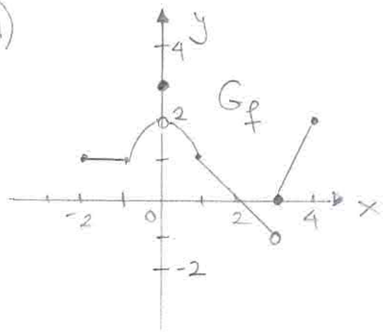
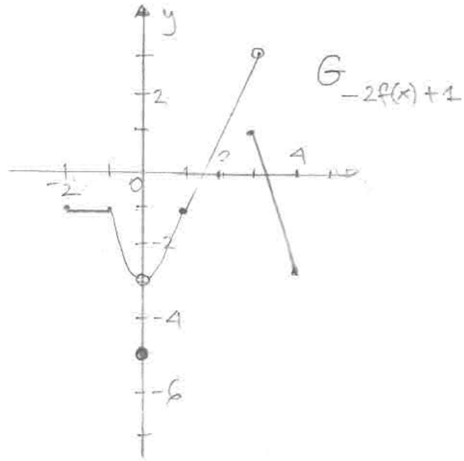
Università degli Studi di Trento – Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione – CdL in Filosofia
VERIFICA SETTIMANALE DI ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2012–2013 — Rovereto, 22–26 ottobre 2012

1)

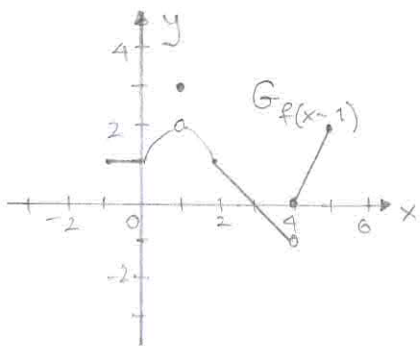


i) $f([-2,4]) =]-1,2] \cup \{3\}$.

ii) $x \longmapsto -2f(x)+1$
$\underset{[-2,4]}{\uparrow}$



$x \longmapsto f(x-1)$
$\underset{[-1,5]}{\uparrow}$



iii) f è <u>limitata inferiormente</u>, poiché $f(x) \geq -1 \quad \forall x \in [-2,4]$;
f è <u>limitata superiormente</u>, poiché $f(x) \leq 3 \quad \forall x \in [-2,4]$.

$\nexists \min\limits_{[-2,4]} f$; $\max\limits_{[-2,4]} f = \underline{\underline{3}}$ $\quad \underline{\underline{x=0}}$ pt. di massimo.

iv) $\min\limits_{[3,4]} f = \underline{\underline{0}}$ $\quad \underline{\underline{x = 3 \text{ pt. di minimo.}}}$

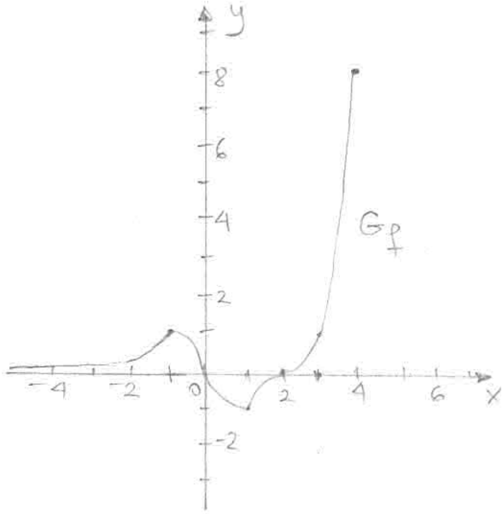
$\max\limits_{[3,4]} f = \underline{\underline{2}}$ $\quad \underline{\underline{x = 4 \text{ pt. di massimo.}}}$

$\min\limits_{]-2,1[} f = \underline{\underline{1}}$ $\quad \underline{\underline{x \in]-2,-1] \text{ sono pt. di minimo.}}}$

$\max\limits_{]-2,1[} f = \underline{\underline{3}}$ $\quad \underline{\underline{x = 0 \text{ pt. di massimo.}}}$

