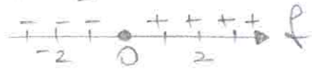
Università degli Studi di Trento — Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie delle Comunicazione — CdL in Filosofia
VERIFICA SETTIMANALE DI ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2012 – 2013 — Rovereto, 10 – 14 dicembre 2012

1) i) $f(x) = x^3 e^{-x}$ $\left(= \dfrac{x^3}{e^x}\right)$

• $\mathrm{dom} f = \mathbb{R} = ]-\infty, +\infty[$

• segno di $f$



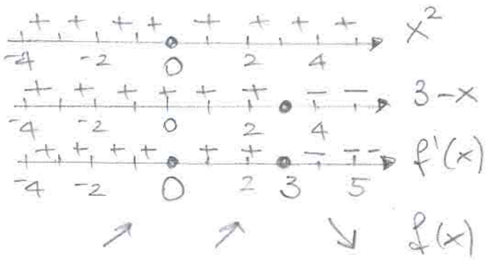

• $\lim_{x \to -\infty} f(x) = -\infty$, $\lim_{x \to +\infty} f(x) = 0$

$y = 0$ è asintoto orizzontale per $f$, per $x \to +\infty$;

• continuità su tutto $\mathbb{R}$ (rapporto di funzioni continue e $e^x \neq 0$ $\forall x \in \mathbb{R}$);

• $\mathrm{dom} f' = \mathbb{R}$ $\quad f'(x) = 3x^2 e^{-x} - x^3 e^{-x} = x^2(3-x)e^{-x}$

$$f'(x) = 0 \iff \left.\begin{array}{l} x = 0 \\ x = 3 \end{array}\right\} \text{ pt. critici di } f$$



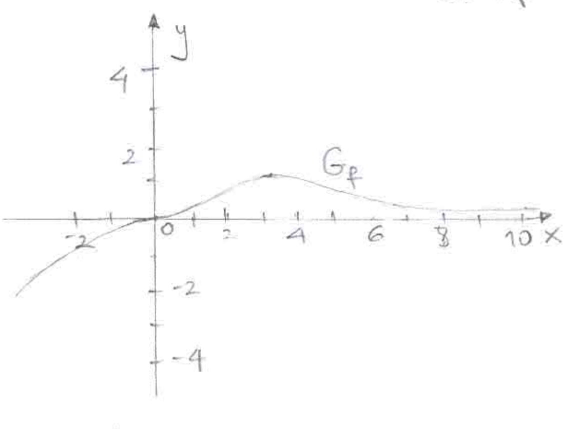

(nota: $e^x > 0$ $\forall x \in \mathbb{R}$)

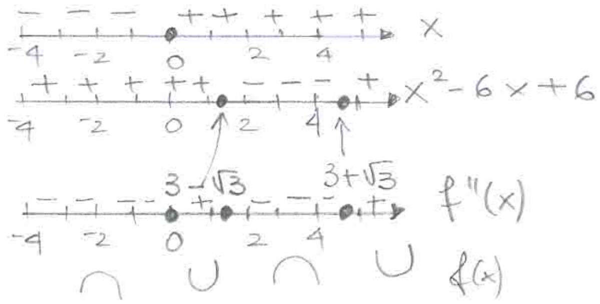
$x = 3$ pt. di massimo loc. stretto per $f$

$$f(3) = \frac{27}{e^3} \ (\sim 1.3) \quad f(0) = 0$$

• $\mathrm{dom} f'' = \mathbb{R}$ $\quad f''(x) = 6x e^{-x} - 3x^2 e^{-x} - 3x^2 e^{-x} + x^3 e^{-x}$

$$= e^{-x} \cdot x\left(x^2 - 6x + 6\right)$$

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ii) $y = \dfrac{27}{e^3}$ è l'eq. della retta $r$ tangente al grafico di $f$ nel punto di coordinata $x = 3$.

