Università degli Studi di Trento – Facoltà di Scienze Cognitive
CdL in Scienze e Tecniche di Psicologia Cognitiva
CdL in Interfacce e Tecnologie della Comunicazione – CdL in Filosofia
ESAME SCRITTO DI ANALISI MATEMATICA (CON ELEMENTI DI ALGEBRA)
a.a. 2012/2013 – Rovereto, 7 gennaio 2013

FILA (B)

1) i) $\mathcal{A}$ = "Nel 2013 l'economia statunitense si riprende."

$\mathcal{B}$ = "Nel 2013 l'economia europea uscirà dall'attuale situazione difficile".

Allora la proposizione data si scrive in matematichese : $\mathcal{A} \Rightarrow \mathcal{B}$. $\square$

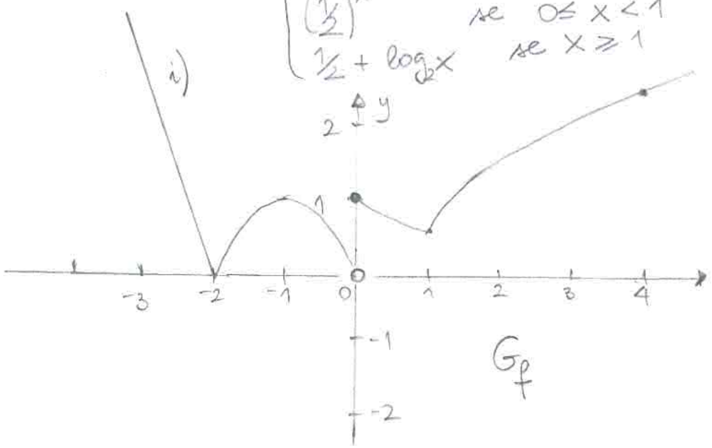
ii) "non $[\mathcal{A} \Rightarrow \mathcal{B}]$" $\Leftrightarrow$ "$\mathcal{A}$ e (non $\mathcal{B}$)".

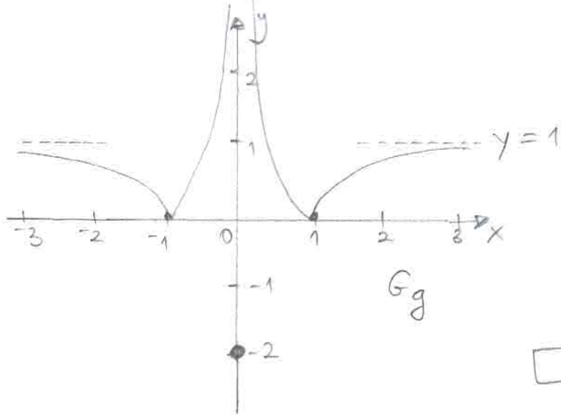
"Nel 2013 l'economia statunitense si riprende e quella europea non uscirà dall'attuale situazione difficile". ■

2) $f, g : \mathbb{R} \to \mathbb{R}$,

$$f(x) = \begin{cases} -3x-6 & \text{se } x<-2 \\ -x^2-2x & \text{se } -2 \le x < 0 \\ \left(\frac{1}{2}\right)^x & \text{se } 0 \le x < 1 \\ \frac{1}{2} + \log_2 x & \text{se } x \ge 1 \end{cases} \qquad g(x) = \begin{cases} -2 & \text{se } x=0 \\ \left|\frac{1}{x^2}-1\right| & \text{se } x \ne 0 \end{cases}$$

i)





$\square$

ii) $g(\mathbb{R}) = \underline{\{-2\} \cup [0, +\infty[}$; g non è suriettiva poiché $g(\mathbb{R}) \subsetneq \mathbb{R}$.

g è limitata inferiormente, ma non superiormente; quindi g non è limitata.

$\square$

iii) $(g \circ f)(-2) = g(f(-2)) = g(0) = \underline{-2}$;

$(f \circ g)(1) = f(g(1)) = f(0) = \underline{1}$;

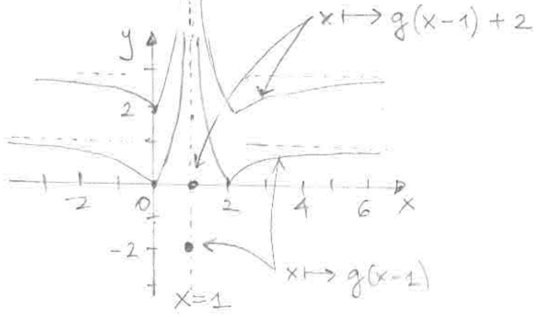
$(f+g)(2) = f(2) + g(2) = \frac{3}{2} + \frac{3}{4} = \underline{\frac{9}{4}}$.

