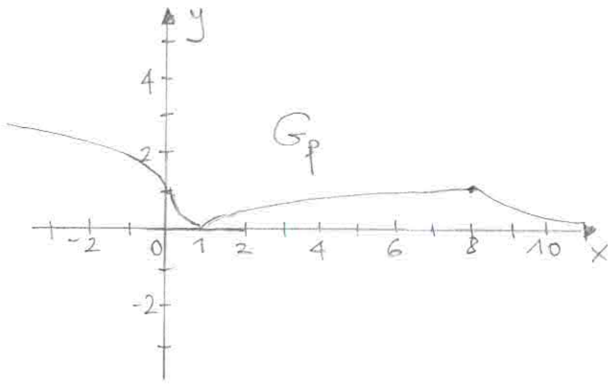
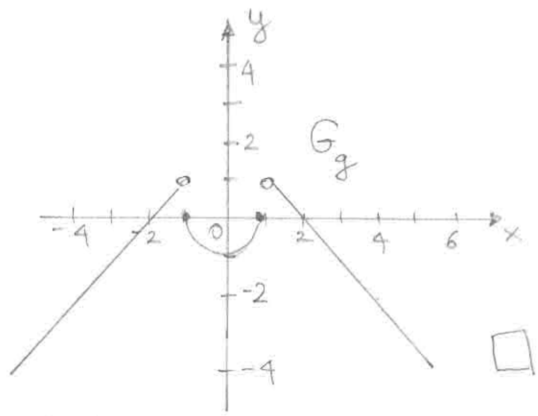
Università degli Studi di Trento — Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione — CdL in Filosofia
VERIFICA SETTIMANALE DI ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2012/13 — Rovereto, 5 – 9 novembre 2012

1) i) $f : \mathbb{R} \to \mathbb{R}$ $\qquad$ $g : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} |\sqrt[3]{x} - 1| & \text{se } x \leq 8 \\ \dfrac{1}{x-7} & \text{se } x > 8 \end{cases} \qquad g(x) = \begin{cases} x^4 - 1 & \text{se } x \in [-1,1] \\ -|x| + 2 & \text{altrimenti} \end{cases}$$





ii) $\max_{[0,8]} f = |-1| = \underline{\underline{1}}$ $\quad$ $\underline{\underline{x=0,\ x=8}}$ punti di massimo

$\min_{[0,8]} f = \underline{\underline{0}}$ $\quad$ $\underline{\underline{x=1}}$ pt. di minimo.

iii) Sia $x \in [-1,1]$, allora $-x \in [-1,1]$ e

$$g(-x) = (-x)^4 - 1 = x^4 - 1 = g(x) \quad \forall x \in [-1,1].$$

Sia $x \notin [-1,1]$, allora anche $-x \notin [-1,1]$ e si ha

$$g(-x) = -|-x| + 2 = -|x| + 2 = g(x), \quad \forall x \notin [-1,1]$$

quindi g è pari.

iv) $f\big|_{]-\infty,1]}$ è decrescente;

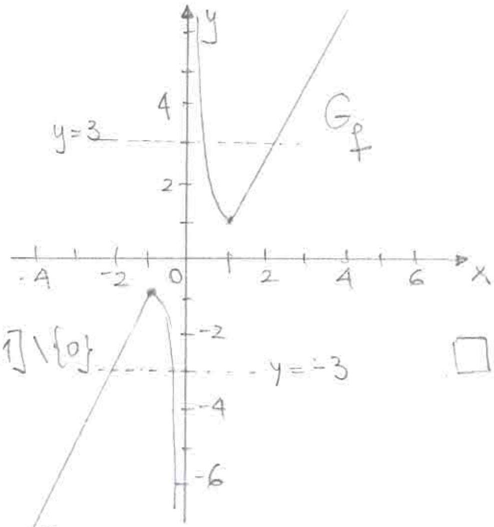
$f\big|_{[1,8]}$ è crescente;

$f\big|_{[8,+\infty[}$ è decrescente.

v) $(f \circ g)(1) = f(g(1)) = f(0) = \underline{\underline{1}}$ $\qquad$ $(g \circ f)(8) = g(f(8)) = g(1) = \underline{\underline{0}}$