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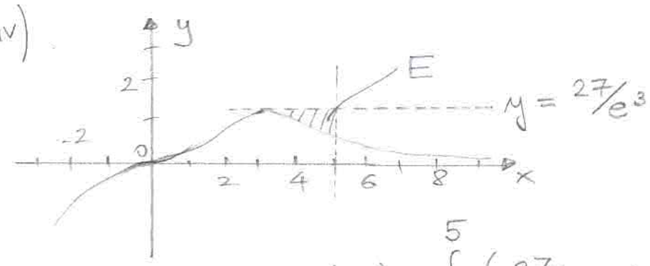
iii) $F(x) = e^{-x}(-x^3-3x^2-6x-6)$ è una funzione derivabile (prodotto di funzioni derivabili) e $F'(x) = -e^{-x}(-x^3-3x^2-6x-6) + e^{-x}(-3x^2-6x-6) = e^{-x}x^3 = f(x) \quad \forall x \in \mathbb{R}$. $\square$

iv)




$$\text{area}(E) = \int_3^5 \left(\frac{27}{e^3} - f(x)\right)dx = \left[\frac{27}{e^3}x\right]_3^5 - \int_3^5 f(x)\,dx$$

$$= \frac{27}{e^3}\cdot 2 - \left(F(5) - F(3)\right) =$$

$$= \frac{54}{e^3} + \frac{(125+75+30+6)}{e^5} + \frac{-27-27-18-6}{e^3}$$

$$= \frac{236}{e^5} - \frac{24}{e^3}.$$ $\blacksquare$

2) i) $\displaystyle\int_0^1 (3x+1)^4 dx = \frac{1}{3}\int_0^1 3(3x+1)^4 dx = \frac{1}{3}\left[\frac{(3x+1)^5}{5}\right]_0^1 =$

$$= \underline{\frac{1}{15}\left[4^5 - 1\right]};$$

$$\int_0^1 \left[(e^x+3)^{-2}e^x + \frac{1}{3x+3}\right]dx = \left[-(e^x+3)^{-1} + \frac{1}{3}\log|3x+3|\right]_0^1 =$$

$$= \left(-(e+3)^{-1} + \frac{1}{3}\log 6\right) - \left(-(4)^{-1} + \frac{1}{3}\log 3\right) =$$

$$= \underline{-\frac{1}{e+3} + \frac{1}{4} + \frac{1}{3}\log 2};$$

$$\int_4^5 4(4x+1)^{-1}dx = \int_4^5 \frac{4}{4x+1}dx = \left[\log|4x+1|\right]_4^5 = \log 21 - \log 17$$

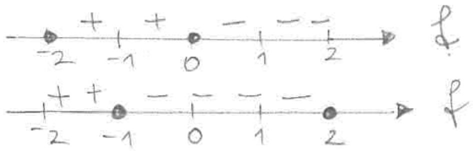
$$= \underline{\log\frac{21}{17}};$$

ii) $\displaystyle\int_0^1 x e^{3x^2}dx = \frac{1}{6}\int_0^1 6x e^{3x^2}dx = \frac{1}{6}\left[e^{3x^2}\right]_0^1 = \underline{\frac{1}{6}(e^3 - 1)};$

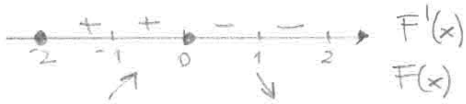
$$\int_{-1}^0 (x^2+2)^{-1}x\,dx = \frac{1}{2}\int_{-1}^0 \frac{2x}{x^2+2}dx = \frac{1}{2}\left[\log|x^2+2|\right]_{-1}^0 = \underline{\frac{1}{2}\log\frac{2}{3}};$$

$$\int_{-2}^2 e^{-|x|}dx = 2\int_0^2 e^{-x}dx = -2\int_0^2 (-e^{-x})dx = -2e^{-x}\Big|_0^2 = \underline{-2e^{-2}+2}.$$ $\blacksquare$