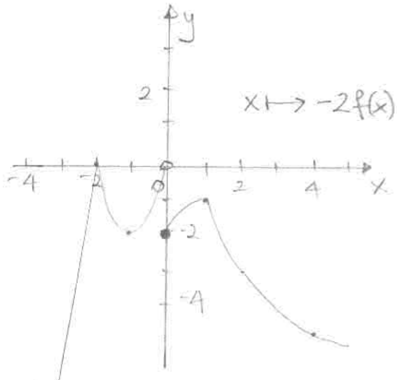
$\square$

iv) $\underline{y = 1}$ è asintoto orizzontale per g, per $x \to -\infty$
per $x \to +\infty$;

$\underline{\underline{x = 0}}$ è asintoto verticale per g.

v)





vi) $\max\limits_{[-2,4]} f = \frac{1}{2} + \log_2 4 = \underline{\underline{\frac{5}{2}}}$ $\qquad$ $\underline{\underline{x=4}}$ pt. di massimo di f su $[-2,4]$.

$\min\limits_{[-2,4]} f = \underline{\underline{0}}$ $\qquad$ $\underline{\underline{x=-2}}$ pt. di minimo di f su $[-2,4]$.

3) i) $\displaystyle\int_{-1}^{0} (3x+2)^6\,dx = \frac{1}{3}\int_{-1}^{0} 3(3x+2)^6\,dx = \frac{1}{3}\left[\frac{(3x+2)^7}{7}\right]_{-1}^{0} =$

$\displaystyle = \frac{1}{3}\left[\frac{2^7}{7} + \frac{1}{7}\right] = \frac{1}{21}\cdot 129 = \underline{\underline{\frac{43}{7}}}$.

$\displaystyle\int_{-3}^{1}\left(\frac{3x}{x^2+1} + x^3\right)dx = \left[\frac{3}{2}\log(x^2+1) + \frac{x^4}{4}\right]_{-3}^{1} = \left(\frac{3}{2}\log 2 + \frac{1}{4}\right) - \left(\frac{3}{2}\log 10 + \frac{81}{4}\right)$

$\displaystyle = -\frac{3}{2}\log\frac{\cancel{10}^{\,5}}{\cancel{2}_{\,1}} - \frac{\cancel{80}^{\,20}}{\cancel{4}_{\,1}} = \underline{\underline{-\frac{3}{2}\log 5 - 20}}$.

$\displaystyle\lim_{x\to 3^-}\left(\underbrace{\frac{1}{x-3}}_{\text{infinitesimo negativo}} - \underbrace{\frac{1}{x^2-9}}_{\text{infinitesimo negativo}}\right) \underset{(-\infty+\infty)}{=} \lim_{x\to 3^-}\left(\frac{x+3-1}{x^2-9}\right) = \lim_{x\to 3^-}\left(\frac{\overset{3}{x+2}}{x^2-9}\right) = \underline{\underline{-\infty}}.$

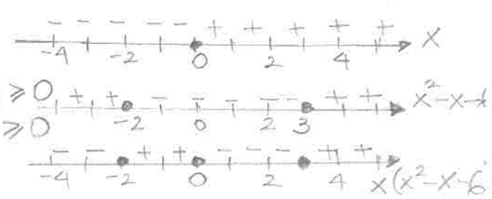
ii) $\displaystyle\sum_{n=0}^{4}\frac{f(-n)}{n!}$ $\quad$ con $f(x) = \begin{cases} 1 & \text{se } x < -2 \\ 2 & \text{se } -2 \le x < 0 \\ 3 & \text{se } x \ge 0 \end{cases}$

$\displaystyle = \frac{f(0)}{0!} + \frac{f(-1)}{1!} + \frac{f(-2)}{2!} + \frac{f(-3)}{3!} + \frac{f(-4)}{4!} =$

$\displaystyle = \frac{3}{1} + \frac{2}{1} + \frac{2}{2!} + \frac{1}{3!} + \frac{1}{4!} = 3 + 2 + 1 + \frac{1}{6} + \frac{1}{24} = \underline{\underline{\frac{149}{24}}}.$

iii) • $x^3\log_2 2 + x^2\log_{1/3} 3 \ge -x\log_{1/2} 64$

$\iff x^3 - x^2 - 6x \ge 0 \iff x(x^2 - x - 6) \ge 0$
$x(x-3)(x+2) \ge 0$



$\underline{\underline{S = [-2,0] \cup [3,+\infty[}}.$

- $\dfrac{2^{-|x-1|}\cdot 2^{x^2}}{4} = 2^x \iff 2^{-|x-1|+x^2-2} = 2^x$

$\iff -|x-1|+x^2-2 = x$

$\iff \begin{cases} x \geq 1 \\ x^2 - x + 1 - 2 - x = 0 \end{cases}$ o $\begin{cases} x < 1 \\ x^2 + \cancel{x} - 1 - 2 - \cancel{x} = 0 \end{cases}$

$\iff \begin{cases} x \geq 1 \\ x^2 - 2x - 1 = 0 \end{cases}$ o $\begin{cases} x < 1 \\ x^2 = 3 \end{cases}$

$\iff \begin{cases} x \geq 1 \\ x_{1/2} = \dfrac{2 \pm \sqrt{4+4}}{2} = \dfrac{2 \pm 2\sqrt{2}}{2} \end{cases}$ o $\begin{cases} x < 1 \\ x = \pm\sqrt{3} \end{cases}$

