Università degli Studi di Trento – Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione – CdL in Filosofia
SECONDA PROVA INTERMEDIA DI ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2012/2013 – Rovereto, 20 dicembre 2012

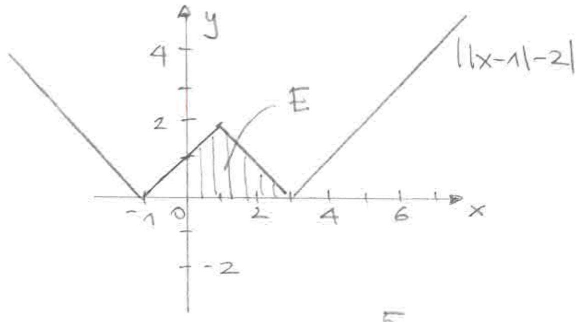
FILA (A)

1) i) $\displaystyle\int_1^2 \frac{3x^2+\sqrt{x}}{x^3}\,dx = \int_1^2 \left(\frac{3}{x} + x^{-5/2}\right)dx = \left[3\log|x| - \frac{x^{-3/2}}{3/2}\right]_1^2 =$

$\displaystyle = \left(3\log 2 - \frac{\cancel{2}}{3}\,\frac{1}{\cancel{2}\sqrt{2}}\right) - \left(3\cancel{\log 1} - \frac{2}{3}\right) = \underline{3\log 2 - \frac{\sqrt{2}}{6} + \frac{2}{3}}.$

$\displaystyle\int_0^3 \big||x-1|-2\big|\,dx =$

$= \text{area}(E) = \underline{\frac{7}{2}}.$



$\displaystyle\sum_{n=1}^{4}\left[\int_n^{n+1}\left(e^{2x} + \frac{1}{x+3}\right)dx\right] =$

$\displaystyle = \int_1^5 \left(e^{2x} + \frac{1}{x+3}\right)dx = \left[\frac{e^{2x}}{2} + \log|x+3|\right]_1^5 =$

$\displaystyle = \left(\frac{e^{10}}{2} + \log 8\right) - \left(\frac{e^2}{2} + \log 4\right) = \frac{e^{10}-e^2}{2} + \log\frac{\cancel{8}^{\,2}}{\cancel{4}} = \underline{\frac{e^{10}-e^2}{2} + \log 2}.$

ii) $\displaystyle\sum_{k=2}^{5} (-1)^k \frac{k}{k^2-1} = \frac{2}{3} - \frac{3}{8} + \frac{4}{15} - \frac{5}{24}$

$\displaystyle = \frac{10+4}{15} - \frac{9+5}{24} = \frac{14}{15} - \frac{14}{24} = \frac{14}{15} - \frac{7}{12} =$

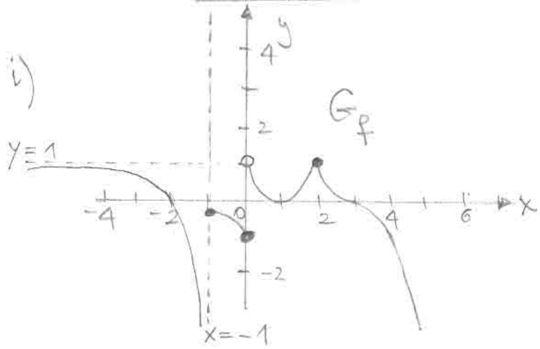
$\displaystyle = \frac{56-35}{60} = \frac{21}{60} = \underline{\frac{7}{20}}$

iii) $\displaystyle\frac{2}{3}x^2 + \frac{4}{5}x^3 + \frac{8}{7}x^4 + \dots + \frac{256}{17}x^9 = \underline{\sum_{n=1}^{8} \frac{2^n}{2n+1}\,x^{n+1}}$

2) $f: \mathbb{R} \to \mathbb{R}$

$f(x) = \begin{cases} \dfrac{1}{x+1} + 1 & \text{se } x < -1 \\ -3^x & \text{se } -1 \le x \le 0 \\ (x-1)^4 & \text{se } 0 < x \le 2 \\ -(x-3)^3 & \text{se } x > 2 \end{cases}$

i)



ii) $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} \left(\frac{1}{x+1} + 1 \right) = \underline{\underline{1}}$;

$\lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1^-} \left(\frac{1}{x+1} + 1 \right) = \underline{\underline{-\infty}}$;

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} (x-1)^4 = \underline{\underline{1}}$;

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} \left(-(x-3)^3 \right) = \underline{\underline{1}}$;

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \left(-(x-3)^3 \right) = \underline{\underline{-\infty}}$. $\square$

iii) f è discontinua in $x = -1$: $\lim\limits_{x \to -1^-} f(x) = -\infty$, $f(-1) = -\frac{1}{3}$.

f è discontinua in $x = 0$: $\lim\limits_{x \to 0^+} f(x) = 1 \neq f(0) = -1$.

$\underline{\underline{y = 1}}$ è asintoto orizzontale per f, per $x \to -\infty$;

$\underline{\underline{x = -1}}$ è asintoto verticale per f. $\square$

iv) $\underline{\underline{x = -1, \; x = 2}}$ pt. di massimo locale stretto per f,

$\underline{\underline{x = 0, \; x = 1}}$ pt. di minimo locale stretto per f. $\square$

v) f non soddisfa le ipotesi del teorema di Weierstrass su $[0,2]$, non essendo continua in $x = 0$.

Comunque, f ha massimo e minimo su $[0,2]$;

$\min\limits_{[0,2]} f = -1$ $\quad x = 0$ pt. di minimo;

$\max\limits_{[0,2]} f = 1$ $\quad x = 2$ pt. di massimo. $\square$

vi) $$\int_{-1}^{2} f(x)\,dx = -\int_{-1}^{0} 3^x\,dx + \int_{0}^{2} (x-1)^4\,dx = \left[-\frac{3^x}{\log 3} \right]_{-1}^{0} + \left[\frac{(x-1)^5}{5} \right]_{0}^{2}$$

$$= \left(-\frac{1}{\log 3} + \frac{1}{3\log 3} \right) + \left(\frac{1}{5} + \frac{1}{5} \right) = \underline{\underline{-\frac{2}{3\log 3} + \frac{2}{5}}} . \quad \blacksquare$$

3) • $x|x-3|+x^2 \le 2 \iff \begin{cases} x \ge 3 \\ x(x-3)+x^2 \le 2 \end{cases}$ o $\begin{cases} x<3 \\ -x(x-3)+x^2 \le 2 \end{cases}$

$\iff \begin{cases} x \ge 3 \\ 2x^2-3x-2 \le 0 \end{cases}$ o $\begin{cases} x<3 \\ +3x \le 2 \end{cases}$

$\iff \begin{cases} x \ge 3 \\ -\frac{1}{2} \le x \le 2 \end{cases}$ o $\begin{cases} x<3 \\ x \le \frac{2}{3} \end{cases}$ $\qquad \underline{S = \left]-\infty, \tfrac{2}{3}\right]}$.

