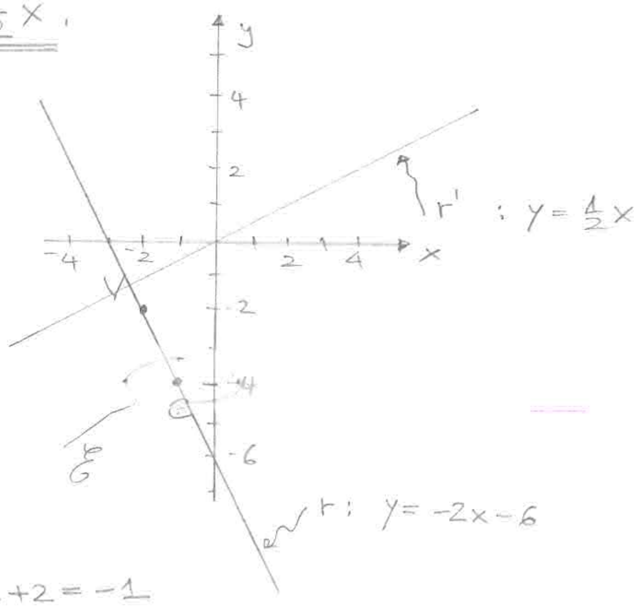
iv) $\frac{y+2}{-4+2} = \frac{x+2}{-1+2} \iff \frac{y+2}{-2} = x+2$

$\iff y+2 = -2x - 4$

$\underline{\underline{y = -2x - 6}}$

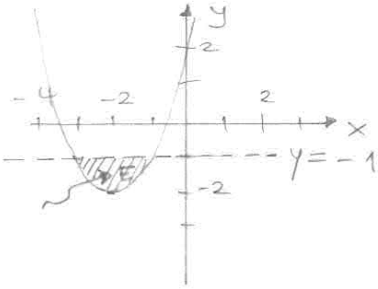
v) $\underline{\underline{y = \frac{1}{2}x}}$.

vi)



vii) $x^2 + 4x + 2 = -1$

$\iff x^2 + 4x + 3 = 0 \iff (x+1)(x+3) = 0 \iff \begin{array}{l} x = -1 \\ x = -3 \end{array}$



$$\text{area } E = \int_{-3}^{-1} (-1 - x^2 - 4x - 2)\, dx =$$

$$= \int_{-3}^{-1} (-x^2 - 4x - 3)\, dx = \left[-\frac{x^3}{3} - 2x^2 - 3x \right]_{-3}^{-1}$$

$$= \left(\frac{1}{3} - 2 + 3\right) - \left(9 - \cancel{18} + 9\right) = \underline{\underline{\frac{4}{3}}}.$$

3) i) $\lim_{x \to +\infty} \dfrac{2x e^{kx} - x}{2x + 1} = \begin{cases} -\frac{1}{2} & \text{se } k = -1 \\ \frac{1}{2} & \text{se } k = 0 \\ +\infty & \text{se } k = 1. \end{cases}$

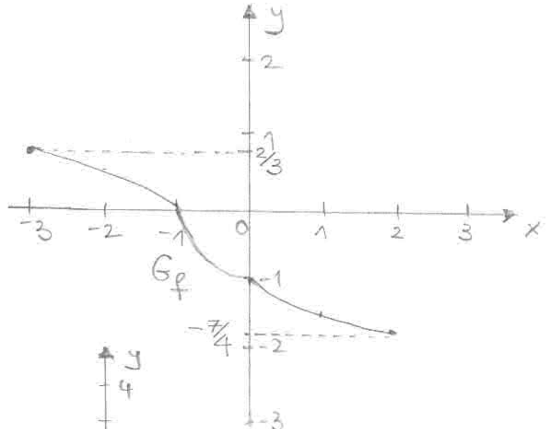
ii) $\displaystyle\sum_{k=1}^{5} \int_{k}^{k+1} \left(\frac{x+3}{x^2}\right) dx = \int_{1}^{6} \left(\frac{1}{x} + \frac{3}{x^2}\right) dx = \left[\log|x| - \frac{3}{x}\right]_1^6 =$