3) i)

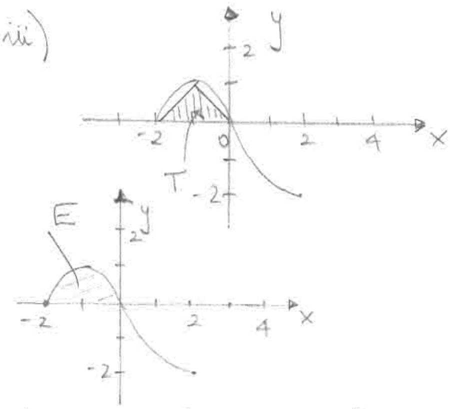



ii) Dal teorema fondamentale del calcolo integrale si ha $F'(x) = f(x)$ $\forall x \in [-2,2]$; quindi



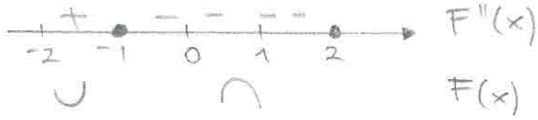

Allora $F(x)$ ristretta all'intervallo $[-2,0]$ è crescente, mentre è decrescente su $[0,2]$.

iii)




area$(E)$ > area$(T) = \frac{\cancel{2}\cdot 1}{\cancel{2}} = 1$.

iv) Poiché $f$ è derivabile su $[-2,2]$, dal teorema fondamentale del calcolo integrale si ottiene $F''(x) = f'(x)$ $\forall x \in [-2,2]$, e quindi




$x = -1$ pt. di flesso per $F$.

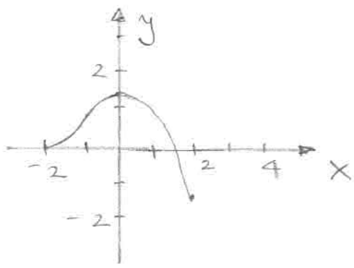
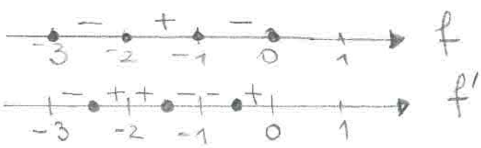


grafico qualitativo di $F(x)$

4) i)




ii) Dal teorema fondamentale del calcolo integrale si ha $F'(x) = f(x)$ $\forall x \in [-3,0]$; quindi i punti critici di $F$ sono $x = -2$, $x = -1$.

iii) Poiché

$$\begin{array}{ccccccccc} & - & & + & & - & & & F' \\ -3 & & -2 & & -1 & & 0 & \quad 1 & \\ & \searrow & & \nearrow & & \searrow & & & F(x) \end{array}$$

si deduce che $x=-2$ è pt. di min. loc. stretto per $F$, mentre $x=-1$ è pt. di max loc. stretto per $F$.

Poiché $f$ è derivabile in $[-3,0]$, dal teorema fondamentale del calcolo integrale si ottiene $F''(x)=f'(x)$ $\forall x \in [-3,0]$ e quindi

$$\begin{array}{cccccccccc} - & & + & & - & & + & & & F'' \\ -3 & & -2 & & -1 & & 0 & & 1 & \\ \cap & & \cup & & \cap & & \cup & & & F(x) \end{array}$$