$= 1 \pm \sqrt{2}$

$\underline{\underline{S = \{-\sqrt{3}, 1+\sqrt{2}\}}}.$

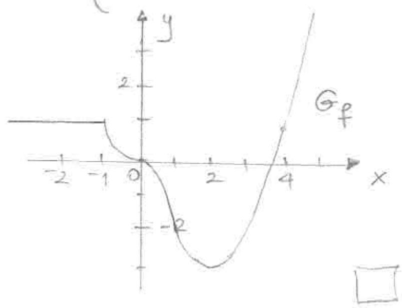
- $\log_{1/4}(|x|-2) \geq 0 \iff \log_{1/4}(|x|-2) \geq \log_{1/4} 1 \underset{\log_{1/4} x \downarrow}{\iff} \begin{cases} |x|-2 > 0 \\ |x|-2 \leq 1 \end{cases}$

$\iff \begin{cases} |x| > 2 \\ |x| \leq 3 \end{cases} \Rightarrow \underline{\underline{S = [-3, 2[\cup]2, 3]}}.$ $\blacksquare$

4) $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} 1 & \text{se } x \leq -1 \\ -x^3 & \text{se } -1 < x \leq 0 \\ -2x^2 & \text{se } 0 < x < 1 \\ (x-2)^2 - 3 & \text{se } x \geq 1 \end{cases}$$

i)



$\square$

ii) $x_0 = -1$: $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} 1 = 1 = f(-1)$

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (-x^3) = 1$

$x_0 = 0$: $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (-x^3) = 0 = f(0)$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (-2x^2) = 0$

$x_0 = 1$: $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (-2x^2) = -2$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} [(x-2)^2 - 3] = -2 = f(1)$.

Quindi, $\forall x_0 \in \{-1, 0, 1\}$ si ha $\lim_{x \to x_0^-} f(x) = \lim_{x \to x_0^+} f(x) = f(x_0)$ e quindi la continuità di f nei punti x_0

$\square$

iii) $\lim_{h\to 0^-} \frac{f(h)-f(0)}{h} = \lim_{h\to 0^-} \frac{-h^3-0}{h} = 0$

$\lim_{h\to 0^+} \frac{f(h)-f(0)}{h} = \lim_{h\to 0^+} \frac{-2h^2-0}{h} = 0$;

quindi $\exists$ finito $\lim_{h\to 0} \frac{f(h)-f(0)}{h} = f'(0)$ e vale 0. Quindi f è derivabile in $x_0 = 0$. $\square$

iv) $\lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^-} \frac{[-2(1+h)^2]-(-2)}{h} = \lim_{h\to 0^-} \frac{\cancel{-2}-4h-2h^2+\cancel{2}}{h} = -4.$

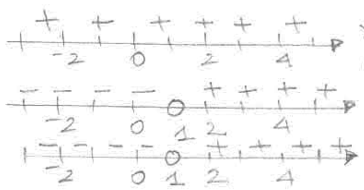
$\lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+} \frac{[(1+h-2)^2-3]-(-2)}{h} = \lim_{h\to 0^+} \frac{h^2-2h+\cancel{1}-\cancel{1}}{h} = -2$

Risulta che $\nexists \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$ (perché il limite sinistro e il limite destro di $\frac{f(1+h)-f(1)}{h}$ non coincidono) e quindi f non è derivabile in $x_0 = 1$. $\square$

5) i) $\boxed{f(x) = \frac{x^2-2x+2}{x-1}}$

• $\operatorname{dom} f = \mathbb{R}\setminus\{1\} =]-\infty, 1[\cup]1, +\infty[$

• segno di f



• comportamento agli estremi del dominio

$\lim_{x\to -\infty} f(x) = -\infty$

$\lim_{x\to +\infty} f(x) = +\infty$

Asintoto obliquo: $\lim_{x\to -\infty\,(+\infty)} \frac{f(x)}{x} = 1$

$\lim_{x\to -\infty\,(+\infty)} [f(x) - x] = -1$

$y = x - 1$

$\lim_{x\to 1^-} f(x) = -\infty$

$\lim_{x\to 1^+} f(x) = +\infty$

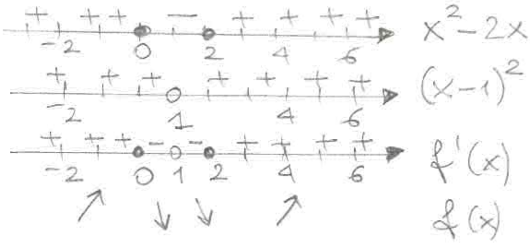
$x = 1$ asintoto verticale per f

• f è continua in $\mathbb{R}\setminus\{1\}$

• $\operatorname{dom} f' = \mathbb{R}\setminus\{1\}$ $\quad f'(x) = \frac{(2x-2)(x-1)-(x^2-2x+2)}{(x-1)^2} =$

$= \frac{2x^2-4x+\cancel{2}-x^2+2x-\cancel{2}}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2}$