• $\dfrac{e^{-x^2} e^{2x}}{e^{|x|}} \le 1 \iff e^{-x^2+2x-|x|} \le e^0 \underset{e^x \nearrow}{\iff} -x^2+2x-|x| \le 0$

$\iff \begin{cases} x \ge 0 \\ -x^2+x \le 0 \end{cases}$ o $\begin{cases} x<0 \\ -x^2+3x \le 0 \end{cases}$

$\iff \begin{cases} x \ge 0 \\ x^2-x \ge 0 \end{cases}$ o $\begin{cases} x<0 \\ x^2-3x \ge 0 \end{cases}$

$\iff \begin{cases} x \ge 0 \\ x \le 0 \text{ o } x \ge 1 \end{cases}$ o $\begin{cases} x<0 \\ x \le 0 \text{ o } x \ge 3 \end{cases}$ $\Rightarrow \underline{S = \left]-\infty, 0\right] \cup \left[1, +\infty\right[}$

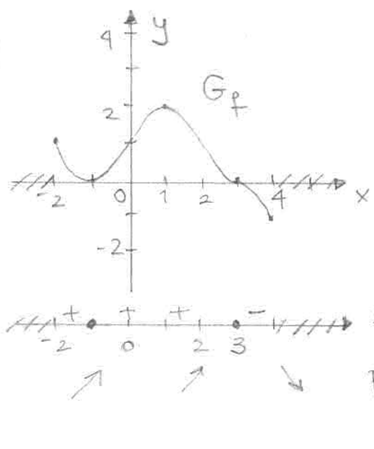
• $\log_{1/3}(x^2-1) + \log_3(x+5) < 0 \iff -\log_3(x^2-1) + \log_3(x+5) < 0$

$\iff \log_3(x+5) < \log_3(x^2-1)$

$\iff \begin{cases} x+5>0 \\ x^2-1>0 \\ x+5 < x^2-1 \end{cases} \iff \begin{cases} x>-5 \\ x<-1 \text{ o } x>1 \\ 0<x^2-x-6 \end{cases} \iff \begin{cases} x>-5 \\ x<-1 \text{ o } x>1 \\ x<-2 \text{ o } x>3 \end{cases}$

$\underline{\underline{S = \left]-5, -2\right[\cup \left]3, +\infty\right[}}$.

4)



Per il Teorema fondamentale del calcolo integrale, $F'(x) = f(x) \ \forall x \in [-2,4]$.

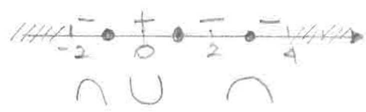
i) F è $\underline{\text{crescente su } [-2,3]}$

F è $\underline{\text{decrescente su } [3,4]}$. □

ii) $\underline{x=-2}$ $\underline{\text{pt. di minimo locale}}$ di F su $[-2,4]$;

$\underline{x=3}$ $\underline{\text{pt. di massimo locale}}$ di F su $[-2,4]$;

$\underline{x=4}$ $\underline{\text{pt. di minimo locale}}$ di F " □

iii)  $f' = F''$

F

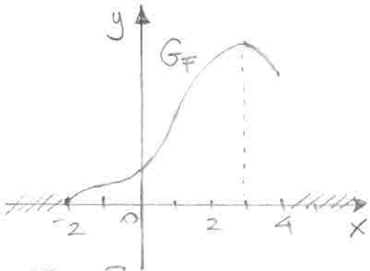
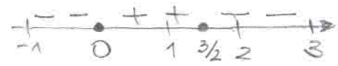


Grafico qualitativo di F.

iv) $\underline{\underline{[-2,3]}}$.

5) i) $f(x) = (3x - 2x^2)e^x$ • $\mathrm{dom} f = \mathbb{R} =]-\infty, +\infty[$

• segno di f  f

• $\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \dfrac{(3x - 2x^2)}{e^{-x}} = 0$ $y = 0$ asint. orizzontale per f per $x \to -\infty$.

$\lim_{x \to +\infty} f(x) = -\infty$.

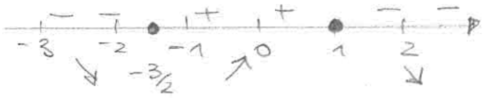
• f è continua su $\mathbb{R}$ (somma, prodotto di funzioni continue su $\mathbb{R}$).

• $\mathrm{dom} f' = \mathbb{R}$ $f'(x) = (3 - 4x)e^x + (3x - 2x^2)e^x = e^x(-2x^2 - x + 3)$

$-2x^2 - x + 3 = 0 \iff x_{1/2} = \dfrac{1 \pm \sqrt{1 + 24}}{-4} = \dfrac{1 \pm 5}{-4} = \begin{cases} -3/2 \\ 1 \end{cases}$.

