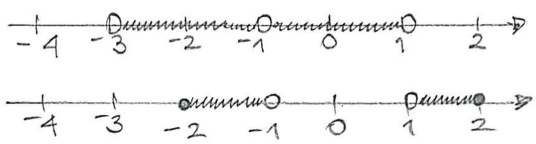
Università di Trento – Dip. di Psicologia e Scienze Cognitive
CDL in Scienze e Tecniche di Psicologia Cognitiva
ESAME SCRITTO DI ANALISI MATEMATICA
a.a. 2013 - 2014 – Rovereto, 9 gennaio 2014

FILA (C)

1) i) $A = \{x \in \mathbb{R} : |x^2 + 2x - 1| < 2\} =]-3, 1[\setminus \{-1\}$ (SPI, Fila (A), Es. 3)

$B = \{x \in \mathbb{R} : \log_3(x^2 - 1) \leq \log_3(|x| + 1)\} = [-2, -1[\cup]1, 2]$

(SPI, Fila (B), Es. 3; una base diversa, ma > 1 non cambia la soluzione).



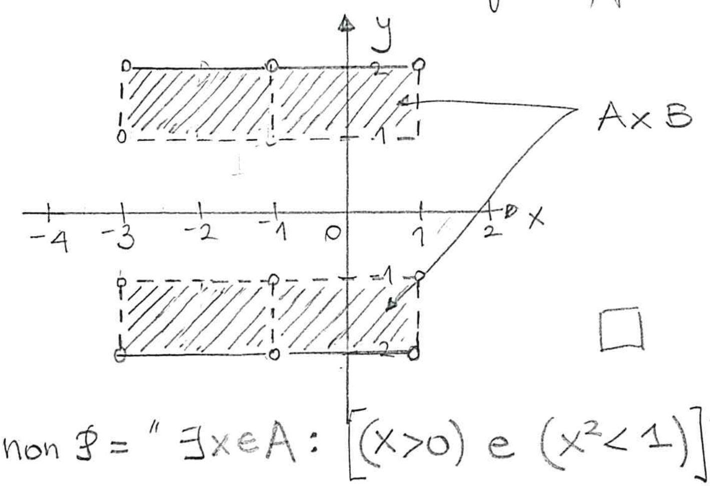
$\square$

ii) $A \cup B =]-3, -1[\cup]-1, 1[\cup]1, 2]$;

$A \setminus B =]-3, -2[\cup]-1, 1[$.

A e B non sono insiemi disgiunti, poiché $A \cap B = [-2, -1[$. $\square$

iii)



$\square$

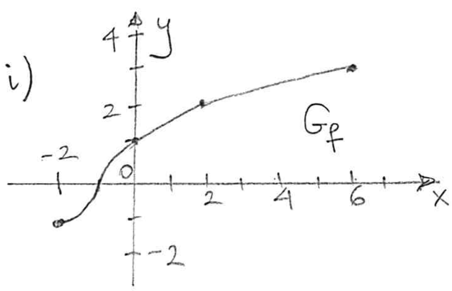
iv) non $\mathcal{P}$ = " $\exists x \in A : [(x > 0) \text{ e } (x^2 < 1)]$ ". Notiamo che non $\mathcal{P}$ è vera; basta prendere $x_0 = \frac{1}{2}$ (ma un qualunque altro $0 < x < 1$ va bene) e si ha $x_0 \in A$, $x_0 > 0$ e $x_0^2 = \frac{1}{4} < 1$.

Quindi $\mathcal{P}$ è falsa. $\blacksquare$

2) $f : [-2, 6] \to \mathbb{R}$

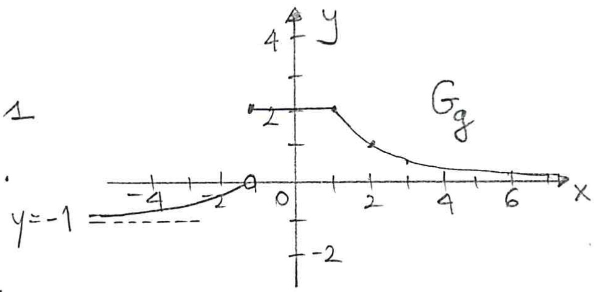
$$f(x) = \begin{cases} \sqrt[3]{x+1} & \text{se } -2 \leq x < -1 \\ \log_2(x+2) & \text{se } -1 \leq x \leq 6 \end{cases}$$

i)