$x=-5/2$, $x=-3/2$, $x=-1/2$ pt. di flesso per $F$

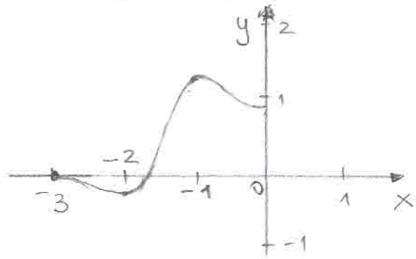
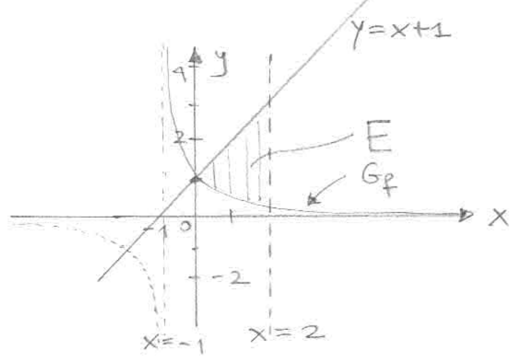



grafico qualitativo di $F$

iv) $\min\limits_{[-3,0]} F(x) = F(-2)$ $\qquad$ $\max\limits_{[-3,0]} F(x) = F(-1)$

(questo risultato si ottiene anche facilmente dall'interpretazione geometrica di $F(x)$).

5) i) $f(x)=\frac{1}{x+1}$ ; $y=x+1$ ; $x=2$

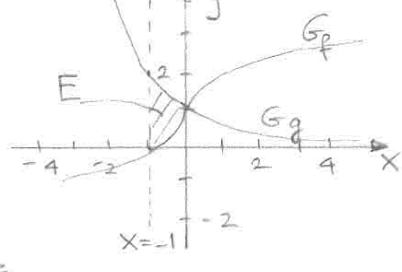



$$\text{area}(E)=\int_0^2\left(x+1-\frac{1}{x+1}\right)dx$$

$$=\left[\frac{x^2}{2}+x-\log|x+1|\right]_0^2$$

$$=\frac{\cancel{4}^{2}}{\cancel{2}}+2-\log 3 = \underline{\underline{4-\log 3.}}$$

ii) $f(x)=\sqrt[3]{x}+1$ ; $g(x)=\left(\frac{1}{2}\right)^x$; $x=-1$




$$\text{area}(E)=\int_{-1}^{0}\left[\left(\frac{1}{2}\right)^x-\sqrt[3]{x}-1\right]dx$$

$$=\left[\frac{(1/2)^x}{\log 1/2}-\frac{x^{4/3}}{4/3}-x\right]_1^0=$$

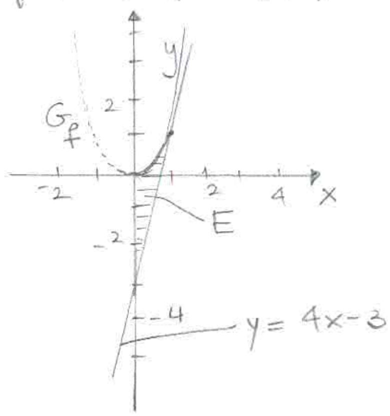
$$=\left(-\frac{1}{\log 2}\right)-\left(-\left(\frac{1}{2}\right)\frac{1}{\log 2}-\frac{3}{4}-1\right)=\underline{\underline{\frac{7}{4}-\frac{1}{2\log 2}.}}$$

iii) $f(x) = x^4$ $\qquad f'(x) = 4x^3$

retta tg al grafico di f in (1,1) : $y = 1 + 4(x-1)$

$y = 4x - 3$

l'asse y



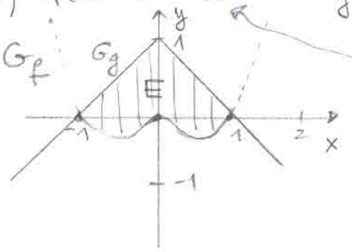

$$\text{area}(E) = \int_0^1 (x^4 - 4x + 3)\,dx$$

$$= \left[\frac{x^5}{5} - \cancel{4}^{\,2}\frac{x^2}{\cancel{2}} + 3x\right]_0^1$$

$$= \frac{1}{5} - 2 + 3 = \underline{\underline{\frac{6}{5}}}$$ $\square$

iv) $f(x) = x^4 - x^2$ $\qquad g(x) = -|x| + 1$

fate un breve studio della funzione !