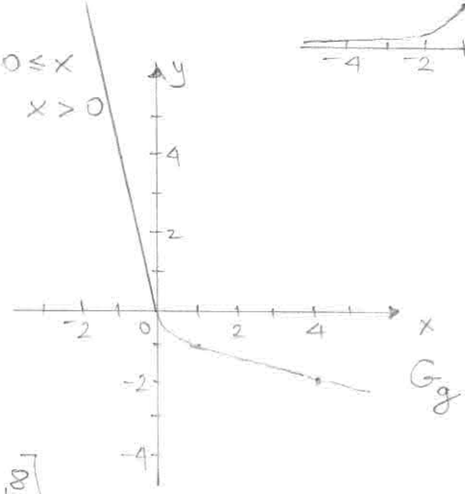
2) i) $f :]-\infty, 4] \to \mathbb{R}$

$$f(x) = \begin{cases} 1/x^2 & \text{se} \quad x < -1 \\ -\sqrt[3]{x} & \text{se} \quad -1 \le x \le 1 \\ (x-2)^3 & \text{se} \quad 1 < x \le 4 \end{cases}$$



$g : \mathbb{R} \to \mathbb{R}$

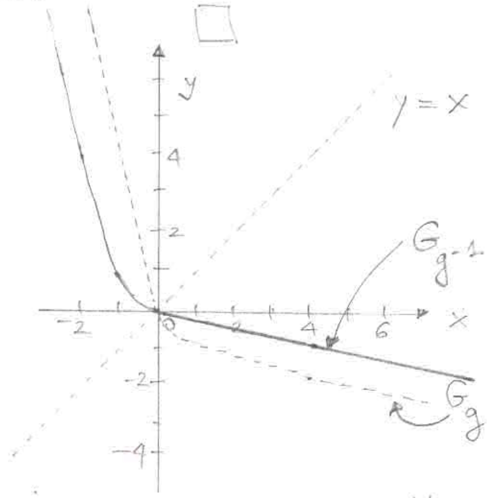
$$g(x) = \begin{cases} -4x & \text{se} \quad 0 \le x \\ -\sqrt{x} & \text{se} \quad x > 0 \end{cases}$$



ii) $f(]-\infty, 4]) = [-1, 8]$

$g(\mathbb{R}) = \mathbb{R}$.

iii) $g^{-1} : g(\mathbb{R}) \to \mathbb{R}$



iv) f è limitata, poiché $-1 \le f(x) \le 8 \quad \forall x \in]-\infty, 4]$;

$g(\mathbb{R}) = \mathbb{R}$ e quindi g non è limitata.

$\min\limits_{]-\infty,4]} f = -1 \qquad x = 1$ pt. di minimo.

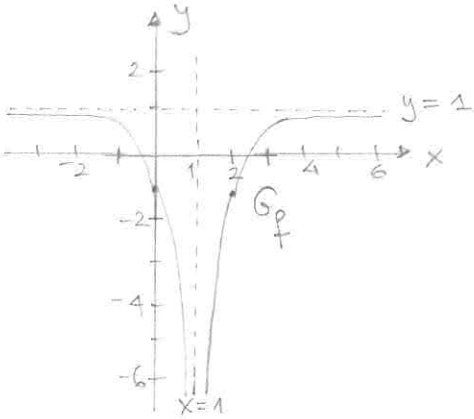
$\max\limits_{]-\infty,4]} f = 8 \qquad x = 4$ pt. di massimo.

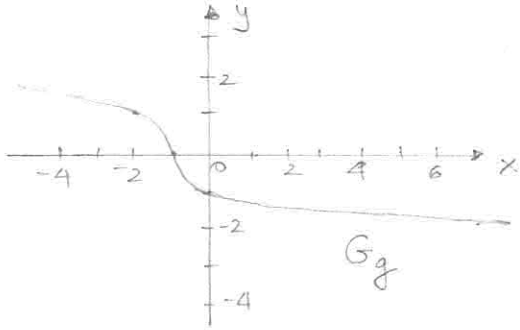
v) $\{x \in \mathbb{R} : 0 \le f(x) \le 1\} =]-\infty, 0] \cup [2, 3]$.