2) i) $f : \mathbb{R}\setminus\{0\} \to \mathbb{R}$,

$$f(x)=\begin{cases} 2x+1 & \text{se } x<-1 \\ 1/x & \text{se } x\in[-1,1]\setminus\{0\} \\ 2x-1 & \text{se } x>1 . \end{cases}$$



ii) Sia $x\in[-1,1]\setminus\{0\}$; allora $-x\in[-1,1]\setminus\{0\}$

$$f(-x)=\frac{1}{-x}=-\frac{1}{x}=-f(x)$$

Sia $x\in\,]-\infty,-1[$, allora $-x\in\,]1,+\infty[$;

si ha

$$f(-x)=2(-x)-1=-(2x+1)=-f(x) \quad \forall x\in\,]-\infty,-1[.$$

Sia $x\in\,]1,+\infty[$, allora $-x\in\,]-\infty,-1[$; si ha

$$f(-x)=2(-x)+1=-(2x-1)=-f(x) \quad \forall x\in\,]1,+\infty[.$$

$\square$

iii) $f|_{]-\infty,-1]}$ è crescente;

$f|_{[-1,0[}$ è decrescente;

$f|_{]0,1]}$ è decrescente;

$f|_{[1,+\infty[}$ è crescente. $\square$

iv) $\min\limits_{[-3,0[} f \quad \nexists$

$\max\limits_{[-3,0[} f = f(-1) = \underline{\underline{-1}}$ $\qquad$ $\underline{\underline{x=-1 \text{ pt. di massimo.}}}$ $\square$

v) $A=\{x\in\mathbb{R}\setminus\{0\} : -3\le f(x)\le 3\} =$

$$= \underline{\underline{\left[-2,-\tfrac{1}{3}\right]\cup\left[\tfrac{1}{3},2\right]}} .$$

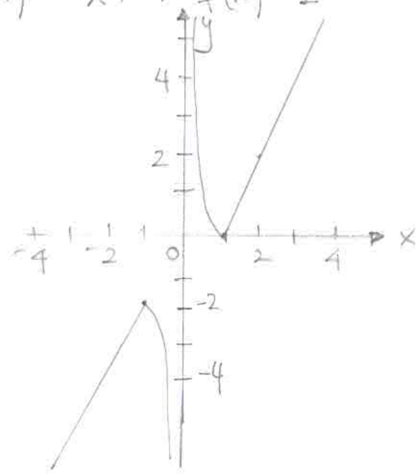
$2x+1=-3 \iff x=-2$

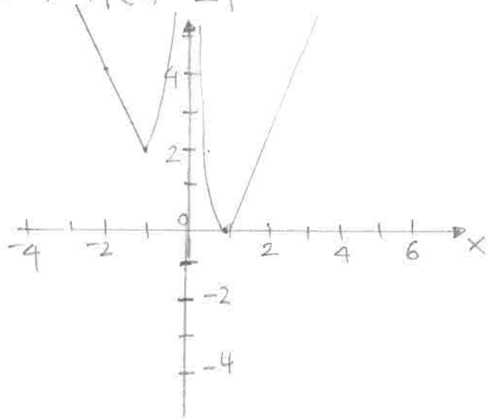
$1/x=-3 \iff x=-1/3$

$1/x=3 \iff x=1/3$

$2x-1=3 \iff x=2$

vi) $x \longmapsto f(x)-1$ $\qquad\qquad$ $x \longmapsto |f(x)-1|$





3) $3|x+1|>6 \iff |x+1|>2 \iff x+1<-2$ o $x+1>2$

$\iff x<-3$ o $x>1$

$\underline{\underline{S =]-\infty,-3[\cup]1,+\infty[}}.$

$|x^2-x^3|=0 \iff x^2-x^3=0 \iff x^2(1-x)=0$

$\underline{\underline{S=\{0,1\}}}.$

$2|x^2-1|\leq 4 \iff |x^2-1|\leq 2 \iff -2\leq x^2-1\leq 2$

$\iff \underbrace{-1\leq x^2}_{\text{sempre vero!}}\leq 3$

$\underline{\underline{S=[-\sqrt{3},\sqrt{3}]}}.$

$||x-1|-3|>0 \iff |x-1|-3\neq 0 \iff |x-1|\neq 3$

$\iff x-1\neq -3$ o $x-1\neq 3$

$\underline{\underline{S=\mathbb{R}\setminus\{-2,4\}}}.$

4) i) $x|x-2|\leq 3 \iff \begin{cases} x-2\geq 0 \\ x^2-2x-3\leq 0 \end{cases}$ o $\begin{cases} x-2<0 \\ -x^2+2x-3\leq 0 \end{cases}$

$\iff \begin{cases} x\geq 2 \\ -1\leq x\leq 3 \end{cases}$ o $\begin{cases} x<2 \\ x^2-2x+3\geq 0 \end{cases}$

$x^2-2x-3=(x-3)(x+1)$ $\qquad \iff \{2\leq x\leq 3$ o $\{x<2$ $\quad$ poiché $x^2-2x+3 \geq$ $\forall x\in\mathbb{R}$