$$= \left(\log 6 - \frac{\cancel{3}^{\,1}}{\cancel{6}_{\,2}}\right) - \left(\log 1 - 3\right) = \underline{\underline{\log 6 + \frac{5}{2}}}.$$

iii) $-\dfrac{2x^2}{3} + \dfrac{4x^3}{6} - \dfrac{8x^4}{9} + \dots - \dfrac{128x^8}{21} = \underline{\underline{\displaystyle\sum_{i=1}^{7} \frac{(-1)^i 2^i x^{i+1}}{3i}}}$

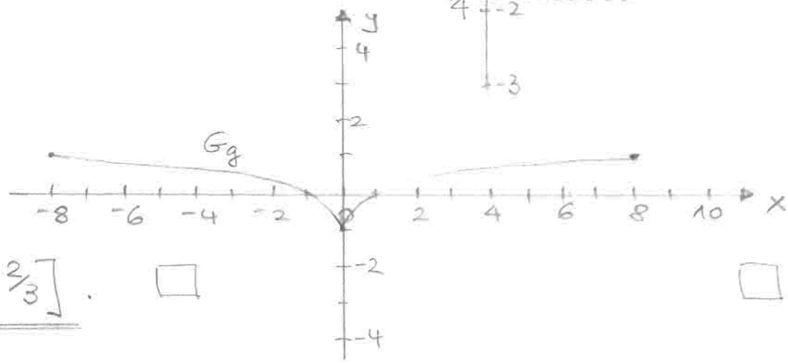
4) i) $f : [-3, 2] \to \mathbb{R}$

$$f(x) = \begin{cases} 1/x + 1 & \text{se } -3 \le x < -1 \\ x^2 - 1 & \text{se } -1 \le x < 0 \\ \left(\frac{1}{2}\right)^x - 2 & \text{se } 0 \le x \le 2 \end{cases}$$



$g : [-8, 8] \to \mathbb{R}$

$g(x) = |\sqrt[3]{x}| - 1$



ii) $f([-3,2]) = \underline{\underline{\left[-\frac{7}{4}, \frac{2}{3}\right]}}$.

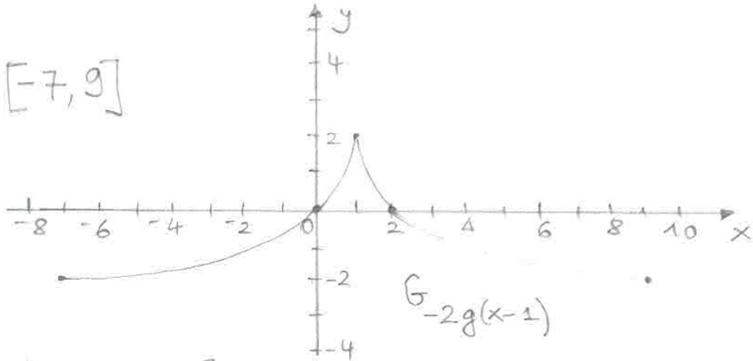
iii) $(f+g)(0) = f(0) + g(0) = -1 - 1 = \underline{\underline{-2}}$

$(fg)(1) = f(1) \cdot g(1) = \left(-\frac{3}{2}\right) \cdot 0 = \underline{\underline{0}}$.