$f(-3/2) = \left(-\frac{9}{2} - \cancel{2} \cdot \frac{9}{\cancel{4}_2}\right)e^{-3/2} = \underline{\underline{-9e^{-3/2}}}$ (~ -2)

$f(1) = (3-2)e = \underline{\underline{e}}$.

 f'

f

$x = -3/2$ pt. di min. loc. stretto per f

$x = 1$ pt. di max. loc. stretto per f ;

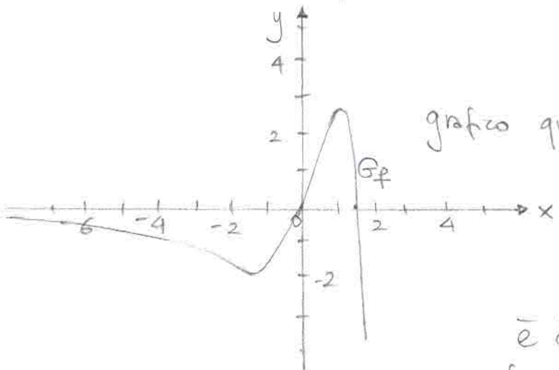


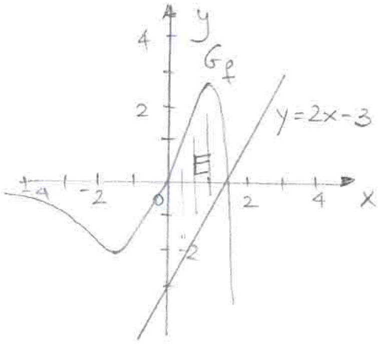
grafico qualitativo di f

ii) $F(x) = (-2x^2 + 7x - 7)e^x$

è derivabile su $\mathbb{R}$ (somma e prodotto di funzioni derivabili su $\mathbb{R}$)

Inoltre $F'(x) = (-4x+7)e^x + (-2x^2+7x-7)e^x = (3x-2x^2)e^x =$

$= f(x) \quad \forall x \in \mathbb{R}.$ $\square$



$$\text{area}(E) = \int_0^{3/2} \left[f(x) - (2x-3) \right] dx =$$

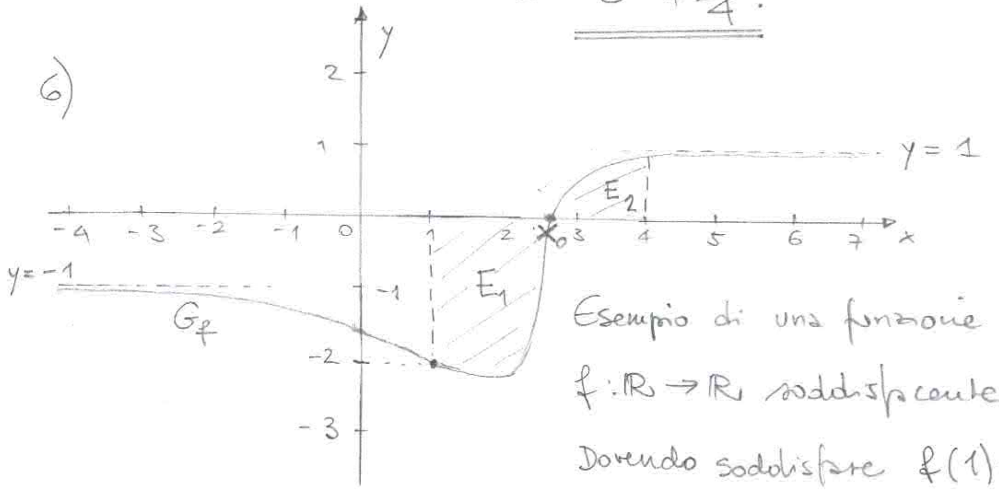
$$= \left[F(x) \right]_0^{3/2} - \int_0^{3/2} (2x-3)\,dx =$$

$$= F(3/2) - F(0) - \left[x^2 - 3x \right]_0^{3/2} =$$

$$= \left(-\cancel{2} \cdot \frac{9}{\cancel{4}_2} + 7 \cdot \frac{3}{2} - 7 \right) e^{3/2} + 7 - \frac{9}{4} + \frac{9}{2}$$

$$= \underline{\underline{-e^{3/2} + \frac{37}{4}}}.$$ ▨

6)



Esempio di una funzione $f : \mathbb{R} \to \mathbb{R}$ soddisfacente i), ii).

Dovendo soddisfare $f(1) = -2$ e f decrescente su $]-\infty, 2[$ (essendo $f'(x) < 0$ su $]-\infty, 2[$) si ha $\int_1^2 f(x)\,dx < -2$. Poiché $\lim_{x \to +\infty} f(x) = 1$ e f è crescente su $]2, +\infty[$ (essendo $f'(x) > 0$ su $]2, +\infty[$) si ha $f(x) < 1 \quad \forall x \in]2, +\infty[$ e si ha

$$\int_1^4 f(x)\,dx = \int_1^{x_0} f(x)\,dx + \int_{x_0}^4 f(x)\,dx = -\text{area}(E_1) + \text{area}(E_2) < 0$$

qualunque sia $x_0 \in]2, 4]$.

Quindi non si trova una funzione $f : \mathbb{R} \to \mathbb{R}$ che soddisfi i) ii) <u>e</u> iii). ▨

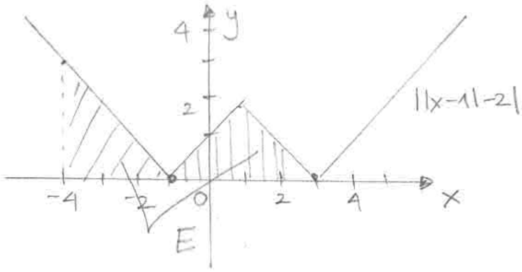
FILA (B)

1) i) $$\int_1^2 \frac{3x-\sqrt{x}}{x^2}\,dx = \int_1^2\left(\frac{3}{x} - x^{-3/2}\right)dx = \left[3\log|x| + \frac{x^{-1/2}}{-\frac{1}{2}}\right]_1^2 =$$

$$= \left(3\log 2 + 2\cdot\frac{1}{\sqrt{2}}\right) - \left(\cancel{3\log 1} + 2\right) = \underline{\underline{3\log 2 + \sqrt{2} - 2.}}$$

$$\int_{-4}^{2} \big||x-1|-2\big|\,dx = \text{area}\,(E)$$

$$= \frac{9}{2} + \frac{7}{2} = \underline{\underline{8.}}$$



$$\sum_{n=1}^{4}\left[\int_n^{n+1}\left(e^x + \frac{1}{2x+1}\right)dx =\right.$$

$$= \int_1^5 e^x\,dx + \frac{1}{2}\int_1^5 \frac{2}{2x+1}\,dx = \left[e^x\right]_1^5 + \frac{1}{2}\left[\log|2x+1|\right]_1^5 =$$

$$= e^5 - e + \frac{1}{2}\left[\log 11 - \log 3\right] = e^5 - e + \log\sqrt{\frac{11}{3}}\,.$$