$g: \mathbb{R} \to \mathbb{R}$

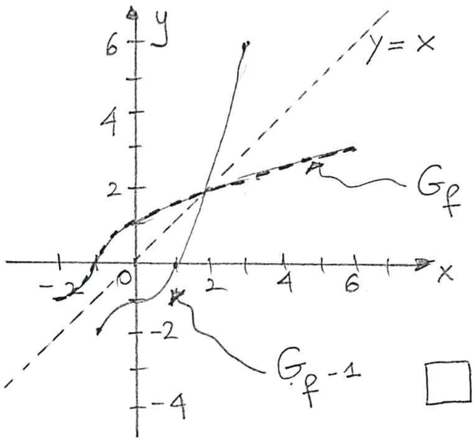
$$g(x) = \begin{cases} \frac{1}{x^2} - 1 & \text{se } x < -1 \\ 2 & \text{se } -1 \le x \le 1 \\ \left(\frac{1}{2}\right)^{x-2} & \text{se } x > 1. \end{cases}$$



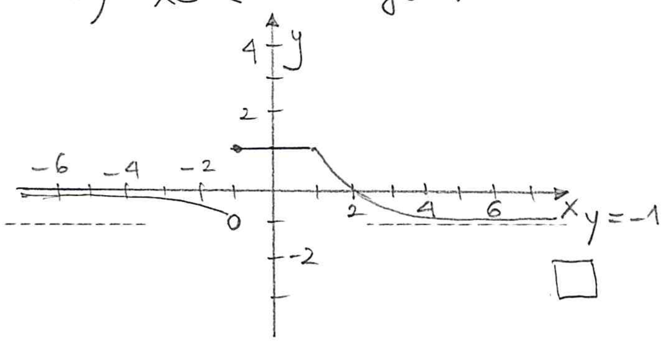
• $-1 \le f(x) \le 3$ $\forall x \in [-2,6]$, e quindi f è limitata.

•• $-1 < g(x) \le 2$ $\forall x \in \mathbb{R}$, e quindi g è limitata.

ii) $f([-2,6]) = \underline{\underline{[-1,3]}}$; $f^{-1}: [-1,3] \to [-2,6]$ esiste poiché f è iniettiva su $[-2,6]$ essendo $f|_{[-2,-1[}$ strettamente crescente, $f|_{[-1,6]}$ strettamente crescente e $f(x_1) < f(x_2)$ $\forall x_1 \in [-2,-1[$ e $\forall x_2 \in [-1,6]$.



iii) $x \in \mathbb{R} \longrightarrow |g(x)| - 1$



iv) $(f+g)(6) \doteq f(6) + g(6) = 3 + \left(\frac{1}{2}\right)^4 = 3 + \frac{1}{16} = \underline{\underline{\frac{49}{16}}}$;

$(fg)(-1) \doteq f(-1)\,g(-1) = 0 \cdot 2 = \underline{\underline{0}}$;

$\left(\frac{g}{f}\right)(-1) \nexists$ poiché $f(-1) = 0$.

v) $(f \circ g)(1) \doteq f(g(1)) = f(2) = \log_2 4 = \underline{\underline{2}}$;

$(g \circ f)(2) \doteq g(f(2)) = g(2) = \underline{\underline{1}}$

3) vedi SPI, Fila Ⓐ, Es. 1) i), iii), iv).

4) vedi SPI, Fila Ⓐ, Es. 2).