$(g \circ f)(-1) = g(f(-1)) = g(0) = \underline{\underline{-1}}$.

iv) $\min_{[-3,2]} f = \underline{\underline{-\frac{7}{4}}}$. $\underline{\underline{x = 2 \text{ pt. di min.}}}$; $\max_{[-3,2]} f = \underline{\underline{\frac{2}{3}}}$ $\underline{\underline{x = -3 \text{ pt. di max.}}}$

v) $\mathrm{dom}\left[-2g(x-1)\right] = [-7, 9]$



5) $\boxed{f(x) = (e^x - 1)^2}$ i)

- $\mathrm{dom} f = \mathbb{R} =]-\infty, +\infty[$
- segno di f

  $+ \ + \ + \ \bullet_0 \ + \ + \ + \ + \ +$ $\quad f(x)$

- $\lim_{x \to -\infty} f(x) = 1$ $\quad y = 1$ asintoto orizzontale per f, per $x \to -\infty$

  $\lim_{x \to +\infty} f(x) = +\infty$
- f è continua su $\mathbb{R}$ essendo composizione di funz. contin
- $\mathrm{dom} f' = \mathbb{R}$

  $f'(x) = 2(e^x - 1)e^x$ $\quad f'(x) = 0 \iff e^x = 1 \iff x = 0$

$x = 0$ pt. critico per f

$x = 0$ è pt. di minimo per f

$- \ - \ - \ - \ \bullet_0 \ + \ + \ + \ +$ $\quad e^x - 1$

$\searrow \qquad \nearrow$ $\quad f(x)$

$f(0) = \underline{\underline{0}}$

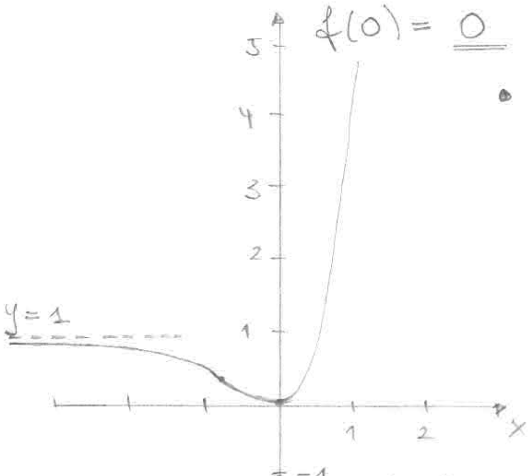


grafico qualitativo di f

- $\mathrm{dom} f'' = \mathbb{R}$

  $f''(x) = 2e^x \cdot e^x + 2(e^x - 1)e^x = 2e^x(2e^x - 1)$

  $f''(x) = 0 \iff e^x = \frac{1}{2} \iff x = \log\frac{1}{2} = -\log 2$

  $- \ - \ - \ \bullet_{-\log 2} \ + \ + \ + \ + \ +$ $\quad 2e^x - 1$

  $\cap \qquad \cup$ $\quad f(x)$

ii) $y = 0 + f'(0) \cdot x \Rightarrow \underline{\underline{y = 0}}$

iii) $\int_0^1 \sqrt{f(x)}\,dx = \int_0^1 (e^x - 1)\,dx = \left[e^x - x\right]_0^1 = (e - 1) - (1) = \underline{\underline{e - 2}}.$

6) i) V Infatti, fissato $x \in \mathbb{N}$ qualsiasi, basta prendere $y = 4 - x$ $(\in \mathbb{Z})$.

ii) non $\left[\forall x \in \mathbb{N}, \exists y \in \mathbb{Z} : x + y = 4\right]$ = "$\exists x \in \mathbb{N} : \forall y \in \mathbb{Z},\ x + y \neq 4$."

MLA (E)

1) (vedi MLa (A)) i) $A = \{x \in \mathbb{R} : \log_2 (2+|x|) \geq 3\} = \underline{]-\infty, -6] \cup [6, +\infty[}$

$B = \{x \in \mathbb{R} : \frac{x^2-2x-3}{|x|} \leq 0\} = \underline{[-1, 3] \setminus \{0\}}$

ii) $A \cup B = \underline{]-\infty, -6] \cup [-1, 0[\cup]0, 3] \cup [6, +\infty[}$