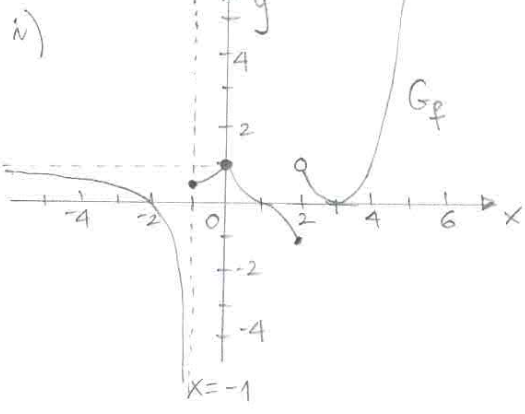
ii) $$\sum_{k=2}^{5} (-1)^k \frac{k}{k^2+1} = \frac{2}{5} - \frac{3}{10} + \frac{4}{17} - \frac{5}{26} =$$

$$= \frac{1}{10} + \frac{4}{17} - \frac{5}{26} = \frac{1}{10} + \frac{19}{442} = \underline{\underline{\frac{632}{4420}}}\,.$$

iii) $$\frac{3}{2}x^2 + \frac{5}{4}x^3 + \frac{7}{8}x^4 + \dots + \frac{17}{256}x^9 = \underline{\underline{\sum_{n=1}^{8} \frac{2n+1}{2^n}\,x^{n+1}}}$$

2) $f:\mathbb{R}\to\mathbb{R}$

i)



ii) $$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}\left(\frac{1}{x+1}+1\right) = \underline{\underline{1}}\,;$$

$$\lim_{x\to-1^-} f(x) = \lim_{x\to-1^-}\left(\frac{1}{x+1}+1\right) = \underline{\underline{-\infty}}\,;$$

$$\lim_{x\to0^+} f(x) = \lim_{x\to0^+} -(x-1)^3 = \underline{\underline{1}}\,;$$

$$\lim_{x\to2^+} f(x) = \lim_{x\to2^+} (x-3)^4 = \underline{\underline{1}}\,;$$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} (x-3)^4 = \underline{\underline{+\infty}}\,.$$

iii) f è discontinua in $x=-1$: $\lim_{x\to -1^-} f(x) = -\infty$, $f(-1) = 1/2$.

f è discontinua in $x=2$: $\lim_{x\to 2^+} f(x) = 1 \neq f(2) = -1$. $\square$

$\underline{\underline{y=1}}$ è asintoto orizzontale per f, per $x\to-\infty$;

$\underline{\underline{x=-1}}$ è asintoto verticale per f. $\square$

iv) $\underline{\underline{x=0}}$ pt. di massimo locale stretto per f,

$\underline{\underline{x=2}}$ pt. di minimo locale stretto per f.

$\underline{\underline{x=3}}$ " . $\square$

v) f soddisfa le ipotesi del teorema di Weierstrass su $[0,2]$, essendo f continua su un intervallo chiuso e limitato.

$\min_{[0,2]} f = -1$ $x=2$ pt. di minimo;

$\max_{[0,2]} f = 1$ $x=0$ pt. di massimo. $\square$

vi) $$\int_{-1}^{2} f(x)\,dx = \int_{-1}^{0} 2^x\,dx - \int_{0}^{2} (x-1)^3\,dx = \left[\frac{2^x}{\log 2}\right]_{-1}^{0} - \left[\frac{(x-1)^4}{4}\right]_{0}^{2} =$$

$$= \left(\frac{1}{\log 2} - \frac{1}{2\log 2}\right) - \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{4}}\right) = \underline{\underline{\frac{1}{2\log 2}}}$$

3) • $x|x-3| + x^2 < 2$ $\underline{\underline{S = \left]-\infty, 2/3\right[}}$ (vedi FILA A Es. 3))

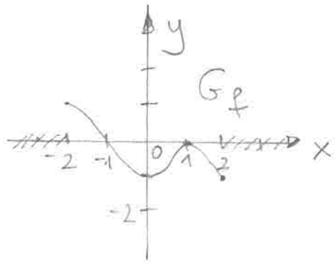
• $\dfrac{e^{-x^2} e^{2x}}{e^{|x|}} \geq 1$ $\underline{\underline{S = [0,1]}}$ "

• $\log_{1/3}(x^2-4) + \log_3(2x+4) < 0 \iff -\log_3(x^2-4) + \log_3(2x+4) < 0$

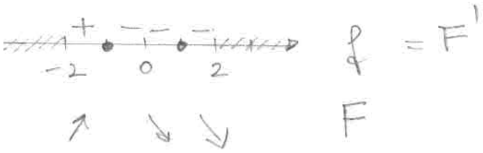
$\iff \log_3(2x+4) < \log_3(x^2-4)$

$$\iff \begin{cases} 2x+4>0 \\ x^2-4>0 \\ 2x+4 < x^2-4 \end{cases} \iff \begin{cases} x>-2 \\ x<-2 \text{ o } x>2 \\ 0 < x^2-2x-8 \end{cases} \iff \begin{cases} x>2 \\ 0<(x-4)(x+2) \end{cases}$$

$$\iff \begin{cases} x>2 \\ x<-2 \text{ o } x>4 \end{cases} \Rightarrow \underline{\underline{S = \left]4, +\infty\right[}} .$$

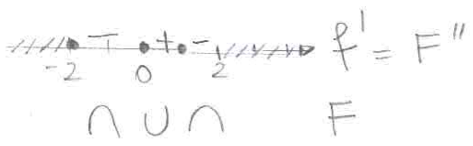
4) 

Per il Teorema fondamentale del calcolo integrale, $F'(x) = f(x)$ $\forall\, x \in [-2, 2]$.



i) F è crescente su $[-2, -1]$,
F è decrescente su $[-1, 2]$. ☐

ii) $x=-2$ pt. di minimo locale di F su $[-2, 2]$;
$x=-1$ pt. di massimo locale di F su " ;
$x=2$ pt. di minimo locale di F " . ☐

iii) 

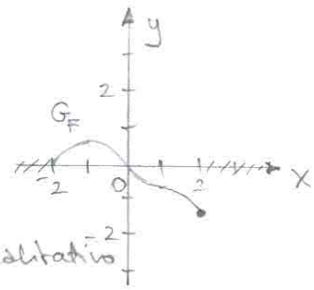
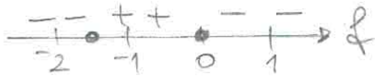


grafico qualitativo di F ☐

iv) $[-1, 2]$. ■

5) i) $f(x) = \dfrac{-3x - 2x^2}{e^x}$ • dom $f = \mathbb{R} =]-\infty, +\infty[$

• segno di f 

• $\lim\limits_{x \to -\infty} f(x) = -\infty$

$\lim\limits_{x \to +\infty} f(x) = 0$ $\quad y=0$ asintoto orizzontale per f, per $x \to +\infty$.