3) $f : \mathbb{R} \setminus \{1\} \to \mathbb{R} \qquad f(x) = 1 - \dfrac{2}{(x-1)^4}$

$g : \mathbb{R} \to \mathbb{R} \qquad g(x) = -\sqrt[3]{x+1}$

i)





$f(\mathbb{R} \setminus \{1\}) = \underline{\underline{]-\infty, 1[}} \qquad g(\mathbb{R}) = \underline{\underline{\mathbb{R}}}.$

ii) $(f+g)(0) = f(0) + g(0) = -1 + (-1) = \underline{\underline{-2}}.$

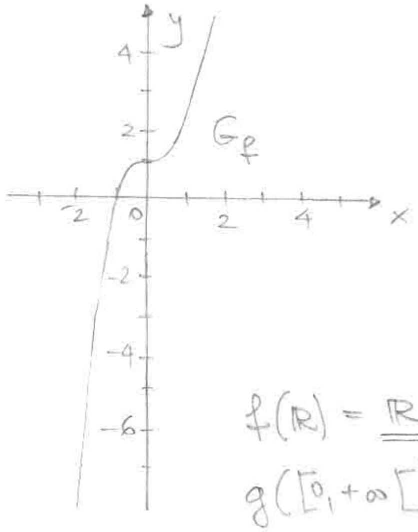
$(fg)(0) = f(0) \cdot g(0) = (-1)\cdot(-1) = \underline{\underline{1}}.$

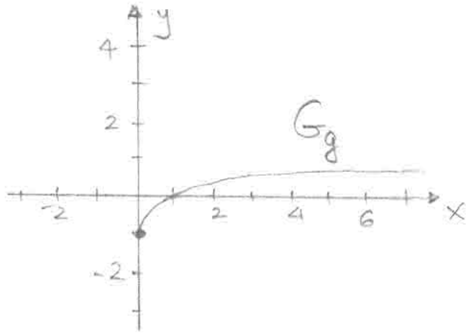
$\left(\dfrac{g}{f}\right)(-1) \overset{\text{poiché } f(-1) \neq 0}{=} \dfrac{g(-1)}{f(-1)} = \dfrac{0}{7/8} = \underline{\underline{0}}$

$\left(\dfrac{f}{g}\right)(-1) \nexists$ poiché $g(-1) = 0$.

4) $f : \mathbb{R} \to \mathbb{R} \qquad f(x) = x^3 + 1$

$g : [0, +\infty[\to \mathbb{R} \qquad g(x) = \sqrt[4]{x} - 1$





$f(\mathbb{R}) = \underline{\mathbb{R}}$

$g([0, +\infty[) = \underline{\underline{[-1, +\infty[}}.$

ii) $(g \circ f)(x) \underset{\uparrow}{=} g(f(x)) \quad \forall\, x \in [-1, +\infty[$

$f([-1, +\infty[) = [0, +\infty[= \operatorname{dom} g$

e quindi l'insieme di def. della funzione $g \circ f$ è $[-1, +\infty[$.

Si ha $(g \circ f)(x) = g(f(x)) = \sqrt[4]{f(x)} - 1 = \sqrt[4]{x^3+1} - 1$
$\forall\, x \in [-1, +\infty[$.

$(f \circ g)(x) \underset{\uparrow}{=} f(g(x)) \quad \forall\, x \in [0, +\infty[$

$g([0, +\infty[) = [-1, +\infty[\subset \operatorname{dom} f = \mathbb{R}$

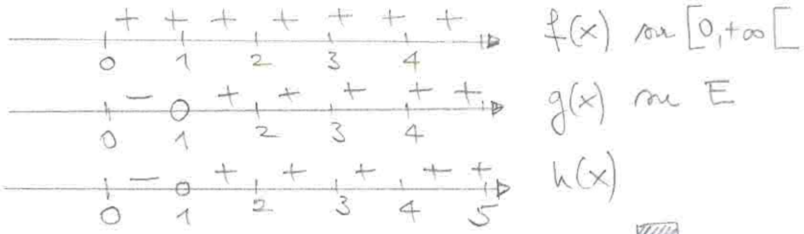
e quindi l'insieme di def. della funzione $f \circ g$ è $[0, +\infty[$.