$f'(x) = 0 \iff x = 0,\ x = 2$ pt. critici di f



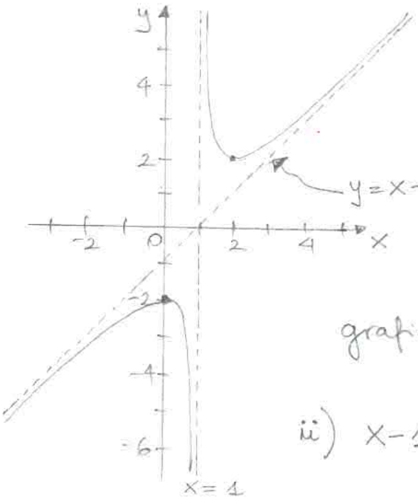
$x=0$ pr. di max. loc. stretto per f

$\underline{\underline{f(0)=-2}}$

$x=2$ pr. di min. loc. stretto per f $\quad \underline{\underline{f(2)=2}}$

• dom $f'' = \mathbb{R}\setminus\{1\}$ $\quad f''(x) = \dfrac{(2x-2)(x-1)^{\cancel{2}} - (x^2-2x)\,2\cancel{(x-1)}}{(x-1)^{\cancel{4}\,3}} =$

$= \dfrac{\cancel{2x^2} - \cancel{4x} + 2 - \cancel{2x^2} + \cancel{4x}}{(x-1)^3} = \dfrac{2}{(x-1)^3}$



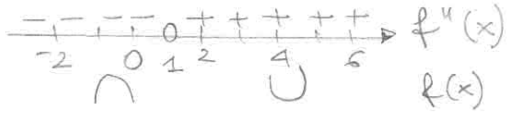
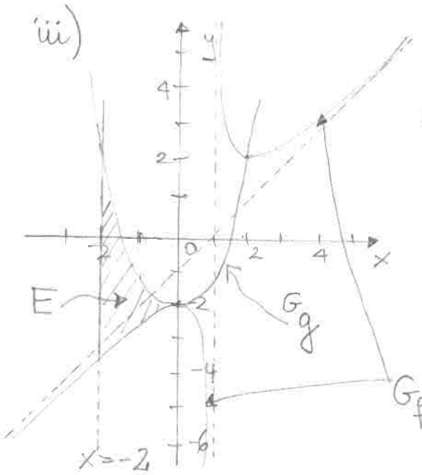


grafico qualitativo di f. $\square$

ii) $x-1+\dfrac{1}{x-1} = \dfrac{(x-1)^2+1}{x-1} = \dfrac{x^2-2x+2}{x-1} = f(x)$ $\quad \forall x \in \mathbb{R}\setminus\{1\}$. $\square$

iii)



$\text{area}(E) = \displaystyle\int_{-2}^{0} [g(x) - f(x)]\,dx =$

$\overset{ii)}{=} \displaystyle\int_{-2}^{0} \left(x^2 - 2 - x + 1 - \frac{1}{x-1}\right) dx =$

$= \left[\dfrac{x^3}{3} - \dfrac{x^2}{2} - x - \log|x-1|\right]_{-2}^{0} =$

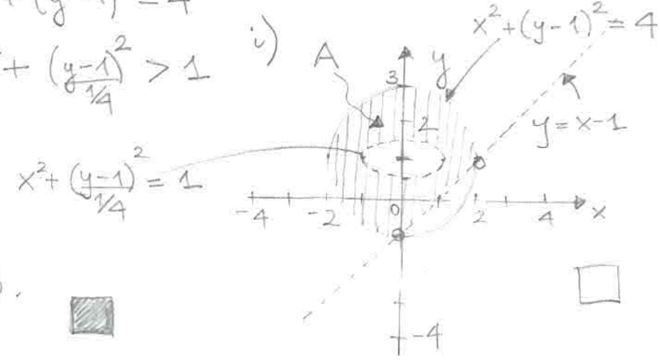
$= -\left(-\dfrac{8}{3} - \cancel{2} + \cancel{2} - \log 3\right) = \underline{\underline{\dfrac{8}{3} + \log 3}}$. $\blacksquare$

6) $x^2+y^2-2y-3 \le 0 \iff x^2+(y-1)^2 \le 4$

$x^2+4(y^2-2y)+3 > 0 \iff x^2 + \dfrac{(y-1)^2}{1/4} > 1$

$y > x-1$

i)



ii) $y=k \quad k \le -1$ o $k > 3$. $\blacksquare$ $\square$

FILA C

1) i) $\mathcal{A}$ = "Nel 2013 l'economia statunitense non si riprende."

$\mathcal{B}$ = "Nel 2013 l'economia europea rimarrà nell'attuale situazione difficile."

Allora la proposizione data si scrive in matematichese : $\mathcal{A} \Rightarrow \mathcal{B}$.