• f continua su $\mathbb{R}$ (somma e prodotto di funzioni continue su $\mathbb{R}$)!

• dom $f' = \mathbb{R}$ $\quad f'(x) = (-3-4x)e^{-x} + (3x + 2x^2)e^{-x}$

$$= (2x^2 - x - 3)e^{-x} = \frac{(2x^2 - x - 3)}{e^x}$$

$$2x^2-x-3=0 \iff x_{1/2}=\frac{1\pm\sqrt{1+24}}{4}=\frac{1\pm 5}{4}\ \begin{matrix}\nearrow 3/2\\ \searrow -1\end{matrix}$$

$$f(3/2)=\left(-9/2-\cancel{2}^{\,1}\cdot \frac{9}{\cancel{4}_{\,2}}\right)e^{-3/2}=\underline{\underline{-9e^{-3/2}}}\qquad(\sim -2)$$

$$f(-1)=(3-2)e=\underline{\underline{e}}$$

	-2	-1	0	1	2	
$+$	$+$	$\bullet$ $-$	$-$	$\bullet$ $+$		$f'(x)$
$\nearrow$		$\searrow$		$\nearrow$		$f(x)$

$x=-1$ pr. di massimo loc. stretto per f

$x=3/2$ pr. di minimo loc. stretto per f;

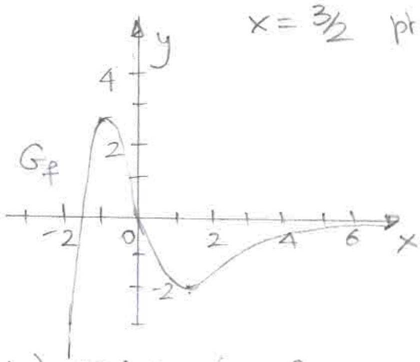


grafico qualitativo di f $\square$

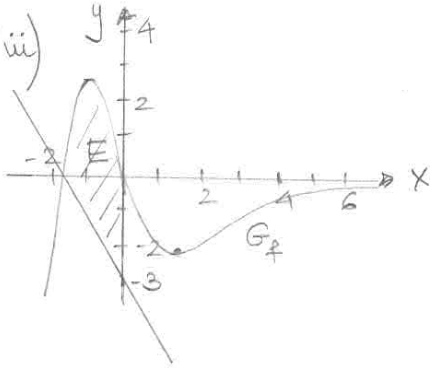
ii) $F(x)=(2x^2+7x+7)e^{-x}$ è una funzione derivabile su $\mathbb{R}$;

inoltre $F'(x)=(4x+7)e^{-x}-(2x^2+7x+7)e^{-x}$

$$=(-2x^2-3x)e^{-x}=\frac{-3x-2x^2}{e^x}=f(x)\quad\forall x\in\mathbb{R}$$

$\square$

iii)



$$\text{area}(E)=\int_{-3/2}^{0}\left[f(x)-(-2x-3)\right]dx$$

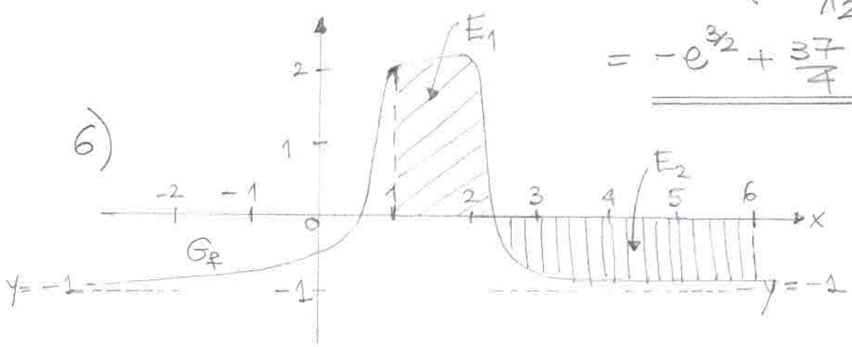
$$=\left[F(x)\right]_{-3/2}^{0}+\left[x^2+3x\right]_{-3/2}^{0}=$$

$$=F(0)-F(-3/2)-\left(\frac{9}{4}-\frac{9}{2}\right)=$$

$$=7-\left(\cancel{2}\cdot\frac{9}{\cancel{4}_2}-\frac{21}{2}+7\right)e^{3/2}+\frac{9}{4}$$

$$=\underline{\underline{-e^{3/2}+\frac{37}{4}}}$$

6)



$$\int_1^6 f(x)\,dx=\text{area}(E_1)-\text{area}(E_2)<0$$

FILA C

1) • $x|x-3|+x^2<2$ $\qquad S = \left]-\infty, \frac{2}{3}\right[$ $\qquad$ (vedi FILA A Es. 3)

• $\dfrac{e^{-x^2}e^{4x}}{e^{|x|}} \geq 1 \iff e^{-x^2+4x-|x|} \geq e^0 \underset{e^x \nearrow}{\iff} -x^2+4x-|x| \geq 0$

$\iff \begin{cases} x \geq 0 \\ -x^2+3x \geq 0 \end{cases}$ o $\begin{cases} x<0 \\ -x^2+5x \geq 0 \end{cases}$

$\iff \begin{cases} x \geq 0 \\ x^2-3x \leq 0 \end{cases}$ o $\begin{cases} x<0 \\ x^2-5x \leq 0 \end{cases}$