Si ha $(f \circ g)(x) = f(g(x)) = (g(x))^3 + 1 = (\sqrt[4]{x} - 1)^3 + 1$
$\forall\, x \in [0, +\infty[$. □

iii) $\dfrac{1}{f(x)}$ è definita $\forall\, x \in \mathbb{R} : f(x) \neq 0$, quindi per $x \in \mathbb{R} \setminus \{-1\}$.

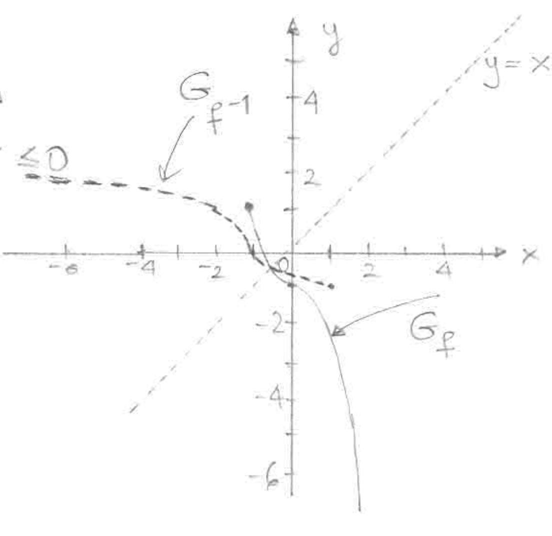
$\dfrac{1}{g(x)}$ è definita $\forall\, x \in [0, +\infty[: g(x) \neq 0$, e quindi per $x \in [0, +\infty[\setminus \{1\}$.

iv) $E = \{x \in \mathbb{R} : f(x)$ è ben definito, $g(x)$ è ben definito, $g(x) \neq 0\}$
$= [0, +\infty[\setminus \{1\}$.



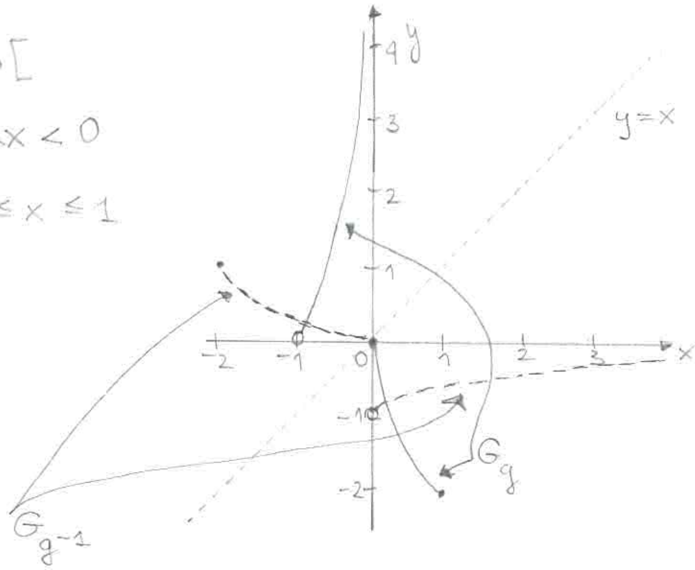
5) $f : [-1, +\infty[\;\to\;]-\infty, 1]$

i) $f(x) = \begin{cases} 2x^2 - 1 & \text{se } -1 \leq x \leq 0 \\ -x^3 - 1 & \text{se } x > 0 \end{cases}$



$g: \,]-1,1] \to [-2,+\infty[$

$$g(x) = \begin{cases} -\frac{1}{x} - 1 & \text{se } -1 < x < 0 \\ -2\sqrt{x} & \text{se } 0 \le x \le 1 \end{cases}$$



ii) $\nexists \min\limits_{[-1,+\infty[} f$; $\max\limits_{[-1,+\infty[} f = \underline{\underline{1}}$ $\underline{\underline{x=-1 \text{ pt. di massimo.}}}$

$\min\limits_{]-1,1]} g = \underline{\underline{-2}}$ $\underline{\underline{x = 1 \text{ pt. di minimo}}}$; $\nexists \max\limits_{]-1,1]} g.$

iii) $\nexists \min\limits_{[-2,+\infty[} g$ $\max\limits_{[-2,+\infty[} g = \underline{\underline{1}}$ $\underline{\underline{x=-2 \text{ pt. di massimo.}}}$