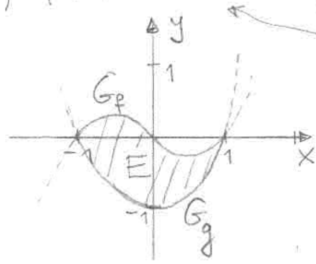

$$\text{area}(E) = \int_{-1}^{1} \left[(-|x|+1) - (x^4 - x^2)\right] dx$$

$$= 2\int_0^1 \left[-x + 1 - x^4 + x^2\right] dx =$$

$$= 2\left[-\frac{x^2}{2} + x - \frac{x^5}{5} + \frac{x^3}{3}\right]_0^1 = 2\left(-\frac{1}{2} + 1 - \frac{1}{5} + \frac{1}{3}\right) = 2\left(\frac{-15+30-6+10}{30}\right) = \underline{\underline{\frac{19}{15}}}$$ $\square$

v) $f(x) = x^3 - x$ $\qquad g(x) = x^2 - 1$

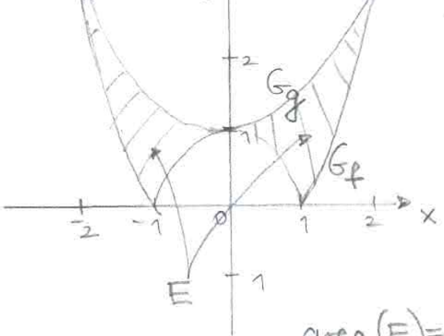
fate un breve studio della funzione !




$$\text{area}(E) = \int_{-1}^{1} \left[(x^3 - x) - (x^2 - 1)\right] dx$$

$$= \left[\frac{x^4}{4} - \frac{x^2}{2} - \frac{x^3}{3} + x\right]_{-1}^{1} = \left(\frac{1}{4} - \frac{1}{2} - \frac{1}{3} + 1\right) - \left(\frac{1}{4} - \frac{1}{2} + \frac{1}{3} - 1\right)$$

$$= -\frac{2}{3} + 2 = \underline{\underline{\frac{4}{3}}}.$$ $\square$




se $x \geq 1$, $|x^2 - 1| = x^2 - 1$ ; osserviamo che

$x^2 - 1 = \frac{x^2}{2} + 1 \iff \frac{x^2}{2} = 2 \iff x = 2$ (se $x \geq 0$)

$$\text{area}(E) = 2\int_0^2 \left[g(x) - f(x)\right] dx = 2\int_0^1 \left[\left(\frac{x^2}{2}+1\right) - \left(-x^2+1\right)\right] dx +$$

$$+ \ 2\int_1^2 \left[\left(\frac{x^2}{2}+1\right) - \left(x^2 - 1\right)\right] dx$$

$$= \cancel{2}\int_0^1 \frac{3}{\cancel{2}} x^2\,dx + 2\int_1^2 \left(-\frac{x^2}{2}+2\right)dx =$$

$$= \left[x^3\right]_0^1 + \left[-\frac{x^3}{3}+4x\right]_1^2 = 1 + \left[\left(-\frac{8}{3}+8\right) - \left(-\frac{1}{3}+4\right)\right]$$

$$= 1 + 4 - \frac{7}{3} = \underline{\underline{\frac{8}{3}}}\,.$$

6) i) $D_{7,2} \cdot P_{13} = \frac{7!}{2!\,5!} \cdot 13!$

$P_{13}^{7,6} = \frac{13!}{7!\,6!}$ .

ii) $C_{9,3} = \frac{9!}{3!\,6!}$ .

iii) $D_{7,4} = \frac{7!}{(7-4)!} = \frac{7!}{3!}$

iv) $P_7^{2,2} = \frac{7!}{2!\,2!}$