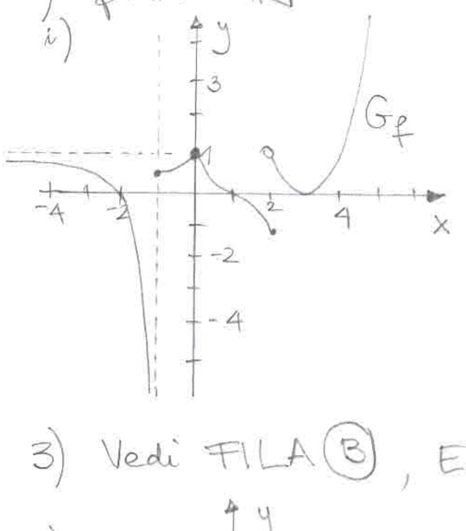
$\iff \begin{cases} x \geq 0 \\ 0 \leq x \leq 3 \end{cases}$ o $\begin{cases} x<0 \\ 0 \leq x \leq 5 \end{cases}$ $\Rightarrow S = [0,3]$.

• $\log_{1/3}(x^2-4) + \log_3(2x+4) \geq 0$. (vedi FILA B Es. 3)

$\iff \begin{cases} x>2 \\ (x-4)(x+2) \leq 0 \end{cases} \iff \begin{cases} x>2 \\ -2 \leq x \leq 4 \end{cases} \Rightarrow S =]2,4]$.

2) $f: \mathbb{R} \to \mathbb{R}$,

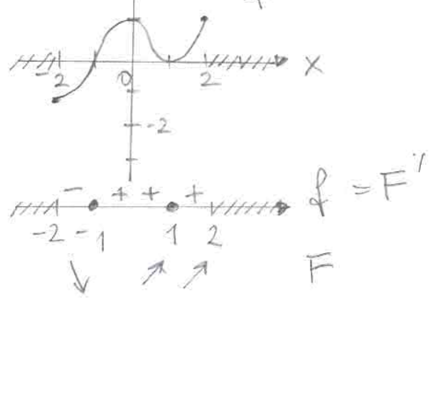
i)



Vedi FILA B Es. 2).

3) Vedi FILA B, Es. 1).

4)



Per il Teorema fondamentale del Calcolo integrale $F'(x) = f(x)$ $\forall x \in [-2,2]$.

i) F è decrescente su $[-2,-1]$,

F è crescente su $[-1,2]$.

ii) $x=-2$ pt. di massimo locale di F su $[-2,2]$;

$x=-1$ pt. di minimo locale di F "

$x=2$ pt. di massimo locale di F ". □

iii) 

$f' = F''$

$\cup \; \cap \; \cup$ $\quad F$

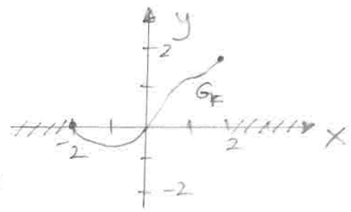
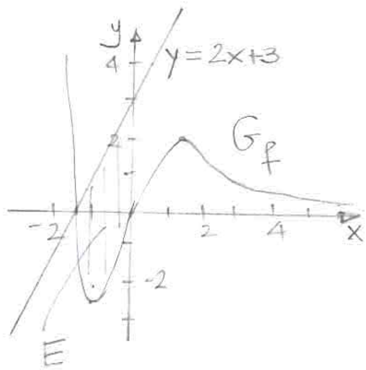


grafico qualitativo di F. □

iv) $[-1,2]$. ■

5) i) $f(x) = \dfrac{3x+2x^2}{e^x}$: questa funzione coincide con la funzione di FILA (B), Es. 5) cambiata di segno!



ii) $F(x) = (-2x^2-7x-7)e^{-x}$ è derivabile su $\mathbb{R}$ e $F'(x) = (-4x-7)e^{-x} + (2x^2+7x+7)e^{-x}$

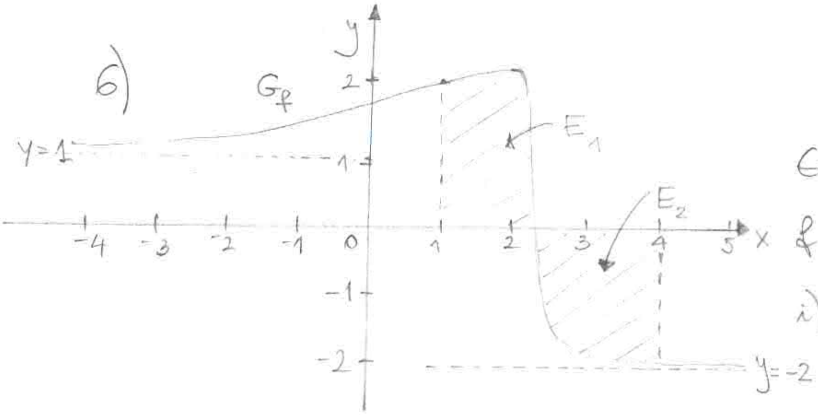
$= (2x^2+3x)e^{-x} = f(x)$

quindi F è una primitiva di f su $\mathbb{R}$. □

Risulta analog. a quanto fatto negli esercizi 5 delle altre file che

$$\text{area}(E) = -e^{3/2} + \frac{37}{4}$$ ■

6)



Esempio di una funzione $f:\mathbb{R}\to\mathbb{R}$ soddisfacente i) ii) e iii) :

$$\int_1^4 f(x)\,dx = \text{area}(E_1) - \text{area}(E_2) < 0$$ ■

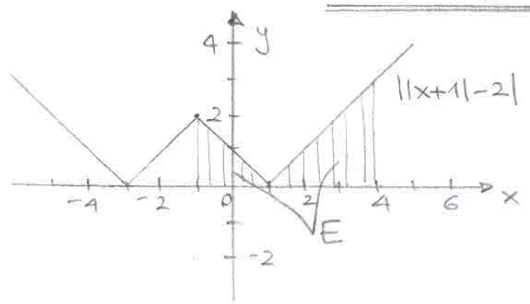
FILA (D)