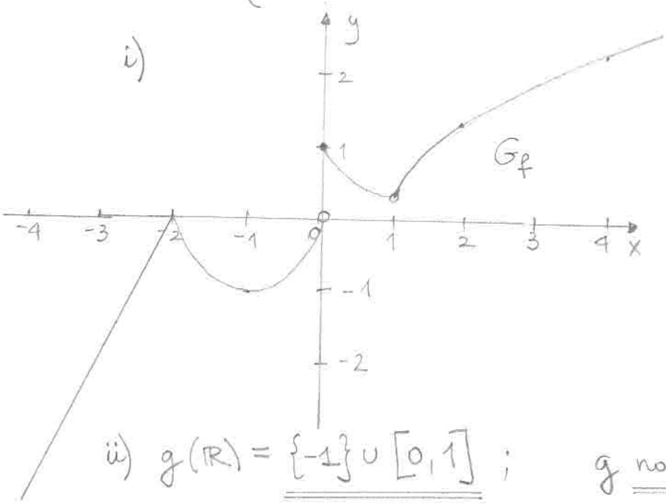
ii) "non $[\mathcal{A} \Rightarrow \mathcal{B}]$" $\Longleftrightarrow$ "$\mathcal{A}$ e (non $\mathcal{B}$)".

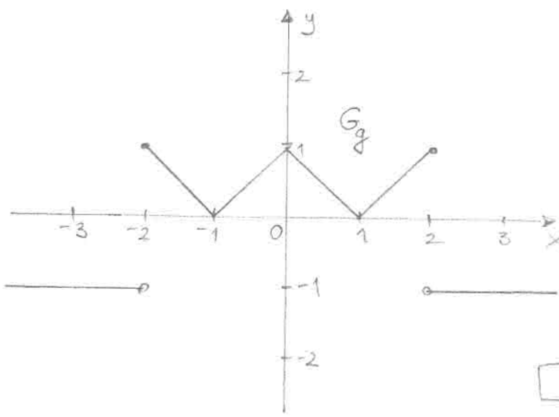
"Nel 2013 l'economia statunitense non si riprende e quella europea non rimarrà nell'attuale situazione difficile."

2) $f, g : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} 2x+4 & \text{se } x < -2 \\ x^2+2x & \text{se } -2 \le x < 0 \\ \left(\frac{1}{3}\right)^x & \text{se } 0 \le x < 1 \\ \frac{1}{3} + \log_2 x & \text{se } x \ge 1, \end{cases} \qquad g(x) = \begin{cases} -1 & x \in \mathbb{R} \setminus [-2,2] \\ \big||x|-1\big| & \text{se } x \in [-2,2]. \end{cases}$$

i)





ii) $g(\mathbb{R}) = \{-1\} \cup [0,1]$; $\quad$ g non è suriettiva, poiché $g(\mathbb{R}) \subsetneq \mathbb{R}$.

g è limitata inferiormente e limitata superiormente ; quindi g è limitata.

iii) $(g \circ f)(-2) = g(f(-2)) = g(0) = \underline{1}$;

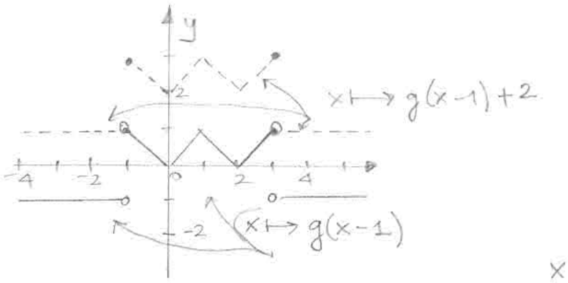
$(f \circ g)(1) = f(g(1)) = f(0) = \underline{1}$;

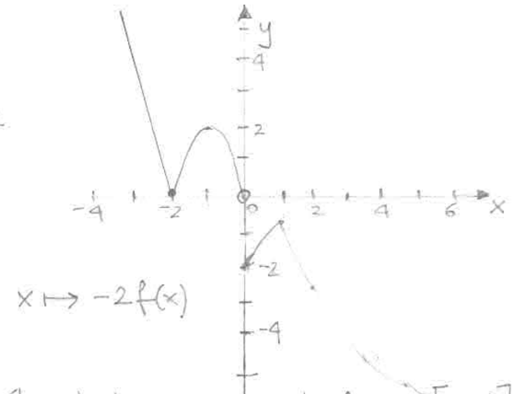
$(f+g)(4) = f(4) + g(4) = \frac{1}{3} + 2 + (-1) = \frac{1}{3} + 1 = \underline{\frac{4}{3}}$.

iv) $\underline{y = -1}$ è asintoto orizzontale per g, per $x \to -\infty$

per $x \to +\infty$

v)





vi) $\max\limits_{[-2,4]} f = \frac{1}{3} + \log_2 4 = \underline{\underline{\frac{7}{3}}}$ $\qquad$ $\underline{x=4}$ pt. di massimo di f su $[-2,4]$

$\min\limits_{[-2,4]} f = \underline{\underline{-1}}$ $\qquad$ $\underline{x=-1}$ pt. di minimo di f su $[-2,4]$.