$\Rightarrow \underline{\underline{S=]-\infty,3]}}.$

$$\frac{x^2-2}{2|x|+1} < 1 \quad \Longleftrightarrow \quad x^2-2 < 2|x|+1 \quad \Longleftrightarrow \quad x^2-2|x|-3<0$$

poiché $2|x|+1>0 \ \forall x \in \mathbb{R}$

$$\Longleftrightarrow \begin{cases} x \geq 0 \\ x^2-2x-3<0 \end{cases} \quad \text{o} \quad \begin{cases} x<0 \\ x^2+2x-3<0 \end{cases}$$

$$\Longleftrightarrow \begin{cases} x \geq 0 \\ (x-3)(x+1)<0 \end{cases} \quad \text{o} \quad \begin{cases} x<0 \\ (x+3)(x-1)<0 \end{cases}$$

$$\Longleftrightarrow \begin{cases} x \geq 0 \\ x \in]-1,3[\end{cases} \quad \text{o} \quad \begin{cases} x<0 \\ x \in]-3,1[. \end{cases}$$

$$\Rightarrow \underline{\underline{S =]-3,3[.}} \qquad \square$$

$$x^2-|x-2| \geq 0 \quad \Longleftrightarrow \begin{cases} x-2 \geq 0 \\ x^2-x+2 \geq 0 \end{cases} \quad \text{o} \quad \begin{cases} x-2<0 \\ x^2+x-2 \geq 0 \end{cases}$$

$$\Longleftrightarrow \begin{cases} x \geq 2 \\ \mathbb{R} \end{cases} \quad \text{o} \quad \begin{cases} x<2 \\ x \leq -2 \ \text{o} \ x \geq 1 \end{cases}$$

$$\Rightarrow \underline{\underline{S =]-\infty,-2] \cup [1,+\infty[.}} \qquad \square$$

$$3x^2-x|x|=4 \quad \Longleftrightarrow \begin{cases} x \geq 0 \\ 3x^2-x^2=4 \end{cases} \quad \text{o} \quad \begin{cases} x<0 \\ 3x^2+x^2=4 \end{cases}$$

$$\Longleftrightarrow \begin{cases} x \geq 0 \\ x^2=2 \end{cases} \quad \text{o} \quad \begin{cases} x<0 \\ x^2=1 \end{cases}$$

$$\Rightarrow \underline{\underline{S = \{-1, \sqrt{2}\}.}} \qquad \square$$

ii) $\left(\frac{1}{3}\right)^{|x+2|} > 3 \quad \Longleftrightarrow \quad 3^{-|x+2|} > 3^1 \quad \underset{3^x \uparrow}{\Longleftrightarrow} \quad -|x+2|>1 \quad \Rightarrow \underline{\underline{S=\emptyset.}}$

$4^{|x(x-1)|} \geq 1 \quad \Longleftrightarrow \quad 4^{|x(x-1)|} \geq 4^0 \quad \Longleftrightarrow \quad |x(x-1)| \geq 0 \quad \Rightarrow \underline{\underline{S=\mathbb{R}.}}$

$2^{-|x^2-2|+1}=1 \quad \Longleftrightarrow \quad -|x^2-2|+1=0 \quad \Longleftrightarrow \quad |x^2-2|=1$

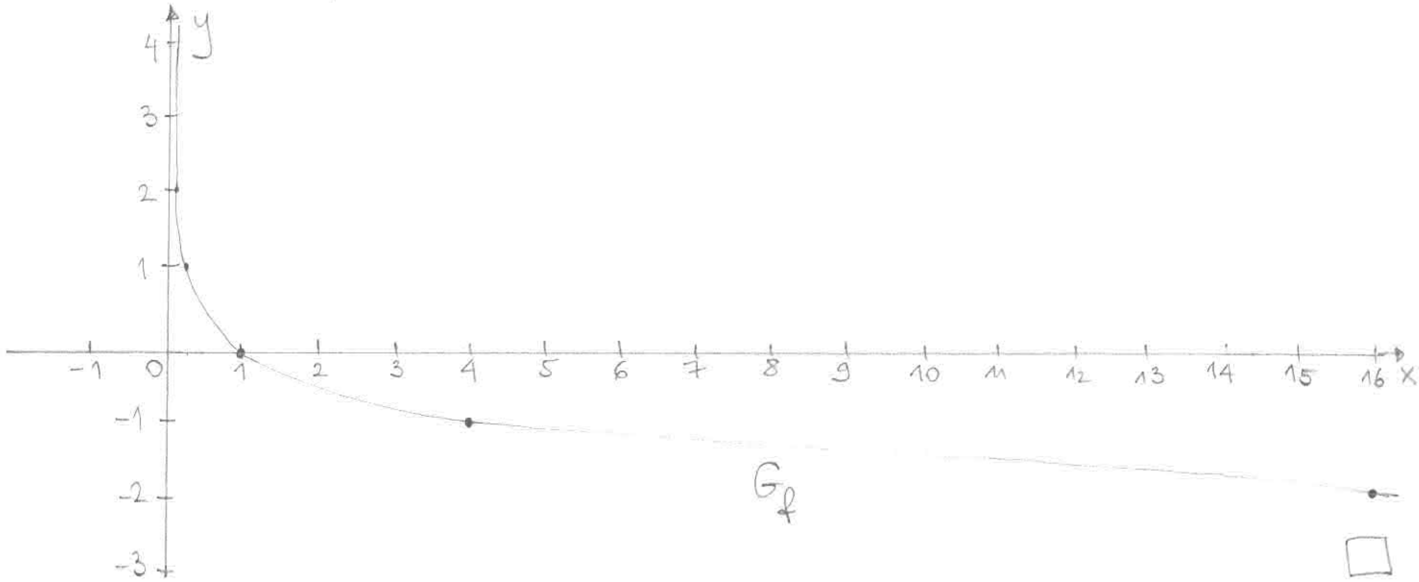
$\Longleftrightarrow \quad x^2-2=-1 \quad \text{o} \quad x^2-2=1$