1) i) $\int_1^2 \frac{4x^2+\sqrt{x}}{x^3}\,dx = \int_1^2 \left(\frac{4}{x} + x^{-5/2}\right)dx = \left[4\log|x| - \frac{x^{-3/2}}{3/2}\right]_1^2 =$

$= \left(4\log 2 - \frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{2}\sqrt{2}}\right) - \left(4\cancel{\log 1} - \frac{2}{3}\right) = 4\log 2 - \frac{\sqrt{2}}{6} + \frac{2}{3}.$

$\int_{-1}^{4} ||x+1|-2|\,dx =$

$= \text{area}(E) = \frac{13}{2}$



$\sum_{n=1}^{4}\left[\int_n^{n+1}\left(e^{3x} + \frac{1}{x+3}\right)dx\right] =$

$= \int_1^5 \left(e^{3x} + \frac{1}{x+3}\right)dx = \left[\frac{e^{3x}}{3} + \log|x+3|\right]_1^5 =$

$= \left(\frac{e^{15}}{3} + \log 8\right) - \left(\frac{e^3}{3} + \log 4\right) = \frac{e^{15}-e^3}{3} + \log\frac{\cancel{8}^{\,2}}{\cancel{4}} = \frac{e^{15}-e^3}{3} + \log 2.$

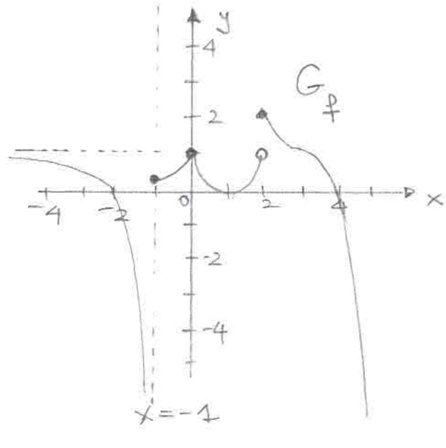
ii) $\sum_{k=2}^{5} (-1)^k \cdot \frac{k}{k^2-1} = +\frac{2}{3} - \frac{3}{8} + \frac{4}{15} - \frac{5}{24} = \frac{7}{20}.$ ☐ ☐

iii) $\frac{2}{3}x^3 + \frac{4}{5}x^4 + \frac{8}{7}x^5 + \dots + \frac{256}{17}x^{10} = \sum_{n=1}^{8} \frac{2^n}{2n+1}x^{n+2}.$ ■

2) $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} \frac{1}{x+1} + 1 & \text{se } x < -1 \\ 3^x & \text{se } -1 \le x \le 0 \\ (x-1)^4 & \text{se } 0 < x < 2 \\ -(x-3)^3 + 1 & \text{se } x \ge 2 \end{cases}$$

i)



ii) $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}\left(\frac{1}{x+1} + 1\right) = 1;$

$\lim_{x\to-1^-} f(x) = \lim_{x\to-1^-}\left(\frac{1}{x+1} + 1\right) = -\infty;$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x-1)^4 = 1;$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x-1)^4 = 1; \quad \lim_{x\to+\infty} f(x) = -\infty.$ ☐

iii) f è discontinua in $x=-1$: $\lim_{x\to -1^-} f(x) = -\infty$, $f(-1) = 1/3$.

f è discontinua in $x=2$: $\lim_{x\to 2^-} f(x) = 1 \neq f(2) = 2$.

$y=1$ è asintoto orizzontale per f , per $x\to -\infty$;

$x=-1$ è asintoto verticale per f. □

iv) $x=0$, $x=2$ pt. di massimo locale stretto per f ,

$x=1$ pt. di minimo loc. stretto per f. □

v) f non soddisfa le ipotesi del teorema di Weierstrass su $[0,2]$, non essendo continua in $x=2$.

Comunque, f ha massimo e minimo su $[0,2]$;

$\min_{[0,2]} f = 0$ $\quad x=1$ pt. di minimo ;

$\max_{[0,2]} f = 2$ $\quad x=2$ pt. di massimo . □

vi) $$\int_{-1}^{2} f(x)\,dx = \int_{-1}^{0} 3^x\,dx + \int_{0}^{2} (x-1)^4\,dx = \left[\frac{3^x}{\log 3}\right]_{-1}^{0} + \left[\frac{(x-1)^5}{5}\right]_0^2 =$$

$$= \left(\frac{1}{\log 3} - \frac{1}{3\log 3}\right) + \left(\frac{1}{5} + \frac{1}{5}\right) = \frac{2}{3\log 3} + \frac{2}{5} .$$ ■

3) $$x|x-3| + x^2 \geq 2 \iff \begin{cases} x \geq 3 \\ x(x-3) + x^2 \geq 2 \end{cases} \text{ o } \begin{cases} x < 3 \\ -x(x-3) + x^2 \geq 2 \end{cases}$$

$$\iff \begin{cases} x \geq 3 \\ 2x^2 - 3x - 2 \geq 0 \end{cases} \text{ o } \begin{cases} x < 3 \\ 3x \geq 2 \end{cases}$$

$$\iff \begin{cases} x \geq 3 \\ x \leq -\tfrac{1}{2} \text{ o } x \geq 2 \end{cases} \text{ o } \begin{cases} x < 3 \\ x \geq \tfrac{2}{3} \end{cases} \implies S = \left[\tfrac{2}{3}, +\infty\right[.$$

$$\frac{e^{-x^2} e^{3x}}{e^{|x|}} \leq 1 \iff e^{-x^2+3x-|x|} \leq e^0 \underset{e^x \uparrow}{\iff} -x^2+3x-|x| \leq 0$$

$$\iff \begin{cases} x \geq 0 \\ -x^2 + 2x \leq 0 \end{cases} \text{ o } \begin{cases} x < 0 \\ -x^2 + 4x \leq 0 \end{cases}$$

$$\iff \begin{cases} x \geq 0 \\ x^2 - 2x \geq 0 \end{cases} \text{ o } \begin{cases} x < 0 \\ x^2 - 4x \geq 0 \end{cases}$$

$$\Leftrightarrow \quad \begin{cases} x \geq 0 \\ x \leq 0 \text{ o } x \geq 2 \end{cases} \quad \text{o} \quad \begin{cases} x < 0 \\ x \leq 0 \text{ o } x \geq 4 \end{cases} \quad \Rightarrow \quad \underline{\underline{S = \left]-\infty, 0\right] \cup \left[2, +\infty\right[}}$$