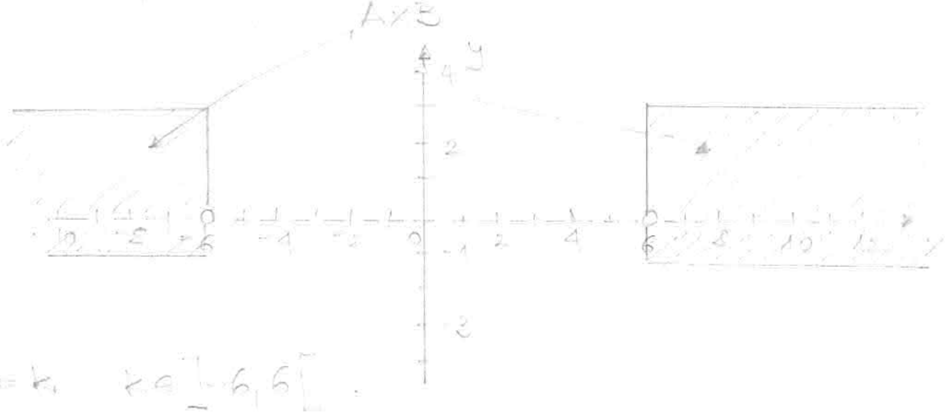
$A \setminus B = A = \underline{]-\infty, -6[\cup]6, +\infty[}$

iii) $A \cap B = \emptyset$: A e B sono insiemi disgiunti.

A non è limitato inferiormente e non è limitato superiormente.

B è limitato.

iv)



v) $x = k, \quad k \in \underline{]-6, 6[}$.

2) i) $f(x) = ax^2 + bx + c$ $\qquad f(0) = 2 \iff c = 2$

$f'(x) = 2ax + b$ $\qquad f'(0) = -4 \iff b = -4$

$f''(x) = 2a$ $\qquad f''(x) = 2 \iff a = 1$

$\underline{f(x) = x^2 - 4x + 2}$ $\qquad x \in \mathbb{R}$.

ii) $f(x) = (x-2)^2 - 4 + 2$

$= (x-2)^2 - 2$

iii) $V = (x_0, f(x_0)) = \underline{\underline{(2, -2)}}$

$\mathcal{E}: \ \frac{(x-1)^2}{3} + \frac{(y+4)^2}{3/4} = 1 \qquad C = (1, -4) \qquad \begin{matrix} a = \sqrt{3} \\ b = \frac{\sqrt{3}}{2} \end{matrix}$

$d(V, C) = \sqrt{(2-1)^2 + (-2+4)^2} = \sqrt{1+4} = \underline{\underline{\sqrt{5}}}$. $\square$

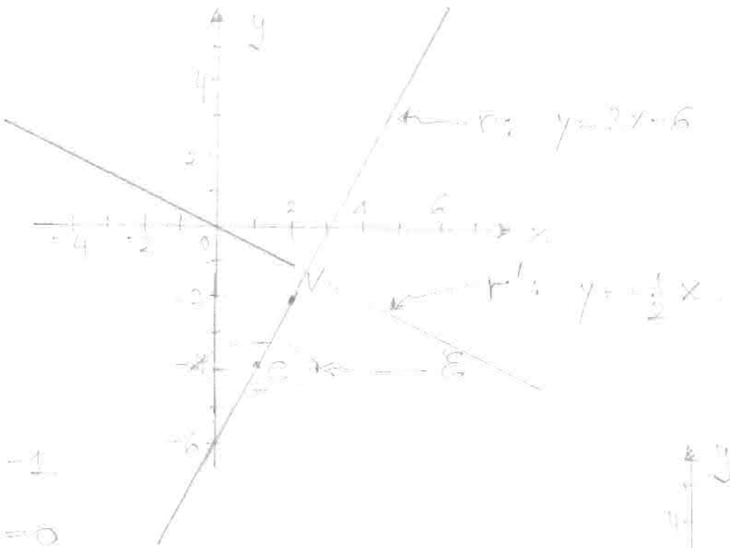
iv) $\frac{y+2}{-4+2} = \frac{x-2}{1-2} \iff \frac{y+2}{-2} = \frac{x-2}{-1}$

$\iff y + 2 = 2x - 4$

$\underline{\underline{y = 2x - 6}}$ $\square$

v) $\underline{\underline{y = -\frac{1}{2}x}}$.

vi)



$\square$

vii) $x^2 - 4x + 2 = -1$

$\iff x^2 - 4x + 3 = 0$

$\iff (x-3)(x-1) = 0 \iff x = 1, \ x = 3$.