3) i) $$\int_{-2}^{0} (2x+2)^4\,dx = \frac{1}{2}\int_{-2}^{0} 2(2x+2)^4\,dx = \frac{1}{2}\left[\frac{(2x+2)^5}{5}\right]_{-2}^{0} =$$

$$= \frac{1}{2}\left[\frac{2^5}{5} + \frac{2^5}{5}\right] = \underline{\underline{\frac{32}{5}}}.$$

$$\int_{-1}^{2}\left(\frac{4x}{x^2+2} + x^2\right)dx = \left[2\log(x^2+2) + \frac{x^3}{3}\right]_{-1}^{2} = \left(2\log 6 + \frac{8}{3}\right) - \left(2\log 3 - \frac{1}{3}\right)$$

$$= 2\log\frac{6}{3} + \frac{9}{3} = \underline{\underline{2\log 2 + 3}}.$$

$$\lim_{x\to 2^+}\left(\frac{1}{\underset{\text{infinitesimo positivo}}{(x-2)}} - \frac{1}{\underset{\text{infinitesimo negativo}}{(4-x^2)}}\right) = +\infty + \infty = \underline{\underline{+\infty}}$$

ii) $\displaystyle\sum_{n=0}^{5} \frac{f(-n)}{n!}$ $\quad$ con $f(x) = \begin{cases} 2 & \text{se } x < -2 \\ 1 & \text{se } -2 \le x < 0 \\ 4 & \text{se } x \ge 0 \end{cases}$

$$= \frac{f(0)}{0!} + \frac{f(-1)}{1!} + \frac{f(-2)}{2!} + \frac{f(-3)}{3!} + \frac{f(-4)}{4!} + \frac{f(-5)}{5!} =$$

$$= \frac{4}{1} + \frac{1}{1!} + \frac{1}{2!} + \frac{2}{3!} + \frac{2}{4!} + \frac{2}{5!} = 4 + 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{12} + \frac{1}{60}$$

$$= 5 + \frac{5}{6} + \frac{1}{12} + \frac{1}{60} = \frac{300+50+5+1}{60} = \frac{\cancel{356}^{\cancel{178}^{89}}}{\cancel{60}_{\cancel{30}_{15}}} = \underline{\underline{\frac{89}{15}}}.$$

iii) • $x^3 \log_4 4 + x^2 \log_{1/3} 3 \le -x \log_{1/2} 64$

$\iff x^3 - x^2 - 6x \le 0$

$\iff x(x^2 - x - 6) \le 0$ $\qquad$ $\underline{\underline{S = \left]-\infty, -2\right] \cup [0, 3]}}$. $\quad$ (vedi FILA (B) Es. 3) iii))

- $\dfrac{2^{-|x-1|}\,2^{x^2}}{2^x} = 16 \iff 2^{-|x-1|+x^2-x} = 2^4$

$\iff -|x-1|+x^2-x = 4$

$\iff \begin{cases} x \geq 1 \\ x^2-2x-3=0 \end{cases}$ o $\begin{cases} x<1 \\ x^2 \cancel{+x} -1 \cancel{-x} = 4 \end{cases}$

$\iff \begin{cases} x \geq 1 \\ x=-1,\ x=3 \end{cases}$ o $\begin{cases} x<1 \\ x=\pm\sqrt{5} \end{cases}$

$$\underline{S=\{-\sqrt{5}, 3\}}$$

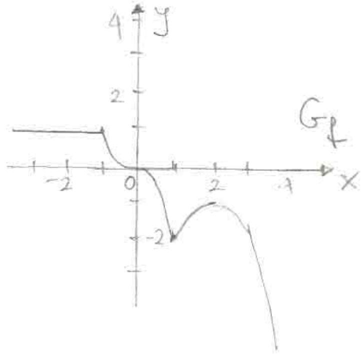
- $\log_{1/4}(|x|-3) > 0 \iff \log_{1/4}(|x|-3) > \log_{1/4} 1$

$\underset{\log_{1/4} x \searrow}{\iff} \begin{cases} |x|-3>0 \\ |x|-3<1 \end{cases} \implies \underline{\underline{S = \,]-4,-3[\,\cup\,]3,4[\,.}}$

4) $f: \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} 1 & \text{se } x \leq -1 \\ x^4 & \text{se } -1<x\leq 0 \\ -2x^2 & \text{se } 0<x<1 \\ -(x-2)^2-1 & \text{se } x \geq 1 \end{cases}$$

i)



ii) $x_0=-1$: $\lim\limits_{x\to -1^-} f(x) = \lim\limits_{x\to -1^-} 1 = 1 = f(-1)$

$\lim\limits_{x\to -1^+} f(x) = \lim\limits_{x\to -1^+} x^4 = 1$ //

$x_0=0$: $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} x^4 = 0 = f(0)$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} (-2x^2) = 0$ //

$x_0=1$: $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} (-2x^2) = -2$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} \left[-(x-2)^2-1\right] = -2 = f(1)$.