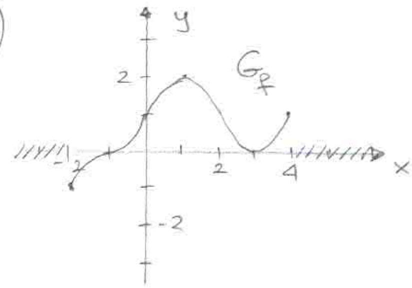
$$\log_{1/3}(x^2-1) + \log_3(x+5) > 0 \Leftrightarrow -\log_3(x^2-1) + \log_3(x+5) > 0$$

$$\Leftrightarrow \log_3(x+5) > \log_3(x^2-1)$$

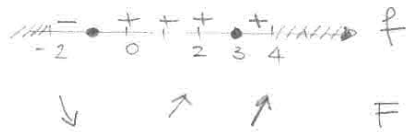
$$\Leftrightarrow \begin{cases} x+5 > 0 \\ x^2-1 > 0 \\ x+5 > x^2-1 \end{cases} \Leftrightarrow \begin{cases} x > -5 \\ x < -1 \text{ o } x > 1 \\ x^2 - x - 6 < 0 \end{cases} \Leftrightarrow \begin{cases} x > -5 \\ x < -1 \text{ o } x > 1 \\ -2 < x < 3 \end{cases}$$

$$\underline{\underline{S = \left]-2, -1\right[\cup \left]1, 3\right[}}.$$

4)



Per il Teorema fondamentale del calcolo integrale $F'(x) = f(x)$ $\forall x \in [-2,4]$.



i) F è decrescente su $\underline{\underline{[-2,-1]}}$

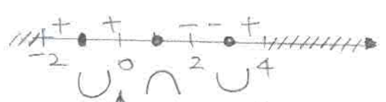
F è crescente su $\underline{\underline{[-1,3]}}$

F è decrescente su $\underline{\underline{[3,4]}}$.

ii) $\underline{\underline{x = -2,\ x = 4}}$ pt. di massimo loc. di F su $[-2,4]$;

$\underline{\underline{x = -1}}$ pt. di minimo loc. di F su $[-2,4]$.

iii)



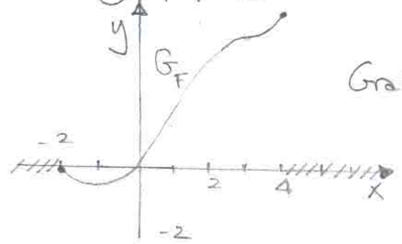


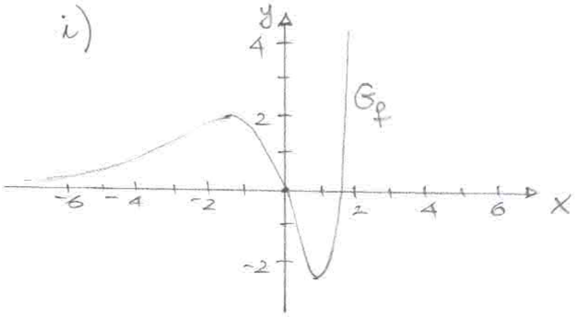
Grafico qualitativo di F

iv) $\underline{\underline{[-1,4]}}$.

5) La funzione f data qui è uguale alla funzione data in FILA (A) Es. 5, ma cambiata di segno.

Quindi il grafico qualitativo di f è

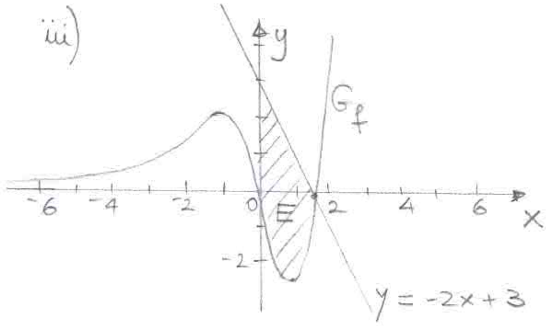
i)



ii) $F(x) = (2x^2 - 7x + 7)e^x$ è derivabile e

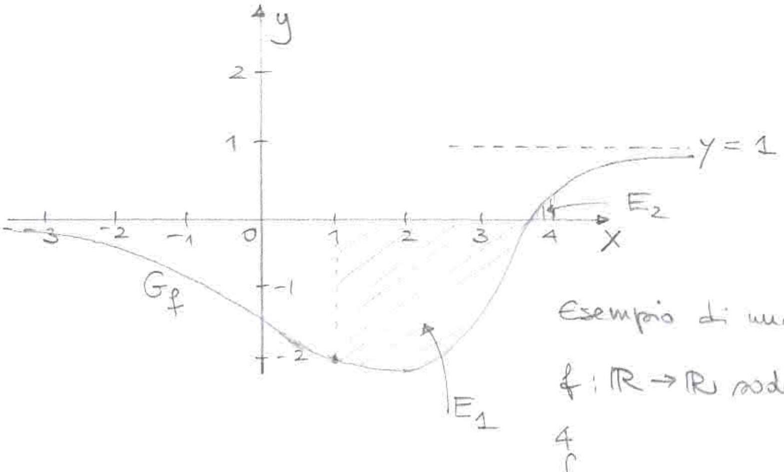
$$F'(x) = (4x-7)e^x + (2x^2 - 7x + 7)e^x$$
$$= (2x^2 - 3x)e^x = f(x) \quad \forall x \in \mathbb{R}.$$

iii)



$$\text{area}(E) = \int_0^{3/2} \left[-2x + 3 - f(x) \right] dx$$
$$= -\left[x^2 - 3x \right]_0^{3/2} - \left[F(x) \right]_0^{3/2}$$
$$= \underline{\underline{-e^{3/2} + \frac{31}{4}}}.$$

6)



Esempio di una funzione $f : \mathbb{R} \to \mathbb{R}$ soddisfacente i) ii) e iii).

$$\int_1^4 f(x)\,dx = -\text{area}(E_1) + \text{area}(E_2) < 0$$