$\text{area } E = \int_1^3 (-1 - x^2 + 4x - 2)\, dx =$

$= \int_1^3 (-x^2 + 4x - 3)\, dx = \left[-\frac{x^3}{3} + 2x^2 - 3x\right]_1^3 =$

$= (-9 + 18 - 9) - \left(-\frac{1}{3} + 2 - 3\right) = \underline{\frac{4}{3}}$.

3) i) $\lim_{x \to +\infty} \frac{2x e^{kx} + x}{2x + 1} = \begin{cases} 1/2 & \text{se } k = -1 \\ 3/2 & \text{se } k = 0 \\ +\infty & \text{se } k = 1 \end{cases}$ $\square$

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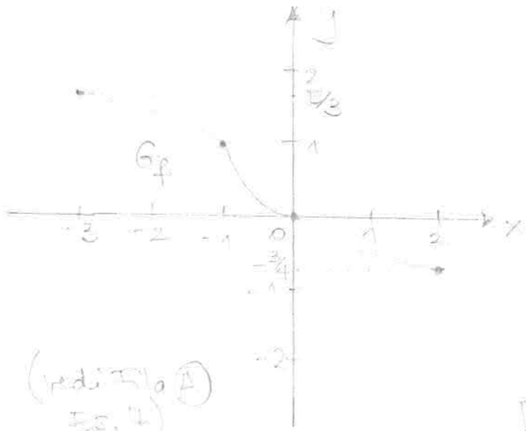
$= \underline{\underline{\log 6 + \frac{10}{3}}}$. $\square$

iii) $\frac{2x^2}{3} - \frac{4x^3}{6} + \frac{8x^4}{9} - \dots + \frac{128x^8}{21} = \sum_{n=1}^{7} (-1)^{n+1} \frac{2^n x^{n+1}}{3n}$.

1) i) $f : [-3,2] \to \mathbb{R}$,

$$f(x) = \begin{cases} \frac{1}{x} + 2 & \text{se } -3 \le x < -1 \\ x^2 & \text{se } -1 \le x < 0 \\ \left(\frac{1}{2}\right)^x - 1 & \text{se } 0 \le x \le 2 \end{cases}$$



$g : [-8,8] \to \mathbb{R}$, $\quad g(x) = |\sqrt[3]{x}| - 1$ (vedi FILA (A) Es. 4)

ii) $f([-3,2]) = \left[-\frac{3}{4}, \frac{5}{3}\right]$.

iii) $(f+g)(0) = f(0) + g(0) = 0 - 1 = -1$.

$(fg)(1) = f(1)\, g(1) = \left(-\frac{1}{2}\right) \cdot 0 = 0$.

$(g \circ f)(-1) = g(f(-1)) = g(1) = 0$.

iv) $\min_{[-3,2]} f = -\frac{3}{4}$, $x = 2$ pt di minimo;

$\max_{[-3,2]} f = \frac{5}{3}$, $x = -3$ pt di massimo.

v) vedi FILA (A), Es. 4;

3) i), ii) vedi FILA (A), Es. 3.

iii) $\int_0^2 \sqrt{f(x)}\, dx = \int_0^2 (e^x - 1)\, dx = \left[e^x - x\right]_0^2 = (e^2 - 2) - (1) =$
$= e - 3$

5) i) (V) Infatti, fissato $x \in \mathbb{N}$ qualsiasi, basta prendere
$y = x - 5$ $(\in \mathbb{Z})$.

ii) non $[\forall x \in \mathbb{N}, \exists y \in \mathbb{Z} : x - y = 5] =$ "$\exists x \in \mathbb{N} : \forall y \in \mathbb{Z}, x - y \ne 5$".