5) i) La funzione $f(x) = -\log(x^2+2x+2)$ è $-$ la funzione f

studiata in SPI, Fila (A), Es. 5) i). $\square$

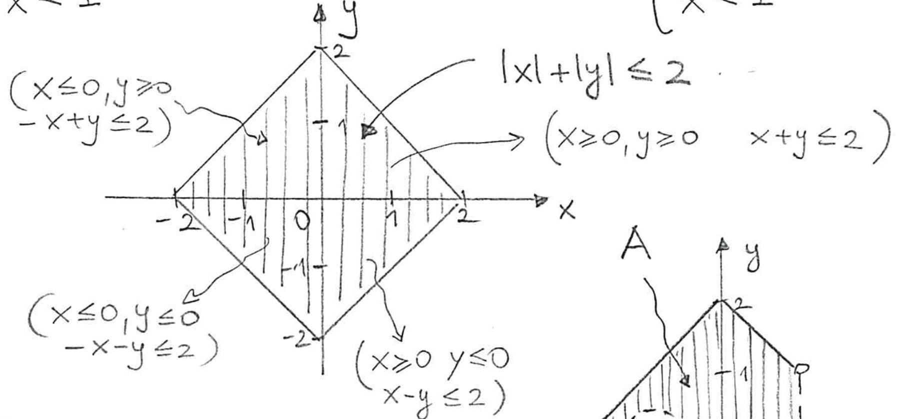
ii) $x=0$, $f(0) = -\log 2$

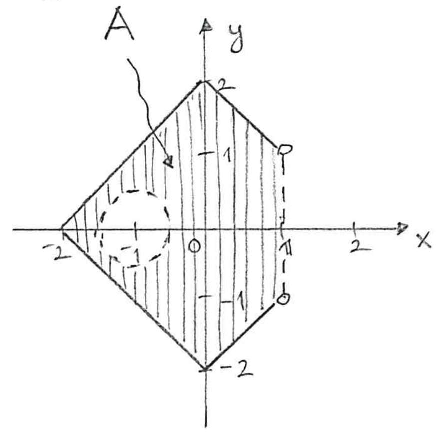
$$f'(x) = \frac{-(2x+2)}{x^2+2x+2} \Rightarrow f'(0) = -1$$

$\underline{\underline{y = -x - \log 2}}$

eq. della retta tg. al grafico di f in $(0, -\log 2)$.

6) i)
$$\begin{cases} |x| + |y| \le 2 \\ 4(x^2+2x) + 4y^2 + 3 > 0 \\ x < 1 \end{cases} \iff \begin{cases} |x| + |y| \le 2 \\ 4(x+1)^2 + 4y^2 > 1 \\ x < 1 \end{cases}$$





ii) $y = k$ con

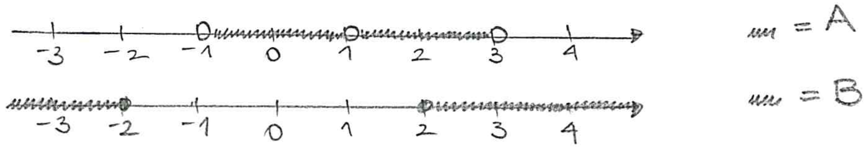
$k < -2$ o $k > 2$. $\square$

FILA (D)

1) i) $A = \{x \in \mathbb{R} : |x^2 - 2x - 1| < 2\} =]-1, 3[\setminus \{1\}$ (SPI, Fila (B), Es. 3)

$B = \{x \in \mathbb{R} : \log_3 (x^2 - 1) \ge \log_3 (|x| + 1)\} =]-\infty, -2] \cup [2, +\infty[$

(SPI, Fila (A), Es. 3; una base diversa, ma >1 non cambia la soluzione).



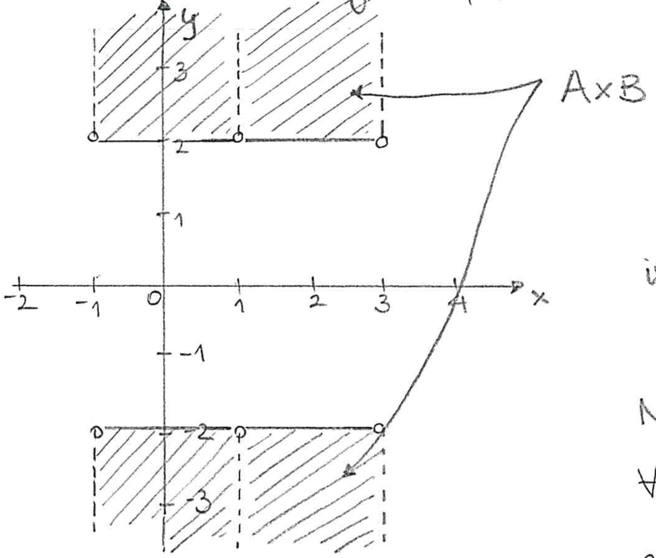
$\square$

ii) $A \cup B = \underline{\underline{]-\infty, -2] \cup]-1, 1[\cup]1, +\infty[}}$;

$A \setminus B = \underline{\underline{]-1, 1[\cup]1, 2[}}$;

A e B non sono disgiunti, poiché $A \cap B = [2,3[$. $\square$

iii)



iv) non $\mathcal{P}$ = " $\exists x \in A$: $[(x<0)$ e $(x^2 \geq 1)]$ ".