Quindi, $\forall x_0 \in \{-1,0,1\}$ si ha $\lim\limits_{x\to x_0^-} f(x) = \lim\limits_{x\to x_0^+} f(x) = f(x_0)$ e quindi la continuità di f nei punti x_0. □

iii) $\lim\limits_{h\to 0^-} \dfrac{f(h)-f(0)}{h} = \lim\limits_{h\to 0^-} \dfrac{h^4-0}{h} = 0$

$\lim\limits_{h\to 0^+} \dfrac{f(h)-f(0)}{h} = \lim\limits_{h\to 0^+} \dfrac{-2h^2-0}{h} = 0$;

quindi $\exists$ finito $\lim\limits_{h\to 0} \dfrac{f(h)-f(0)}{h}$ e vale 0. Quindi f è derivabile in $x_0 = 0$. □

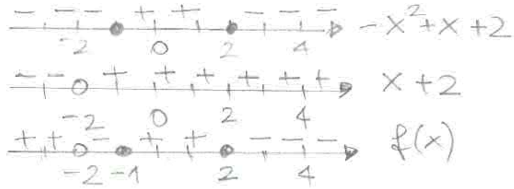
iv) $\lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^-} \frac{\left[-2(1+h)^2\right]-(-2)}{h} = \lim_{h\to 0^-} \frac{\cancel{-2}-4h-2h^2\cancel{+2}}{h} = -4.$

$\lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+} \frac{\left[-(1+h-2)^2-1\right]-(-2)}{h} = \lim_{h\to 0^+} \frac{-h^2+2h}{h} = \underline{2}$

Risulta che $\nexists \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$ (perché il limite sinistro e il limite destro di $\frac{f(1+h)-f(1)}{h}$ non coincidono) e quindi f non è derivabile in $x_0 = 1$. □

5) i) $\boxed{f(x) = \frac{-x^2+x+2}{x+2}}$

- $\text{dom} f = \mathbb{R}\setminus\{-2\} =]-\infty,-2[\,\cup\,]-2,+\infty[$
- segno di f



- comportamento agli estremi del dominio

$\lim_{x\to -\infty} f(x) = +\infty$

$\lim_{x\to +\infty} f(x) = -\infty$

Asintoto obliquo: $\lim_{x\to -\infty\,(+\infty)} \frac{f(x)}{x} = -1$

$\underline{y=-x+3}$

$\lim_{x\to -\infty\,(+\infty)} \left[f(x)+x\right] = 3$

$\lim_{x\to -2^-} f(x) = +\infty$

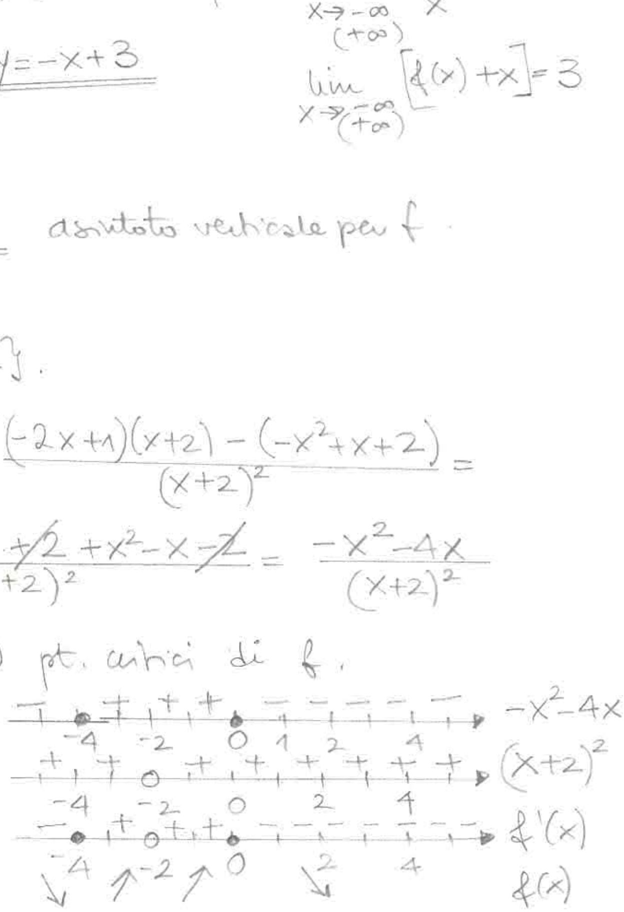
$\lim_{x\to -2^+} f(x) = -\infty$

$\underline{x=-2}$ asintoto verticale per f.

- f è continua su $\mathbb{R}\setminus\{-2\}$.
- $\text{dom} f' = \mathbb{R}\setminus\{-2\}$ $\quad f'(x) = \frac{(-2x+1)(x+2)-(-x^2+x+2)}{(x+2)^2} =$