$\Longleftrightarrow \quad x^2=1 \quad \text{o} \quad x^2=3 \qquad \Rightarrow \underline{\underline{S=\{-\sqrt{3},-1,1,\sqrt{3}\}.}}$

$3^{-|x|-1} \geq -\frac{1}{3} \qquad \underline{\underline{S=\mathbb{R}}}$, poiché $3^x>0 \ \forall x \in \mathbb{R}.$ $\blacksquare$

5) i) $\log_{1/4} \frac{1}{16} = 2$ $\log_{1/4} \frac{1}{4} = 1$ $\log_{1/4} 1 = 0$ $\log_{1/4} 4 = -1$

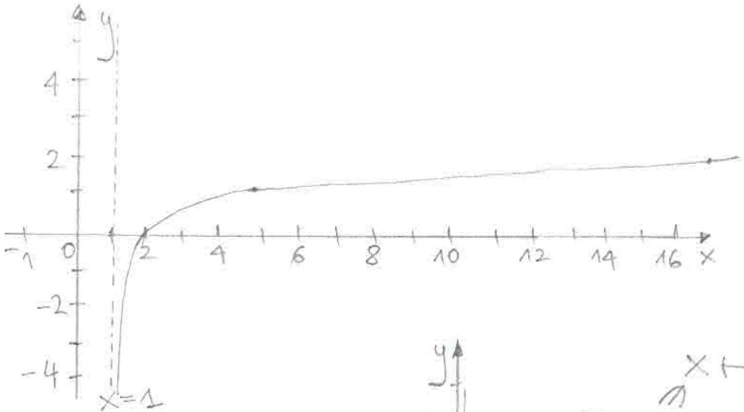
$\log_{1/4} 16 = -2$



$\square$

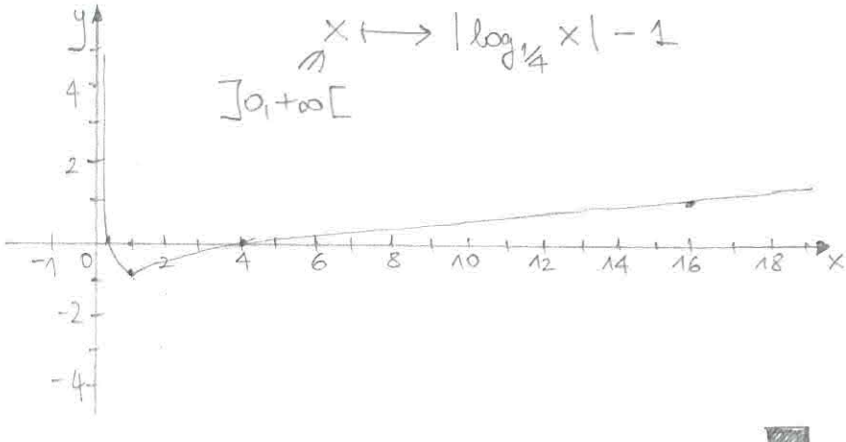
ii) $x \mapsto -\log_{1/4}(x-1)$

$\rotatebox{90}{$\in$}$ $]1, +\infty[$



$x \longmapsto |\log_{1/4} x| - 1$

$\rotatebox{90}{$\in$}$ $]0, +\infty[$



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