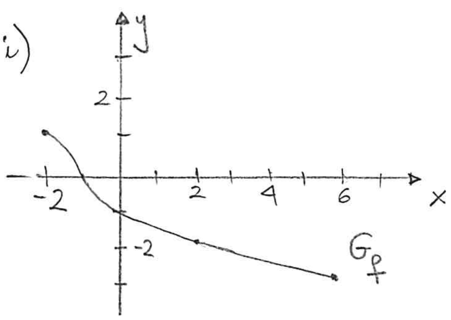
Notiamo che $\mathcal{P}$ è vera; infatti, $\forall x \in A$, $x<0$ si ha $-1<x<0$ e quindi $x^2<1$.

Quindi non $\mathcal{P}$ è falsa. $\blacksquare$

2) $f : [-2,6] \to \mathbb{R}$

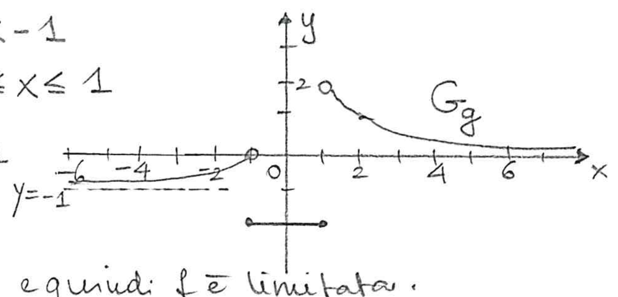
$$f(x) = \begin{cases} -\sqrt[3]{x+1} & \text{se } -2 \leq x < -1 \\ \log_{\frac{1}{2}}(x+2) & \text{se } -1 \leq x \leq 6 \end{cases}$$

i)



$g : \mathbb{R} \to \mathbb{R}$

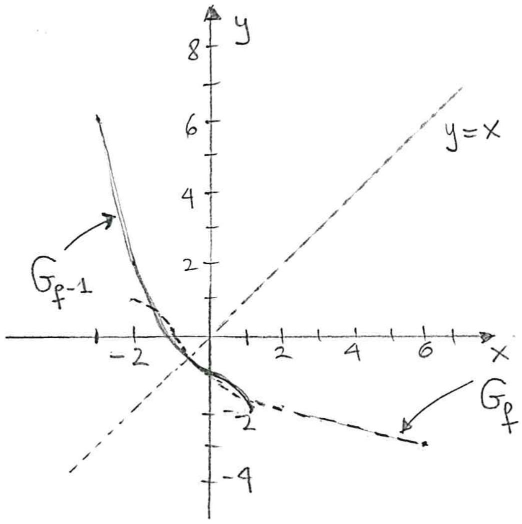
$$g(x) = \begin{cases} \frac{1}{x^2} - 1 & \text{se } x < -1 \\ -2 & \text{se } -1 \leq x \leq 1 \\ \left(\frac{1}{2}\right)^{x-2} & \text{se } x > 1 \end{cases}$$



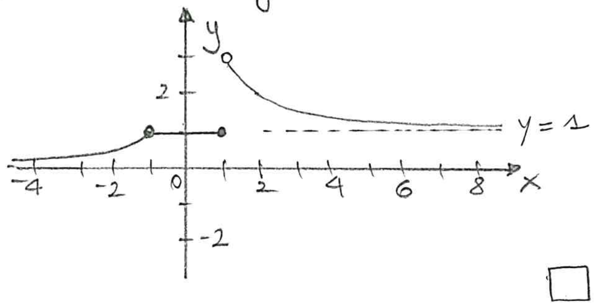
- $-3 \leq f(x) \leq 1$ $\forall x \in [-2,6]$, e quindi f è limitata.