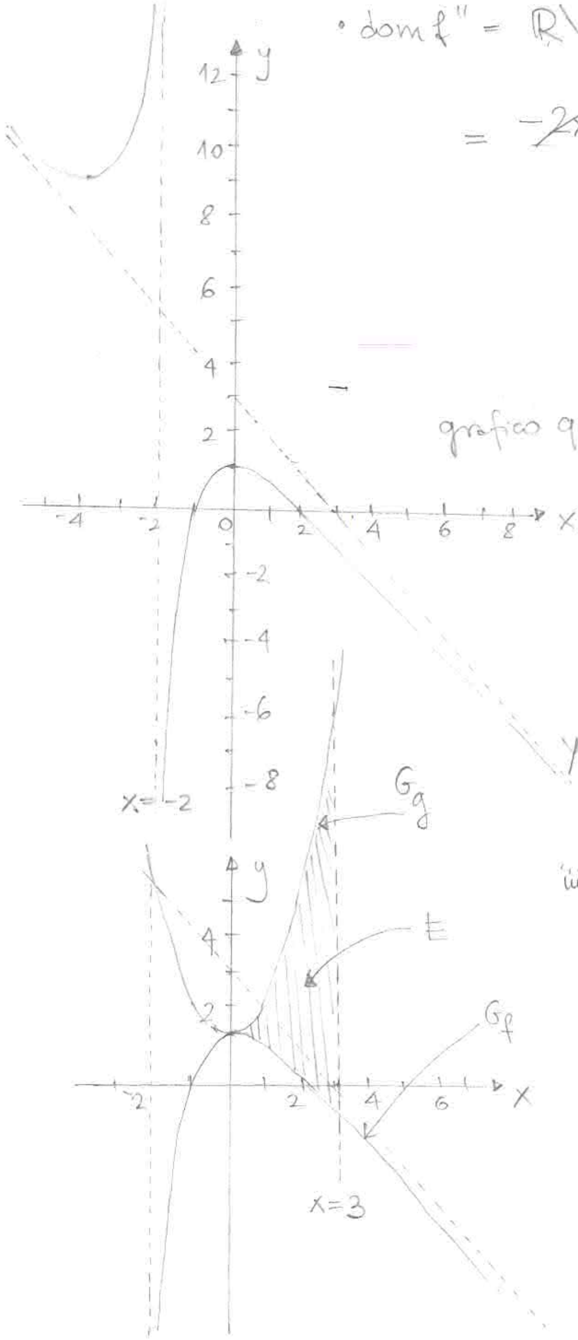
$= \frac{-2x^2-3x\cancel{+2}+x^2-x\cancel{-2}}{(x+2)^2} = \frac{-x^2-4x}{(x+2)^2}$

$f'(x) = 0 \iff x=-4,\ x=0$ pt. critici di f.



$x=-4$ pt. di min. loc. stretto per f $\quad \underline{f(-4)=9}$

$x=0$ pt. di max. loc. stretto per f, $\underline{f(0)=1}$.



• $\mathrm{dom}\, f'' = \mathbb{R}\setminus\{-2\}$ $\quad f''(x) = \dfrac{(-2x-4)(x+2)^{\cancel{2}} + (x^2+4x)\,2\cancel{(x+2)}}{(x+2)^{\cancel{4}\,3}}$

$= \dfrac{-\cancel{2x^2}-\cancel{8x}-8+\cancel{2x^2}+\cancel{8x}}{(x+2)^3} = \dfrac{-8}{(x+2)^3}$

$+ \; + \; + \; 0 \; - \; - \; - \; - \; - \; - \; - \; - \quad f''(x)$

$-4 \quad -2 \quad 0 \quad 2 \quad 4$

$\cup \qquad \cap \qquad f(x)$

grafico qualitativo di f.

ii) $-x+3-\dfrac{4}{x+2} = \dfrac{-x^2+x+6-4}{x+2}$
$= f(x) \quad \forall x \in \mathbb{R}\setminus\{-2\}.$

iii) $\mathrm{area}(E) = \displaystyle\int_0^3 [g(x)-f(x)]\,dx =$

$\overset{\text{ii)}}{=} \displaystyle\int_0^3 \left(x^2+1+x-3+\frac{4}{x+2}\right) dx =$

$= \left[\dfrac{x^3}{3}+\dfrac{x^2}{2}-2x+4\log|x+2|\right]_0^3$

$= 3+\dfrac{9}{2}-6+4\log 5-4\log 2$

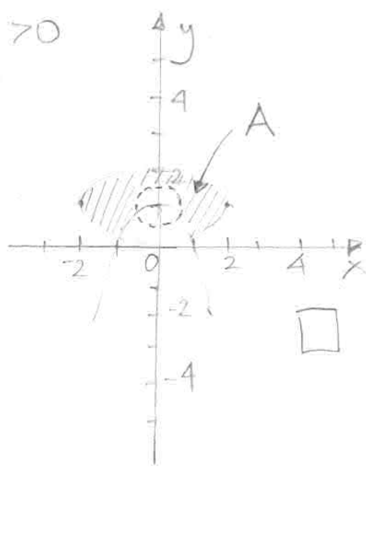
$= \dfrac{3}{2}+4\log\dfrac{5}{2}.$

6) $4x^2+4y^2-8y+3>0 \iff 4x^2+4(y^2-2y)+3>0$
$\iff 4x^2+4(y-1)^2>1$

$x^2+4y^2-8y\le 0 \iff x^2+4(y^2-2y)\le 0$
$\iff x^2+4(y-1)^2\le 4$
$\iff \dfrac{x^2}{4}+(y-1)^2\le 1$

$y \ge -x^2+1$

i)



ii) $x=k, \quad k<-2$ o $k>2.$