$-2 \leq g(x) < 2$ $\forall x \in \mathbb{R}$, e quindi g è limitata. $\square$

ii) $f([-2,6]) = [-3,1]$; $f^{-1} : [-3,1] \to [-2,6]$ esiste poiché f è iniettiva su $[-2,6]$ essendo $f|_{[-2,-1[}$ strettamente decrescente, $f|_{[-1,6]}$ strettamente decrescente e $f(x_1) > f(x_2)$ $\forall x_1 \in [-2,-1[$ e $\forall x_2 \in [-1,6]$.



iii) $x \in \mathbb{R} \rightarrow |g(x)+1|$



iv) $(f+g)(6) \doteq f(6)+g(6) = -3+\left(\frac{1}{2}\right)^4 = -3+\frac{1}{16} = \underline{\underline{-\frac{47}{16}}}$;

$(fg)(-1) \doteq f(-1)\,g(-1) = 0 \cdot (-2) = \underline{\underline{0}}$;

$\left(\frac{g}{f}\right)(-1) \ \nexists$ poiché $f(-1)=0$.

v) $(f \circ g)(1) \doteq f(g(1)) = f(-2) = 1$;

$(g \circ f)(2) \doteq g(f(2)) = g(-2) = \frac{1}{4} - 1 = \underline{\underline{-\frac{3}{4}}}$.

3) i) vedi SPI, Fila (B), Es. 1) i) ;

ii) vedi SPI, Fila (A), Es. 1) iii) ;

iii) vedi SPI, Fila (B), Es. 1) iv).

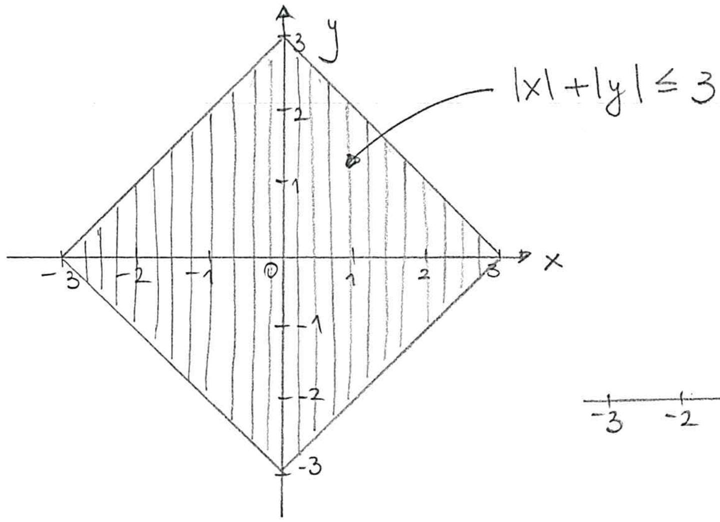
4) vedi SPI, Fila (B), Es. 2).

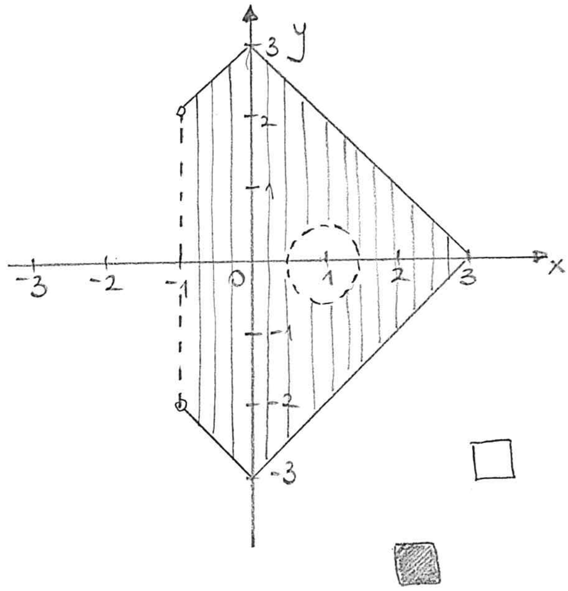
5) i) La funzione $f(x) = -\log(x^2-2x+2)$ è $-$la funzione f studiata in SPI, Fila (B), Es. 5) i).

ii) $x=0 \qquad f(0) = -\log 2$

$f'(x) = \dfrac{-2x+2}{x^2-2x+2} \Rightarrow f'(0) = 1$ $\qquad$ $\underline{\underline{y = x - \log 2}}$ eq. retta tg al grafico di f in $(0, -\log 2)$

6) i) $\begin{cases} |x|+|y| \leq 3 \\ 4(x^2-2x)+4y^2+3>0 \\ x>-1 \end{cases} \iff \begin{cases} |x|+|y| \leq 3 \\ 4(x-1)^2+4y^2>1 \\ x>-1 \end{cases}$





ii) $x = k$ con $k \leq -1$ o $k > 3$.